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ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS
School of Information Sciences and Technology
Department of Informatics

Incentives-Based Power Control in Wireless Networks of Autonomous Entities with Various Degrees of Cooperation

A dissertation submitted in partial satisfaction of the
requirements for the degree
Doctor of Philosophy

in

Computer Science

by

Vaggelis G. Douros

Advising Committee:

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PH.D. DISSERTATION

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ACKNOWLEDGEMENTS

The Ph.D. journey completed successfully and it is the right moment to share some thoughts and sentiments for some special persons for me.

My advisor was Professor George C. Polyzos. Back in 2007, when I was looking for an advisor for my master thesis (I had just met him once in the postgraduate seminar where he had presented to us his research interests), I shared my thoughts with some senior friends and the unanimous feedback that I received was: "Go to work with Polyzos. He knows..." I did it and, after a couple of months, I had a clear answer to the question: "What do you want to do after your graduation?" I wanted to pursue a Ph.D. (actually, this was in my mind even before starting my M.Sc.) under his guidance. And I did it. Undoubtedly, this was not easy. But as he says (and, in retrospective, I tend to agree), "A Ph.D. is more a matter of perspiration than inspiration." Throughout these years, we have built a strong relationship. In terms of game theory, we have reached an efficient Nash Equilibrium point (the proof is omitted ;-)). We had some very interesting meetings that influenced my way of thinking. He also made various challenging questions and some of my answers can be found in the following chapters.

Besides Professor Polyzos, I also worked closely with Assistant Professor Stavros Toumpis. Stavros read carefully (many times!) every manuscript that I submitted (including this thesis). He (always) corrected various errors and inaccuracies, he (always) pointed out inconsistencies and he (always) suggested changes that improved significantly each manuscript. I should try hard to be so meticulous when it will be my turn to mentor someone. Besides enjoying doing research with Stavros, I enjoy discussing with him non-academic issues and I am happy to discover that we share similar thoughts on a vast variety of subjects.

During my postgraduate studies in Computer Science in AUEB, I met Professor George D. Stamoulis, who was the director of the program. Due to him, the program ran like a swiss watch, having created an environment so that each student could maximize his creativity. I would like to thank him for this, as well as for his three very interesting and well-organized graduate courses that I attended and, of course, for his interest for my progress all these years.

I spent four years as an undergraduate student in the Department of Informatics and Telecommunications of the National and Kapodistrian University of Athens, where I had the opportunity to interact with the majority of the professors. I attended some very interesting lectures focusing on telecommunications (it was clear that this will be my field from the first days of my studies). Special thanks go to Associate Professor Stathes Hadjiefthymiades.

He was my teacher in the course of Operating Systems that I enjoyed very much and, after a while, he offered me the opportunity to work for some months in his research group (there, I earned my first money as a researcher). I would like to thank him for this, as well as for arguing on why to choose the postgraduate program of Computer Science in AUEB (he was right!) and, of course, for his interest for my progress all these years.

All these years, I was a member of the Mobile Multimedia Laboratory, where I shared the same office (and became friends) with: (now Dr.) Konstantinos Katsaros, (now Dr.) Pantelis Frangoudis, (now Dr.) Nikos Fotiou, Charilaos Stais, Xenofon Vasilakos, Christos Tsilopoulos, Yannis Thomas, and Dr. Merkourios Karaliopoulos. All of them are talented researchers and I am sure that they will have a very successful career (for the more senior ones, this has already become clear). In the lab, we had many vivid discussions that had covered a vast variety of (non-)academic aspects. Special thanks to Nikos, Xenofon, and Christos for the moments that we shared together. Nikos has the talent to make succinct, to-the-point arguments; his clear mind in complex situations is admirable! Xenofon has a deep knowledge in a broad spectrum of issues; you never get bored to interact with him! Christos is excellent in making concrete plans; he knows what he wants and how to achieve it. I am sure that they will miss me during my military service (note that my "instructions" hold during my absence as well ;-)).

Besides my colleagues in the Mobile Multimedia Laboratory, (now Dr.) Stamatina Thomaidou, a talented colleague and a good friend, deserves a special mention for being a great listener during the last three years. Moreover, I would like to thank Petroula Karagianni for her exceptional administrative support for my fellowship.

A necessary condition for the fulfilment of this journey (that had actually started 12 years ago, when I came to Athens for undergraduate studies) was the support of my family: My parents, George and Angeliki, were, are, and will be there. Besides being exemplary parents, they were my great teachers. My father taught me Physics, Chemistry, Biology, Algebra, and Geometry. I love(d) these courses and I built a strong background due to the countless hours that we spent together. My mother taught me all the greek language-related courses; her influence to the development of my writing and verbal skills was critical. This thesis is dedicated to them, as well as to my brother, Ilias (who is the "boss" of the house in Athens, an amateur craftsman/singer/chef etc. with notable achievements). Last, but not least, I have no doubt that my grandparents Vaggelis, Smaragda, Ilias, and Paraskevi would be very happy and proud of me today.

Vaggelis G. Douros

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PUBLICATIONS

Journal Papers

Vaggelis G. Douros, Stavros Toumpis, and George C. Polyzos, **"Power Control and Bargaining for Cellular Operator Revenue Increase under Licensed Spectrum Sharing,"** submitted for journal publication.

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Vaggelis G. Douros, Stavros Toumpis, and George C. Polyzos, **"Channel Access Competition in Linear Multihop Device-to-Device Networks,"** Proc. 10^{th} International Wireless Communications and Mobile Computing Conference (IWCMC), Nicosia, Cyprus, August 2014.

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ABSTRACT OF THE DISSERTATION

Incentives-Based Power Control in Wireless Networks of Autonomous Entities with Various Degrees of Cooperation

Wireless networks have grown tremendously in recent years: wireless communications are now available anytime, anywhere, and with varying degrees of Quality-of-Service (QoS). More and more smartphones and tablets come into the market, making the dream for ubiquitous connectivity a reality. To continue this trend in the forthcoming fifth generation (5G) era (and beyond), new communication paradigms are expected to arise and be exploited. In particular, next generation multi-tier cellular networks (consisting of traditional cellular networks, small cell networks, device-to-device networks, etc.) are expected to be the norm. To ensure scalability, devices on these heterogeneous networks should be autonomous; this means that they will be controlling their transmission parameters (notably, their radio channel and power level) rather than have them dictated by a centralized entity. Therefore, the choices of each device will have a direct impact on the performance of (some of) the devices with which they share the same portion of the spectrum, and the network as a whole.

Motivated by the above trends, the fundamental goal of this dissertation is to design efficient distributed radio resource management methods for the smooth deployment of these emerging wireless network architectures. We apply two of the most powerful resource allocation methods: power control, *i.e.*, what transmission power a device should choose, and channel access control, *i.e.*, when to transmit. We study settings under a variety of practical scenarios such as the coexistence of small cells and traditional macrocells with different QoS targets, the channel access competition in device-to-device networks (where devices communicate directly without a Base Station or Access Point) and licensed spectrum sharing scenarios (where operators share their spectrum, combining power control with bargaining to improve their revenues).

We analyse these challenging settings under the prism of game theory, which is a natural choice for modelling scenarios where players with conflicting interests interact with each other. We formulate non-cooperative games where the devices are the players, focusing on the solution concept of the Nash Equilibrium. We explore the existence and uniqueness of Nash Equilibria, we devise distributed schemes that converge to them, and we study their performance through analysis and simulations. In cases where the resulting Nash Equilibria are suboptimal, meaning that the devices are not satisfied with their performance at these points, we introduce bargaining as a means for creating incentives to the devices to change their transmission parameters. Then, we propose schemes that are guaranteed to lead to operating points more efficient than the Nash Equilibria obtained without bargaining.

Περιληψη Διατριβης

Ρύθμιση Ισχύος Εκπομπής Βάσει Κινήτρων σε Ασύρματα Δίκτυα Αυτόνομων Οντοτήτων με Διάφορους Βαθμούς Συνεργασίας

Τα ασύρματα δίκτυα αναπτύσσονται με ραγδαίους ρυθμούς τα τελευταία χρόνια: Οι ασύρματες επικοινωνίες είναι διαθέσιμες οπουδήποτε και οποτεδήποτε, παρέχοντας διάφορα επίπεδα ποιότητας υπηρεσίας στους χρήστες. Ολοένα και περισσότερες έξυπνες κινητές συσκευές καθώς και μικροί φορητοί υπολογιστές διεισδύουν στην αγορά, πραγματοποιώντας το όνειρο του απανταχού υπολογίζειν. Αναγκαία συνθήκη για να συνεχιστεί αυτή η τάση καθώς οδεύουμε προς την εποχή των δικτύων 5ης γενιάς (και πέραν αυτών) είναι η μετεξέλιξη των ασυρμάτων τηλεπικοινωνιακών προτύπων. Πιο συγκεκριμένα, τα κυψελωτά δίκτυα πολλαπλών επιπέδων που περιλαμβάνουν τόσο τις παραδοσιακές κυψέλες όσο και μικροκυψέλες, δίκτυα επικοινωνιών συσκευής προς συσκευή κλπ. αναμένεται να έχουν δεσπόζουσα θέση τα προσεχή χρόνια. Τα ετερογενή αυτά δίκτυα αποτελούνται από αυτόνομες οντότητες οι οποίες ελέγχουν και αποφασίζουν μόνες τους για τις παραμέτρους λειτουργίας τους (για παράδειγμα, σε ποιο κομμάτι του φάσματος και με ποια ισχύ θα εκπέμψουν), αντί να εξαρτώνται από τις αποφάσεις κάποιας κεντρικής οντότητας. Το γεγονός αυτό συνεπάγεται ότι οι επιλογές μιας ασύρματης συσκευής έχουν άμεσο αντίκτυπο τόσο στις επιδόσεις κάποιων (τουλάχιστον) συσκευών (που χρησιμοποιούν το ίδιο κομμάτι του φάσματος) όσο και στη συνολική απόδοση του δικτύου.

Με αφετηρία τις άνω συνθήκες, ο θεμελιώδης στόχος της διδακτορικής διατριβής είναι η σχεδίαση αποδοτικών κατανεμημένων σχημάτων διαχείρισης ραδιοπόρων με στόχο την αρμονική συνύπαρξη των συσκευών που συνυπάρχουν σε αυτά τα αναδυόμενα πρότυπα ασυρμάτων δικτύων. Εφαρμόζουμε δύο κλασσικές τεχνικές διαχείρισης ραδιοπόρων: Τη ρύθμιση ισχύος εκπομπής (power control), δηλαδή με ποια ισχύ πρέπει να μεταδώσει η συσκευή και την πρόσβαση στο κανάλι (channel access), δηλαδή πότε να μεταδώσει. Μελετούμε μια σειρά από πρακτικά σενάρια που περιλαμβάνουν *(α)* την αρμονική συνύπαρξη συσκευών που συνδέονται με παραδοσιακά δίκτυα κινητής τηλεφωνίας και συσκευών που συνδέονται με μικροκυψέλες, με τα δύο αυτά είδη συσκευών να έχουν διαφορετικούς δικτυακούς στόχους, *(β)* το πρόβλημα του ανταγωνισμού για πρόσβαση στο κανάλι σε δίκτυα επικοινωνιών συσκευής προς συσκευή, όπου οι αυτόνομες ασύρματες συσκευές επικοινωνούν απευθείας μεταξύ τους, χωρίς τη μεσολάβηση του σταθμού βάσης ή κάποιου ασυρμάτου σημείου πρόσβασης και *(γ)* σενάρια από κοινού χρήσης του αδειοδοτημένου φάσματος, στα οποία οι πάροχοι κινητής τηλεφωνίας δεν έχουν την αποκλειστική χρήση σε τμήματα φάσματος και εφαρμόζουν ρύθμιση ισχύος εκπομπής με τεχνικές διαπραγμάτευσης για να βελτιώσουν τα έσοδά τους.

Αναλύουμε τις παραπάνω καταστάσεις υπό το πρίσμα της θεωρίας παιγνίων, η οποία είναι μια κλασσική και πετυχημένη επιλογή για τη μοντελοποίηση σεναρίων όπου οι οντότητες έχουν αντικρούόμενα συμφέροντα και οι στρατηγικές της μιας έχουν άμεσο αντίκτυπο στις επιδόσεις της άλλης. Χρησιμοποιούμε τη μη-συνεργατική θεωρία παιγνίων με τους παίκτες να είναι οι ασύρματες συσκευές και εστιάζουμε στην εύρεση σημείων ισορροπίας κατά Nash, μια από τις κεντρικότερες έννοιες στη θεωρία παιγνίων. Εξετάζουμε την ύπαρξη και τη μοναδικότητα τέτοιων σημείων ισορροπίας και προτείνουμε μια σειρά από κατανεμημένα σχήματα που συγκλίνουν σε κάποιο από αυτά. Επιπλέον, αποτιμούμε τις επιδόσεις των σχημάτων μέσω θεωρητικής ανάλυσης και εκτενών προσομοιώσεων. Σε περιπτώσεις που οι ισορροπίες είναι υποβέλτιστες, υπό την έννοια ότι οι ασύρματες συσκευές δεν είναι ευχαριστημένες με τις επιδόσεις τους, εισαγάγουμε τεχνικές διαπραγμάτευσης με στόχο τη δημιουργία κινήτρων στις συσκευές για να αλλάξουν περαιτέρω τις παραμέτρους μετάδοσής τους. Εκμεταλλευόμενοι αυτές τις τεχνικές, προτείνουμε κατανεμημένα σχήματα και αποδεικνύουμε ότι οδηγούν σε καλύτερα σημεία λειτουργίας από τις ισορροπίες κατά Nash χωρίς αυτά.

Συνεισφορές Διατριβής

Ο κεντρικός στόχος της διατριβής είναι η σχεδίαση, με χρήση εργαλείων από τη θεωρία παιγνίων, αποδοτικών κατανεμημένων αλγορίθμων ρύθμισης ισχύος εκπομπής και πρόσβασης στο κανάλι που επιτρέπουν στις αυτόνομες οντότητες σύγχρονων ετερογενών ασυρμάτων δικτύων να συνυπάρξουν αποδοτικά.

Προς την κατεύθυνση εκπλήρωσης αυτού του στόχου, στο Κεφάλαιο 2, προχωράμε σε μια εκτενή επισκόπηση θεμελιωδών προσεγγίσεων της ρύθμισης ισχύος εκπομπής σε ασύρματα δίκτυα, κατηγοριοποιώντας και συγκρίνοντας τις σχετικές προσεγγίσεις. Τα Κεφάλαια 3-6 αντιστοιχούν στον πυρήνα της έρευνάς μας, όπου πραγματοποιήσαμε τις ακόλουθες κεντρικές συνεισφορές:

- Στο Κεφάλαιο 3, μοντελοποιούμε ένα μη-συνεργατικό παίγνιο ρύθμισης ισχύος εκπομπής σε ασύρματα δίκτυα αυτόνομων οντοτήτων, όπου οι οντότητες στοχεύουν να πετύχουν μια εγγυημένη ποιότητα υπηρεσίας για κλήσεις φωνής, χρησιμοποιώντας τον περίφημο αλγόριθμο των Foschini-Miljanic [32] (τον οποίο αναλύουμε διεξοδικά στο Κεφάλαιο 2). Αποδεικνύουμε μέσω προσομοιώσεων ότι το μοναδικό σημείο ισορροπίας κατά Nash αυτού του παιγνίου είναι υποβέλτιστο, υπό την έννοια ότι κάποιες συσκευές δεν πετυχαίνουν την επιθυμητή ποιότητα υπηρεσίας. Το φαινόμενο αυτό παρατηρείται ακόμη και σε ασύρματα δίκτυα με λίγες συσκευές. Για να αντιμετωπίσουμε αυτό το πρόβλημα, εισαγάγουμε τεχνικές διαπραγμάτευσης και τις συνδυάζουμε με τη ρύθμιση ισχύος εκπομπής, παρέχοντας κίνητρα στις ασύρματες συσκευές για να επιλέξουν αποδοτικότερα σημεία λειτουργίας από το σημείο λειτουργίας κατά Nash χωρίς τις τεχνικές διαπραγμάτευσης. Προτείνουμε ένα κατανεμημένο σχήμα που εντοπίζει τέτοια σημεία, απαιτώντας μικρά επίπεδα συνεργασίας μεταξύ των ασυρμάτων συσκευών. Επιπλέον, δείχνουμε μέσω προσομοιώσεων ότι το σχήμα μας είναι πιο αποδοτικό και δίκαιο έναντι κλασσικών τεχνικών που εντοπίζουν καλύτερα σημεία λειτουργίας από το σημείο λειτουργίας κατά Nash.

- Στο Κεφάλαιο 4, μελετούμε ένα μοντέρνο ασύρματο δίκτυο δύο επιπέδων, αποτελούμενο από συσκευές που έχουν συνδεθεί στο παραδοσιακό δίκτυο κινητής τηλεφωνίας και από συσκευές που έχουν συνδεθεί σε μικροκυψέλες. Μοντελοποιούμε το συγκεκριμένο δίκτυο με τη βοήθεια της μη-συνεργατικής θεωρίας παιγνίων με τις συσκευές να εφαρμόζουν ρύθμιση ισχύος εκπομπής και να στοχεύουν σε διαφορετικούς δικτυακούς στόχους. Σε αντίθεση με τις συνήθεις προσεγγίσεις, προτείνουμε διαφορετικές συναρτήσεις χρησιμότητας για τους δύο τύπους συσκευών ώστε να μοντελοποιήσουμε ακριβέστερα τους στόχους τους. Αποδεικνύουμε ότι το παίγνιο έχει ένα τουλάχιστον σημείο ισορροπίας κατά Nash και αναλύουμε τις συνθήκες που πρέπει να πληρούνται για τη μοναδικότητά του. Προτείνουμε ένα κατανεμημένο σχήμα που συγκλίνει γρήγορο στο μοναδικό σημείο ισορροπίας και αποτιμούμε την αποδοτικότητα του σημείου μέσω εκτενών προσομοιώσεων, οι οποίες δείχνουν ότι οι δύο τύποι συσκευών συνυπάρχουν αρμονικά στην πλειονότητα των σεναρίων.

- Στο Κεφάλαιο 5, μελετούμε το θεμελιώδες πρόβλημα του ανταγωνισμού για την πρόσβαση στο κανάλι στα ασύρματα δίκτυα επικοινωνιών συσκευής προς συσκευή. Οι αυτόνομες αυτές συσκευές πρέπει να αποφασίσουν από μόνες τους πότε να στείλουν τα δεδομένα τους. Εστιάζουμε σε γραμμικά και δενδρικά δίκτυα και προτείνουμε μια μη-συνεργατική παιγνιοθεωρητική μοντελοποίηση του προβλήματος, εξετάζοντας δύο παραλλαγές για τις συναρτήσεις χρησιμότητας των συσκευών. Αποδεικνύουμε ότι το παίγνιο έχει σημεία ισορροπίας κατά Nash, αναλύουμε τη δομή τους και προτείνουμε δύο κατανεμημένους σχήματα με διαφορετικά επίπεδα συνεργασίας μεταξύ των συσκευών τα οποία συγκλίνουν σε κάποιο σημείο ισορροπίας. Συγκρίνουμε την αποδοτικότητα των σημείων ισορροπίας και αναλύουμε τις διαφορές της προσέγγισής μας με το κλασσικό πρόβλημα του προγραμματισμού των μεταδόσεων (transmission scheduling), εξηγώντας ότι το τελευταίο μπορεί να οδηγήσει σε σημεία λειτουργίας που δεν είναι αποδεκτά για τις αυτόνομες οντότητες.

- Στο Κεφάλαιο 6, μελετούμε σενάρια μη αποκλειστικής χρήσης του αδειοδοτημένου φάσματος και ορίζουμε ένα μη-συνεργατικό παίγνιο στο οποίο οι πάροχοι κινητής τηλεφωνίας στοχεύουν να μεγιστοποιήσουν τα έσοδά τους. Δείχνουμε ότι το μοναδικό σημείο ισορροπίας κατά Nash είναι υποβέλτιστο, με τα έσοδα των παρόχων να είναι μικρά. Για να αντιμετωπίσουμε το πρόβλημα αυτό, συνδυάζουμε τη ρύθμιση ισχύος εκπομπής με τεχνικές διαπραγμάτευσης, αναλύοντας τις συνθήκες που πρέπει να πληρούνται για την εύρεση σημείων λειτουργίας που είναι πιο αποδοτικά από το σημείο ισορροπίας. Για την περίπτωση ύπαρξης ακριβώς δύο παρόχων, υπολογίζουμε το σημείο λειτουργίας που βελτιστοποιεί την κοινωνική ευημερία, δηλαδή μεγιστοποιεί το άθροισμα των εσόδων των παρόχων. Επιπλέον, ορίζουμε απλές στρατηγικές διαπραγμάτευσης που οδηγούν εγγυημένα στο σημείο βελτιστοποίησης της κοινωνικής ευημερίας, απαιτώντας χαμηλότερο βαθμό συνεργασίας μεταξύ των παρόχων σε σχέση με άλλες προσεγγίσεις της βιβλιογραφίας. Παράλληλα, αποδεικνύουμε ότι η προσέγγισή μας υπερτερεί των σχημάτων γραμμικής τιμολόγησης της ισχύος εκπομπής [33] που υιοθετούνται κατά κόρον για την εύρεση τέτοιων σημείων λειτουργίας, τόσο σε ό,τι αφορά τα έσοδα ανά πάροχο όσο και στο άθροισμα των εσόδων τους.

Chapter 1

Introduction and Fundamentals

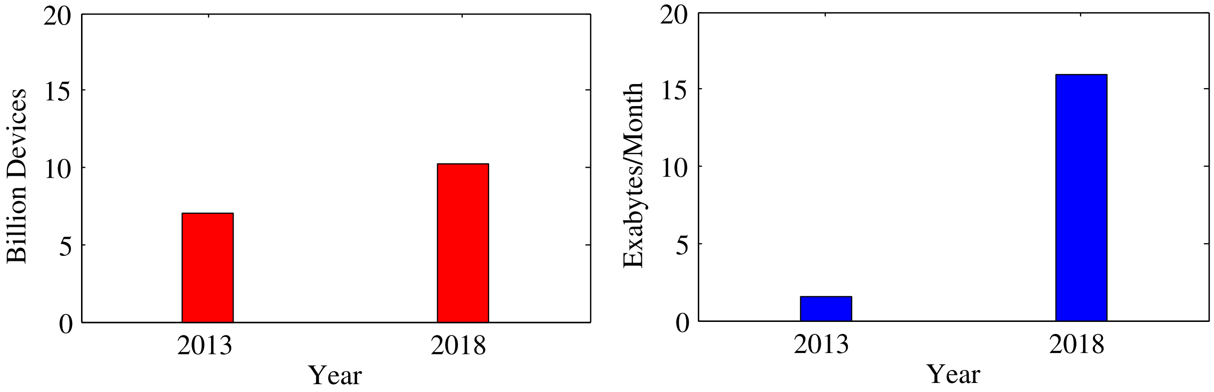
1.1 Motivation for the Dissertation

Wireless communications technology has developed rapidly in the last 25 years. Firstly, it was cellular communications that satisfied the need for untethered mobile real time communication. After the widespread market adoption of laptop computers, the dream for ubiquitous Internet data connectivity became a reality with the success of wireless local area network technology, and in particular the IEEE 802.11 family of standards. The next step was the shrinkage of the laptop and its merging with the cellular telephone, resulting in today's smartphones and tablets. Due to them, we enjoy access to every kind of information in any situation and at any place.

The trend towards ubiquitous connectivity of ever-increasing quality will remain strong in the near future; according to Cisco's 2014 forecast [1], the number of Internet-connected mobile devices will exceed the world's population by 2014. Moreover, in 2013, mobile data traffic was nearly 18 times the size of the entire global Internet traffic of 2000; as shown in Fig. 1.1, it is expected that, by 2018, mobile data traffic will reach the level of 15.9 exabytes per month (1 exabyte is 1 billion gigabytes). In the same year, traffic from wireless devices will exceed traffic from wired devices.

Nowadays, the Long Term Evolution (LTE) system, embodying the fourth generation (4G) set of standards, has been extensively deployed and is reaching maturity, offering a DSL-like experience in the mobile broadband era, with nominal speeds that make it feasible to download an 1-hour High Definition (HD) movie in just six minutes [1].

However, existing wireless systems will not be able to adequately support the increase in mobile broadband data that is expected in the next years. Therefore, research around the world is currently undergoing towards a fifth generation (5G) standard. A key expectation is to provide a fibre-like ubiquitous connectivity experience with nominal speeds that could



(a) Mobile devices. (b) Mobile data traffic.

Fig. 1.1: Forecast for mobile devices and mobile data traffic (based on statistics provided by Cisco [1]).

make it feasible to download an 1-hour HD movie in just six seconds, *i.e.*, 60 times faster than the current nominal LTE data rates [1]. In fact, 5G networks are currently considered one of the "hottest" topics among wireless networks researchers. Although it is still unclear what exactly a 5G network will look like and what services it will offer, there are two issues on which there is a broad consensus: Firstly, to support the massive growth of connected devices, 5G networks should be denser [2] and, secondly, new communication paradigms should arise and be exploited.

To satisfy the tremendous increase in traffic and the addition of different devices and services, more spectrum beyond what was previously allocated for 4G networks is sought for. Since the traditional spectrum availability is scarce and the process of clearing spectrum that is used for other purposes is time-consuming, many more systems and devices are expected to coexist and share the same portion of the spectrum. As the number of devices and their heterogeneity increases, it may be hard for some of them to achieve their Quality-of-Service (QoS) targets due to the interference from the transmission signals of the other devices. Note that the devices will be autonomous in the sense that they may either use unlicensed spectrum or use licensed spectrum in non-exclusive mode for each operator. Therefore, no unique external entity can dictate to them whether to transmit or not, which spectrum band to occupy, and which transmission power to use. Such scenarios are expected to be the norm in the 5G era, making the successful deployment and operation of all systems and devices a challenging task [3].

There are various radio resource management methods (e.g., power control, channel allocation, admission control) that have been used extensively for interference mitigation in wireless networks. However, these approaches should be revisited by taking into consideration the unique features of these challenging environments so as to provide efficient solutions.

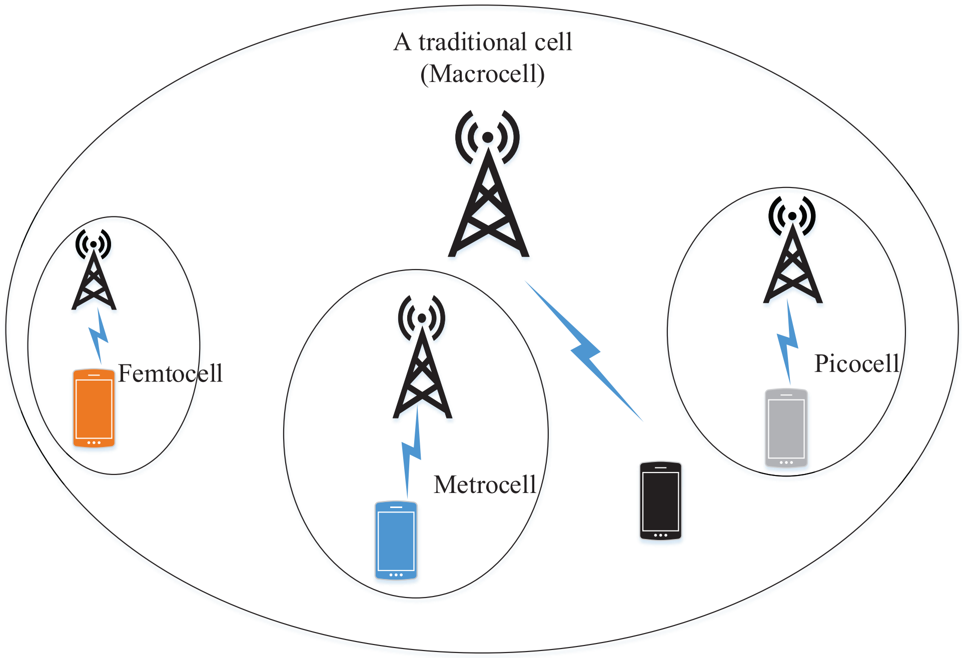


Fig. 1.2: A multi-tier small cell network.

1.2 5G Fundamentals

In this section, we shall briefly discuss 3 communications paradigms that are expected to play a significant role in forthcoming 5G standards: *Small Cells*, *Device-to-Device Communications*, and *Licensed Spectrum Sharing*. For each one communication paradigm, we will devote a separate chapter of this thesis where we will present a game-theoretic approach for efficient radio resource management.

1.2.1 Small Cells

The most straightforward approach to increase the data rates available to each user is to deploy base stations more densely. Thus, the distances covered by wireless transmissions are reduced, and as a consequence more users can be packed in the same geographical area, without any reduction in their QoS. However, the high deployment costs of the traditional cellular networks constitute a serious limitation. The only viable way to support the large demand for mobile data traffic is to make cells smaller, denser, and smarter.

"Small cells" is an umbrella term for operator-controlled, low-powered radio access nodes that operate in licensed spectrum and coexist with the traditional cellular networks (an example is provided in Fig. 1.2). Types of small cells include femtocells, picocells, metrocells, and microcells, broadly increasing in size from femtocells (that have a range from 10 metres) to microcells (that have a range of several hundred metres). Small cells provide improved cellular coverage, increased capacity, and many interesting applications for homes and enterprises as well as metropolitan and rural public spaces [4].

A small cell base station (SCBS) is connected to the mobile operator network using residential DSL or cable broadband connections and is able to support a small number of smartphones [5]. Its installation is very easy (plug & play) and the owner of the SCBS can control who can use it (for example, he might add authorized mobile phone numbers using a web page or SMS messages). This is the so-called closed access model, as opposed to the open access model where all users within the range of the SCBS can connect to it (in this case, owners of SCBSs should be given incentives to share their hardware) [6].

Residential mobile phones that are using small cells experience full third/fourth generation (3G/4G) connectivity with excellent quality voice calls and fast downloads at very low transmit powers, since the mobile phone works at a lower power level while being connected to a nearby SCBS. This dramatically increases their battery life. Enterprise small cells enable business users to take advantage of high-quality mobile services in the office, while improving coverage, accelerating data rates, and significantly reducing capital costs. Finally, in remote rural areas with little or no terrestrial network infrastructure, operators can deploy SCBSs to improve local coverage, increase capacity, and offload traditional cellular network traffic [4].

Small cells are expected to play a significant role in forthcoming 5G standards since they lead to the densification of the networks by increasing the spatial reuse. How to manage the extra interference that arises (from small cells to small cells, from small cells to the traditional cellular network, and vice versa) is the key to their successful deployment [7].

1.2.2 Device-to-Device Communications

In a traditional cellular network, all communications must go through the Base Station (BS) even if both communicating parties are close enough to have direct communication. Even in the case of small cells, no direct communication can be possible. Device-to-device (D2D) communication in cellular networks is defined as direct communication between two Mobile Nodes (MNs) without traversing the BS or core network [8]. An example is provided in Fig. 1.3. Note that a similar idea has already appeared in unlicensed spectrum technologies, in particular Wi-Fi Direct [9] and Bluetooth. However, these traditional D2D communication models provide questionable QoS guarantees and the operators could hardly make a profit from them if they were applied in a cellular context.

With D2D communications in the licensed spectrum, direct peer-to-peer transmission to support context-aware applications and machine-to-machine applications take place.

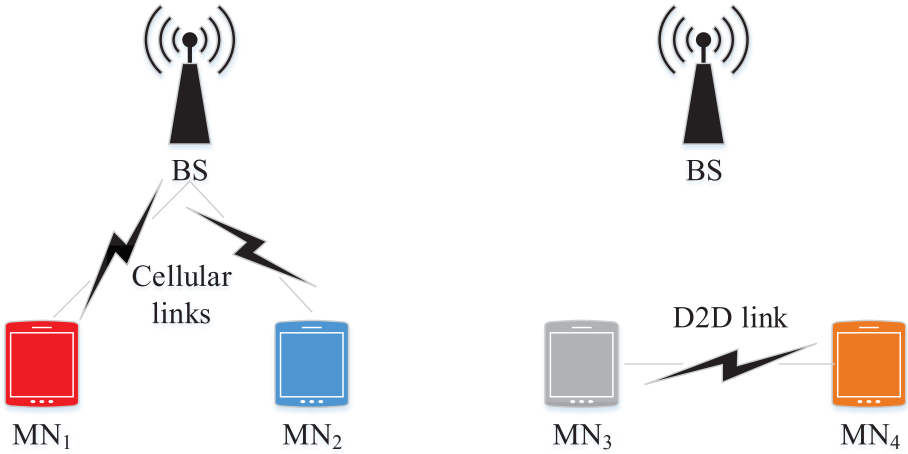


Fig. 1.3: In the traditional cellular network (left), Mobile Nodes (MNs) communicate through the Base Station (BS). In a D2D network (right), MNs are able to communicate directly, coexisting with the cellular links.

Furthermore, D2D communication can be critical in natural disasters. For example, in the case of an earthquake or hurricane, an urgent communication network can be set up using D2D functionality in a short time, replacing the damaged communication network and Internet infrastructure [8].

In general, this communication paradigm provides the following advantages: extended coverage, offloading from cellular networks, increased throughput, and spectrum efficiency. In addition, as D2D communications are short-range transmissions, the MN power consumption can be very low; hence, the battery lifetime of MNs with D2D communications can be extended [10]. For these reasons, the operators are exploring the possibilities of introducing D2D functionality in cellular networks. For example, LTE-Direct [11] has been recently standardized in 3GPP-R12, enabling discovery of thousands of devices in a range of about 500 m. However, while the spectral efficiency and system capacity are improved, extra interference arises for the MNs that communicate with the BS. Therefore, methods for efficient interference management and coordination should be developed to achieve the target performance levels for both the cellular and the D2D links [10].

1.2.3 Licensed Spectrum Sharing

Due to the constant need for ever-increasing spectrum efficiency, the original "licensed vs. unlicensed" spectrum usage model is being revisited. Extending LTE-Advanced to the unlicensed spectrum [12] and licensed spectrum sharing approaches, where no exclusive rights are given to any single operator, are receiving increasing attention as a complementary way of spectrum use.

These ideas are becoming key concepts of 5G networks [13]. We will focus on the concept of licensed spectrum sharing that gives the opportunity to a limited number of licensees in a frequency band, already allocated to one or more incumbent users, to use jointly the spectrum. The 3.5 GHz band in the USA and the 2.3 GHz band in Europe are potential candidates for licensed spectrum sharing. A primary benefit is providing additional capacity in congested areas, especially in indoor locations. A secondary benefit is that the shared band could be available across all operators, opening up increased opportunities for national roaming between operators [14]. In any case, the operators should analyse the economic benefits from spectrum sharing to decide upon the level of their investments.

Under this setting, the operators, though still selfish, have motivation to cooperate so as to control the resulting interference aiming at providing high QoS to their customers. Towards that direction, developing distributed spectrum sharing techniques that allow faster decisions with less control overhead becomes very important.

1.3 Power Control Fundamentals

In this thesis, we focus on power control, *i.e.*, controlling the transmission power, which has always been one of the most important radio resource management techniques in wireless networks, as it addresses two fundamental limitations of wireless networks:

- Radio spectrum is a scarce resource. This makes the mitigation of interference from devices that transmit in the same spectrum band critically important.
- Mobile wireless devices, such as smartphones, tablets, etc., have significant limitations on the duration of their “talk time,” as the life of their battery is limited. As technology improvements in the direction of prolonging battery life are slower than advances in communications, this constraint continues to have a dramatic impact, particularly for uplink transmissions (from Mobile Nodes to Base Stations). Therefore, designing energy efficient wireless networks is very important [15].

For these reasons, applying transmitter power control is a well-known and widely adopted practice. Furthermore, power control, by its nature, can be smoothly combined with other interference mitigation techniques (such as channel assignment, admission control, and directional antennas). These joint radio resource management schemes can further improve the performance of the nodes in the wireless network.

The results from the adoption of power control algorithms in terms of mitigating the interference and increasing the network capacity are significant. Although the quantitative analysis of the benefits from power control techniques needs to be done carefully because the results depend critically on the assumptions and parameters of the various techniques and environments, we would like to briefly present some simple illustrative examples.

It was shown early on that applying power control doubles the capacity of a second generation (2G) Code Division Multiple Access (CDMA) network compared to the non-power controlled case [16]. Further improvements (up to 50%) can be achieved by suitably adjusting the update rate of a power control algorithm [17]. More recent studies by Olama et al. [18] estimated that the Signal-to-Interference plus Noise Ratio (SINR) gains from the adoption of power control exceed 10 dB compared to a policy without power control, for various interesting values of the outage probability (*i.e.*, the probability of a node achieving SINR lower than a threshold required for communication).

This result is similar to early findings on the advantages of using power control in Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA) networks [19]. Moreover, the combination of power control with base station assignment (and beamforming) increases two to four times the capacity of a CDMA network, compared to a network that uses only power control techniques [20]. As far as the energy consumption or battery lifetime is concerned, studies show that power control offers a significant improvement (orders of magnitude) compared to the constant power approach. The exact value of the gain strongly depends on the transmission rate [21]. For mobile ad hoc networks, the adoption of power control leads to an over 50% improvement on the energy expended compared to the IEEE 802.11 standard [22].

As discussed, power control is generally adopted for at least one of the following reasons: *(i)* to mitigate the interference in order to increase the capacity of the network and/or provide QoS, *(ii)* to conserve energy in order to prolong battery life and–nowadays– to "green" the Internet/mobile networks. The first one is correlated with the Signal-to-Interference (plus Noise) Ratio–SI(N)R–metric. We shall introduce the SI(N)R in the context of wireless networks without explicitly defining the type of the network. The only assumption is that the nodes that form the wireless network can obtain feedback (which is the case in modern wireless networks).

Fig. 1.4 presents a single channel wireless network with N transmitter-receiver (Tx-Rx) pairs. As the same channel is used for the transmission of all Txs considered, all signals interfere with each other. Using the standard notation, we denote the path gain coefficient from Tx_i to Rx_j as $G_{ij} \in (0, 1)$. Note that we assume that the path gains do not change due to mobility, traffic arrivals etc. For such dynamic systems (which are out of the scope of this thesis), stochastic power control using, e.g., robust H^∞ control theory [23], annealed Gibbs sampling [24], and Kalman filters [25], should be applied.

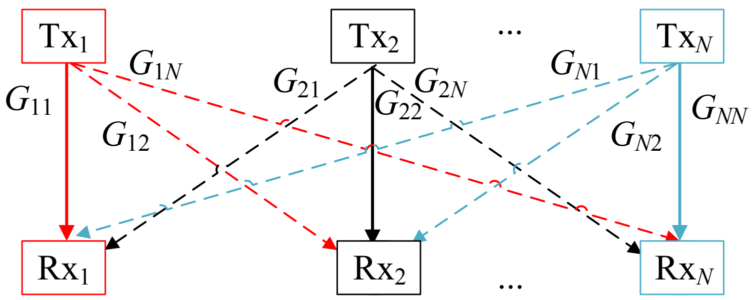


Fig. 1.4: Each transmitter i, Tx$_i$, serves one receiver i, Rx$_i$. We denote the path gain between Tx$_i$ and Rx$_j$ as G_{ij}.

We now state the definition of SINR for Rx$_i$, denoted as γ_i:

$$\gamma_i \triangleq \text{SINR}_i = \frac{G_{ii}P_i}{\sum\limits_{j \neq i}^{N} G_{ji}P_j + n}. \tag{1.1}$$

The numerator $G_{ii}P_i$ expresses the power that Rx$_i$ receives from Tx$_i$, whereas the denominator $\sum\limits_{j \neq i}^{N} G_{ji}P_j + n$ is the sum of the received power from the remaining N-1 Txs plus the thermal noise power of the channel, n.

To compute the value of γ_i, Tx$_i$ and Rx$_i$ exchange information about the interference that Rx$_i$ receives. This is a standard procedure that is adopted in each power control scheme. Note that, in general, Tx$_i$ does not need to know the exact level of interference that each other Tx creates to Rx$_i$.

The higher the SINR value, the better the quality of the communication. As suggested by (1.1), Tx$_i$ can apply power control to improve γ_i. This will be the case if, e.g., it increases its power P_i and the remaining N-1 Txs keep their powers constant. Of course, it is entirely possible that the other Txs will respond by increasing their own powers, possibly leading us back to where we started. This conundrum lies in the heart of power control research and also this thesis.

1.4 Game Theory in Wireless Networks

As we discussed in the previous section, when a node applies power control, it modifies not only its own SINR, but also the SINR of all other nodes that share the same portion of the spectrum. This situation can be efficiently analysed by game theory, *i.e.*, "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers" as defined in [26].

Game theory has emerged as an important tool in the design of future wireless networks. Three indicative examples follow:

- In multihop communications, whether a node should act as a relay (forwarding some data to another node) or not can be directly transformed into a game [27].
- In multichannel networks, game theory can be used to find optimal channel assignments with distributed schemes [28].
- In spectrum sharing scenarios, multiple non-cooperative wireless nodes compete for spectrum access [29]. Game theory is a natural choice for deciding upon who is going to transmit and with what power.

In wireless networks that consist of autonomous nodes, the branch of non-cooperative game theory should be applied. Contrary to coalitional game theory, where decisions are based on the formation of teams, in a non-cooperative game each node decides on its own.

Two important properties that characterize the nodes of such wireless networks are *rationality* and *selfishness*. Rationality means that the decisions of each node are the best possible for the satisfaction of its target. Selfishness means that each node aims at achieving its target without being interested in the way that it may affect the communication capabilities of other nodes. Note that this does not mean that it wants to harm other nodes.

To define a non-cooperative game, we need to specify the set of players and, for each player[1], its strategy and its utility function that expresses its (dis)satisfaction with the current state of the game. Since players are rational and selfish, they aim at maximizing their own utility functions. A formal definition follows:

Definition 1. *A strategic (or normal form) non-cooperative game G with a finite number of players consists of the following triplet: A set of players $\mathbf{N} = \{1, 2, ..., N\}$ and, for each player i, a set of strategies $\mathbf{S}_i$, and a utility function $U_i(\cdot)$.*

A powerful solution concept in non-cooperative game theory is the pure Nash Equilibrium (NE) [30] which predicts outcomes of games that are stable, in a sense described below. A formal definition follows:

Definition 2. *The strategy vector $\mathbf{s}^\star = [s_1^\star, s_2^\star, \ldots, s_N^\star]^T$ is a pure NE for a game G if $\forall i \in N$ and $\forall \acute{s}_i \in \mathbf{S}_i$:*

$$U_i(s_i^\star, \mathbf{s}_{-i}^\star) \geq U_i(\acute{s}_i, \mathbf{s}_{-i}^\star), \text{ where } \mathbf{s}_{-i}^\star = [s_1^\star, s_2^\star, \ldots, s_{i-1}^\star, s_{i+1}^\star, \ldots, s_N^\star]^T.$$

Consequently, a pure NE corresponds to a steady state of a game in the sense that no player has an incentive to change unilaterally its own strategy. In this thesis, we deal with pure Nash Equilibria only (and we will not deal with mixed Nash Equilibria [30]), so we omit the term "pure".

[1]Throughout the thesis, we use the pronoun "it" to refer to a device that acts as a player, rather than "he" or "she".

Table 1.1: A general form of a non-cooperative power control game.

Set of players	Set of nodes $\mathbf{N} = \{1, 2, \ldots, N\}$
Strategy s_i of player i	$P_i \in [0, P_{i,\max}]$
Utility function U_i for player i	Various choices

1.5 A General Non-Cooperative Power Control Game in Wireless Networks

In this section, we discuss a general non-cooperative power control game that will be used extensively in Chapters 3, 4, and 6.

We consider wireless networks such as the one in Fig. 1.4 that consists of N directly interfering links (transmitter-receiver pairs) that share the same channel. Under this broad definition, various types of wireless networks may be considered: Traditional cellular networks, Wi-Fi networks, multi-tier heterogeneous cellular networks that consist of traditional cellular networks overlaid with small cells, device-to-device networks, etc. We focus on scenarios that include autonomous nodes, e.g., nodes that belong to different operators. These nodes have full control of their own equipment, therefore centralized solutions are difficult to be adopted in practice. Nodes apply power control, updating their transmission powers to achieve their QoS target.

In Table 1.1, we present a general form of a non-cooperative power control game G for this setup. The players are the N nodes that act as transmitters. Throughout the thesis, we use the terms "nodes" and "entities" interchangeably to refer to the "transmitter nodes" and "transmitter entities", *i.e.*, we omit the term "transmitter" when it is clear from the context. The strategy that each player i decides on is its transmission power P_i that belongs to $[0, P_{i,\max}]$. Each player i aims at maximizing a utility function $U_i(P_1, P_2, \ldots, P_N)$. We will study various utility functions in the next chapters, typically related with the SINR, which is a key performance metric in every wireless network [31]. In each case, after the definition of the game G, we follow a general roadmap:

- Existence of a Nash Equilibrium: Has the game G at least one Nash Equilibrium (NE) power vector $\mathbf{P}^\star = [P_1^\star, P_2^\star, \ldots, P_N^\star]^T$?

- Uniqueness of the NE: Are there conditions that guarantee the existence of a *unique* NE for the game G?

- Algorithm for finding a NE: Can we find a distributed (iterative) algorithm that converges to a NE of the game G?

- Efficiency and optimality of the operating points: Can we find more efficient operating points by introducing bargaining among some of the entities? Have the players incentives to end up at these points?

When we propose an algorithm[2], we take into consideration the fact that each entity has limited knowledge of the parameters of the whole wireless network. Therefore, we look for distributed schemes, so that the transmission power can be updated by using information that is available only to the transmitter and its associated receiver. We discuss scenarios where nodes have various degrees of cooperation, in the sense that even though in principle they are non-cooperative, they may still exchange information with other nodes. The exact level of information exchanged and the set of nodes with whom messages are exchanged influences significantly the performance of the proposed schemes.

1.6 Contributions

The fundamental goal of this dissertation is to design efficient distributed power control and channel access schemes for modern heterogeneous wireless networks using various tools from game theory, aiming at the seamless coexistence of wireless nodes.

As a preliminary step towards this goal, in Chapter 2, we review fundamental approaches for power control in wireless networks. We present key classifications pointing out relationships and differences in approaches and their consequences and applicability.

Chapters 3-6 constitute the main part of our research, where we make the following key contributions:

- In Chapter 3 we formulate a power control game where autonomous wireless nodes initially aim at achieving their SINR targets by applying the famous Foschini-Miljanic algorithm [32] (which we describe and discuss in Chapter 2). We show through simulations that the resulting unique Nash Equilibrium (NE) of the game is quite often inefficient, since, even in small networks, many nodes are unsatisfied with the payoff that they receive there. To tackle this issue, we introduce bargaining and combine it with power control as a way to provide incentives to the wireless nodes to find operating points that are more efficient than the NE. We show that our distributed scheme can indeed find such points, demanding minimal cooperation among the nodes. Moreover, we show through simulations that it outperforms well-adopted approaches in terms of finding fair and efficient operating points.

[2]Throughout the thesis, we use the terms “scheme” and “algorithm” interchangeably.

- In Chapter 4 we study a two-tier wireless cellular network that consists of traditional cellular nodes overlaid with small cell nodes. A key challenge in this network is that nodes are heterogeneous and aim at different objectives. We model this setting as a non-cooperative game with all nodes applying power control, but, contrary to typical formulations, we propose that the two types of nodes have different utility functions. We show the existence of a NE and derive conditions that guarantee its uniqueness. We present a distributed scheme that converges fast at the NE and we evaluate its efficiency through simulations showing that the payoff that the nodes receive at that NE is satisfactory in most scenarios.

- In Chapter 5 we study the fundamental problem of channel access competition in device-to-device networks. Nodes are autonomous and decide on their own whether to transmit or not. Focusing on linear and tree device-to-device networks, we present a non-cooperative game formulation with two variations of the payoff of the nodes. We show the existence of Nash Equilibria, we analyse their structural properties, and we propose two distributed schemes with different levels of cooperation among the nodes that converge to a NE. We evaluate by simulation the resulting Nash Equilibria in terms of fairness and efficiency and compare the performance of our non-cooperative game against classical scheduling approaches showing that these approaches do not always lead to incentive-compatible operating points.

- In Chapter 6 we study licensed spectrum sharing scenarios and define a non-cooperative power control game where operators aim at maximizing their revenues. We show that this game admits a unique NE that is inefficient. We then combine power control with bargaining and derive conditions that guarantee that the operators will end up at a point that is more efficient than the NE. For the case of 2 operators, we compute the socially optimal operating point, where the sum of the revenues of the operators is maximized, and we define bargaining strategies that guarantee that our scheme will lead to that point and also exhibit lower communication overhead than the state-of-the-art. Moreover, we show that our approach strictly outperforms the idea of linear pricing of the transmission power [33] (a well-adopted approach for finding efficient operating points), in terms of both the payoff per operator and the sum of payoffs.

Chapter 2

Related Work

2.1 Power Control in Cellular Networks: The Big Picture[1]

The goal of this chapter is two-fold:

1. To provide a taxonomy of approaches to power control in cellular networks into some fundamental sub-areas.
2. For each sub-area, to review and comment on key power control approaches.

We focus our discussion on cellular networks, as this is the type of wireless networks considered in the majority of power control schemes in the literature and also the type with the most significant commercial and societal impact at present. Furthermore, our approaches in the contexts of small cell networks (Chapter 4) and licensed spectrum sharing scenarios (Chapter 6), which are expected to be critical in the forthcoming 5G era, are mostly influenced by works that focus on cellular networks. However, we do mention and discuss issues and applications of power control to other modern wireless networks.

Fig. 2.1 illustrates the power control taxonomy that we are going to discuss in the following sections. The left part of Fig. 2.1 corresponds to power control techniques that emerged in 2G networks, where voice applications are the norm and each node aims at achieving a target SINR. We distinguish two broad categories:

- Interference-limited networks, where the level of the thermal noise power n is too small and can be neglected. In this case, the SINR definition in (1.1) is modified to:

$$\gamma_i \triangleq \mathrm{SIR}_i = \frac{G_{ii}P_i}{\sum\limits_{j\neq i}^{N} G_{ji}P_j}. \tag{2.1}$$

[1]This chapter is based on paper [31].

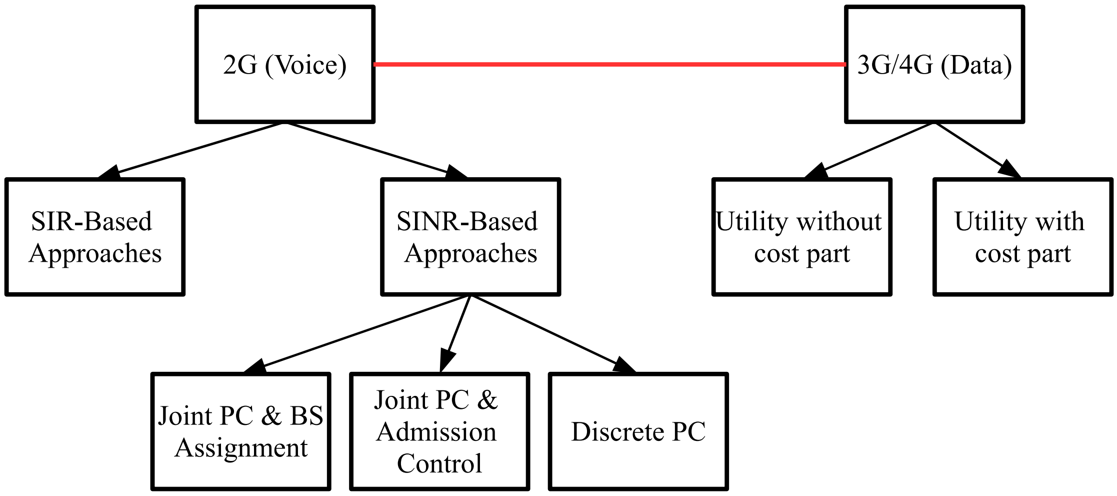


Fig. 2.1: A taxonomy of power control (PC) approaches.

- Networks where the level of the thermal noise power cannot be neglected. Apart from presenting basic algorithms, we will study three approaches of special interest: *Joint Power Control and Base Station Assignment*, *Joint Power Control and Admission Control*, and *Discrete Power Control*. The first two approaches combine power control with other radio resource management techniques so as to further improve the performance of the network. The third approach takes into account that the power levels are not continuous, but take only discrete (predefined) values.

The right part of Fig. 2.1 depicts power control approaches that have emerged in 3G/4G networks, where data applications are the most prominent ones. It is natural to consider utility functions for these cases; furthermore a basic distinction is whether a cost function is used in the definition of that utility function, in order to demotivate the user from transmitting or not. When no such cost part is considered, each entity tries selfishly to maximize its own utility function without having to "pay" for the power that it will choose.

Apart from a thorough description of some representative papers for each area, we shall provide both inter-area and intra-area comparisons: *(i)* power control in 3G/4G networks vs. power control in 2G networks, *(ii)* SIR based approaches vs. SINR based approaches, and *(iii)* utility-based approaches without a cost function vs. utility-based approaches with a cost function.

As a final comment, note that there are power control approaches that can be applied for both voice and data applications. This is the reason that we have joined the first two rectangles in Fig. 2.1 with a horizontal line.

In the following sections, for generality of exposition, we use the terms "node" and "entity" interchangeably to refer to a transmitter node/entity, without explicitly mentioning whether it is a Mobile Node (MN) or a Base Station (BS). When necessary, we shall explicitly mention whether the node is a MN (*i.e.*, the transmission is uplink) or a BS (*i.e.*, the transmission is downlink).

2.2 Power Control in Interference-Limited 2G Cellular Networks

In this section, we consider interference-limited 2G cellular networks. Zander is one of the pioneers of this research area, being among the first, in the early nineties, that studied power control techniques in such networks [19], [34]. In these successive papers, he considers a TDMA/FDMA network where N nodes, with common SIR target γ^t for successful communication, share the same channel. He is interested in applying power control so as to find a power vector $\mathbf{P}^* = [P_1^*, P_2^*, \ldots, P_N^*]^T$ (without placing a limit on the maximum power of each node) that maximizes the minimum SIR of the nodes denoted by γ^*.

In [19], a centralized power control scheme is proposed. By computing the spectral radius λ^* (*i.e.*, the maximum eigenvalue) of the normalized gain matrix G:

$$G = \begin{cases} \frac{G_{ij}}{G_{ii}}, & i \neq j, \\ 0, & i = j. \end{cases} \tag{2.2}$$

he shows that there is a unique solution, which is always feasible and leads to SIR balancing, *i.e.*, all nodes converge to the same SIR, γ^*:

$$\gamma^\star = \frac{1}{\lambda^\star - 1}. \tag{2.3}$$

Consequently, independently of the initial power vector, knowledge of all path gains from (2.2) is a sufficient condition to compute γ^*. Of course, an important drawback of this scheme is its centralized nature, because knowledge of the full path gain matrix is difficult to be achieved in practice.

Even though it is always possible to maximize the minimum SIR in these networks, this γ^* may be below the SIR target γ^t which is the threshold for successful communication. In this case, all nodes would suffer from unacceptable performance. To combat this problem, the notion of "node removal" is introduced [19]. The idea is to find the optimal set of nodes that should power off so that the remaining ones fulfil the condition: $\gamma^* \geq \gamma^t$. As the optimal policy increases dramatically the complexity of this process, a suboptimal but faster solution is to remove in each round one node (*i.e.*, one node–the one with the worst SIR–powers off each time) until the remaining ones fulfil the above condition. This process goes on until an acceptable solution is achieved.

In [34], a partially distributed iterative power control scheme is proposed. Each node i updates its power at round $k+1$ (2.4) by taking into account the following parameters from round k: its power, its SIR, and a positive normalization parameter, b, which is the inverse of the sum of the powers of the N nodes (2.5). This scheme converges to γ^* from every initial strictly positive power vector (*i.e.*, the initial power of each node should be positive).

$$P_i(k+1) = b(k)P_i(k)\left(1 + \frac{1}{\mathrm{SIR}_i(k)}\right), \tag{2.4}$$

$$b(k) = \frac{1}{\sum\limits_{i=1}^{N} P_i(k)}. \tag{2.5}$$

This synchronous (*i.e.*, all nodes update concurrently their power) iterative power control scheme is similar to the power method of numerical analysis [35]. This is not surprising, since γ^* is dependent on the maximum eigenvalue (and the power method is a classical method to compute it). A drawback of the scheme is that although it should be applied autonomously from each entity in each round k, it is not fully distributed; the normalization parameter b (which is necessary to avoid extremely high transmit powers) demands cooperation among nodes in each transmission round (as all powers should be known to each node). The way to choose b is not unique, see [36] for another choice.

In [34], node removal is applied if, after a predefined number of iterations, the partially distributed algorithm (2.4) has not converged to an acceptable solution. Then, the node with the worst SIR powers off and the iterative scheme is reapplied for the N-1 nodes. It is worth mentioning that, to decide which node should power off, cooperation among nodes is, again, necessary.

Both schemes proposed by Zander increase the capacity of the network, as the outage probability (*i.e.*, the probability of a node to achieve SIR lower than γ^t) is decreased compared with non-power control policies.

In [37], Lee and Lin present the following fully distributed power control scheme:

$$P_i(k+1) = \frac{\min(\mathrm{SIR}_i(k), Y)}{\mathrm{SIR}_i(k)} P_i(k), \tag{2.6}$$

that leads to γ^* starting from the maximum power vector where all nodes transmit at $P_{\max}$.

If the value of the positive constant Y is predetermined, nodes can update their powers autonomously. However, if a node needs to power off, the cooperation among nodes is inevitable, as the target is not to remove a random node (which could be done with a predefined criterion too), but the "worst" node, and this cannot be done autonomously. Simulations show that this scheme, for various choices of Y, is faster than Zander's distributed scheme [34], as the latter is based on the power method, whose convergence to the maximum eigenvalue is–generally–slow.

Table 2.1: A classification of power control approaches in interference-limited 2G networks.

Paper	TDMA FDMA	CDMA	Uplink & Downlink	Centralized	Partially Distr.	Fully Distr.	$P_{\max}$ constraint	Different γ_i^t
Lee et al. [37]	✓		✓			✓	✓	
Wu [38]	✓	✓	✓	✓				
Wu [36]	✓	✓	✓	✓	✓			✓
Zander [19]	✓		✓	✓				
Zander [34]	✓		✓		✓			

In [38], Wu extends Zander's centralized scheme [19] in the case of CDMA. Contrary to a TDMA/FDMA network, in CDMA there is only one channel, that is shared by all nodes in each cell. He proposes a centralized scheme so that the entity that powers off is the one that leads to the maximum γ^* for the remaining N-1 entities.

In [36], Wu studies topologies where the SIR thresholds of the nodes are heterogeneous (*i.e.*, each node has its own target γ_i^t). He reapplies the centralized scheme in [38] for this case. Moreover, he proposes a partially distributed scheme, where each node i updates its power as follows:

$$P_i(k+1) = b(k)P_i(k)\frac{\gamma_i^t}{\mathrm{SIR}_i(k)}, \tag{2.7}$$

$$b(k) = \frac{1}{\max\limits_{i=1}^{N} P_i(k)}. \tag{2.8}$$

This scheme leads to $\gamma_i^* = d\gamma_i^t$, where d is a positive constant. In case that $d > 1$, no node needs to power off. Otherwise, a node needs to power off so that other nodes can achieve their targets. Again, nodes need to cooperate to compute parameter $b(k)$ (2.8).

Finally, Table 2.1 depicts our taxonomy of this sub-area by checking with which criteria each paper is compatible. Besides our comments on the previous paragraphs, it is worth mentioning that all approaches can be applied for both uplink and downlink.

2.3 Power Control in 2G Cellular Networks with Noise

2.3.1 Basic Algorithms

In 2G networks where the noise level at each node cannot be neglected, the normalized gain matrix (2.2) cannot be used to compute γ^* (2.3). This is because of the noise power term in the denominator of the SINR definition (1.1). So, the central question is modified as follows: Which is the power P_i^t that each node i should transmit to achieve its SINR target γ_i^t? In practice, most papers set the same γ^t for each node i during the performance evaluation of their method. This is reasonable because, in voice applications (which is the traditional case in 2G networks), the QoS target and the need for resources are (practically) the same for all nodes. However, this assumption is no longer acceptable in today's networks.

In [32], Foschini and Miljanic are the first who answered this question by providing a fully distributed scheme that computes the power that each node i should use to achieve its target. This algorithm is fully distributed, as there is no need for cooperation among the wireless nodes to compute their powers. In [39] this algorithm is further simplified to:

$$P_i(k+1) = \gamma_i^t \frac{P_i(k)}{\gamma_i(k)}, \tag{2.9}$$

which we refer to as the simplified Foschini-Miljanic algorithm. When a feasible solution exists, each node has achieved its γ_i^t target. However, the authors do not discuss the conditions that guarantee the convergence of the scheme. Actually, when nodes cannot achieve their γ_i^t targets, their powers will diverge to infinity. Moreover, no maximum power $P_{\max}$ limitation is imposed.

In [40], Mitra shows that a sufficient and necessary condition for the convergence of the power vector $\mathbf{P^t} = [P_1^t, P_2^t, \ldots, P_N^t]^T$ that arises after the application of (2.9) to $\gamma^{\mathbf{t}} = [\gamma_1^t, \gamma_2^t, \ldots, \gamma_N^t]^T$ is that the spectral radius of the gain matrix (2.2) to be smaller than 1. Moreover, he shows that this power vector $\mathbf{P^t}$ is Pareto optimal, in the sense that any power vector $\mathbf{P}$ that satisfies the target for all nodes demands at least as much power for every node and at least one node's power to be greater, *i.e.*, $\mathbf{P} \geq \mathbf{P^t}$ component-wise. Furthermore, he proposes an asynchronous (*i.e.*, all nodes do not necessarily have to update their power concurrently) version of the Foschini-Miljanic algorithm where nodes satisfy their targets under the above mentioned condition.

In [41], Grandhi, Zander, and Yates incorporate a $P_{\max}$ constraint for each node and restate the Foschini-Miljanic algorithm with a $P_{\max}$ constraint:

$$P_i(k+1) = \min \left\{ P_{\max}, \gamma_i^t \frac{P_i(k)}{\gamma_i(k)} \right\}. \tag{2.10}$$

We call (2.10) the FM algorithm. A version with asynchronous updates is provided as well. Moreover, the authors propose a centralized algorithm that, finds the maximum common γ^t that can be achieved by all nodes (in that case, clearly $\gamma^t = \gamma^*$).

In [42], Yates studies the following interesting problem: If somebody devises an iterative power update scheme $\mathbf{P}(k+1) = I(\mathbf{P}(k))$, where $I(\mathbf{P})$ stands for the interference that each node must overcome, is there any way to know whether this scheme is going to converge to a power vector $\mathbf{P}$ (if that exists) that satisfies the γ_i^t target for each node i? The answer is positive, provided that the–so called–Interference function $I(\mathbf{P})$ is standard, *i.e.*, it fulfils the following properties:

1. Positivity: $I(\mathbf{P}) > 0$.

2. Monotonicity: $\mathbf{P} \geq \mathbf{P}' \Rightarrow I(\mathbf{P}) \geq I(\mathbf{P}')$.

3. Scalability: $aI(\mathbf{P}) > I(a\mathbf{P})$, $a > 1$.

Yates shows that this framework holds for:

- Power Control in both the uplink and the downlink, under fixed BS assignment.

- Power Control and BS Assignment in the uplink.

In the above cases, the framework is valid under very general settings, including support for $P_{\max}$, $P_{\min}$, or no power constraints, synchronous or even asynchronous updates, as well as for joint power control and admission control techniques. The importance of this result is that it functions as a "convergence guarantee" for every proposed power control algorithm that is valid for that framework.

2.3.2 Power Control and Base Station Assignment in the Uplink

Yates and Huang in [43] and Hanly in [44] study the joint power control and BS assignment problem for a single-channel cellular network in the uplink. They are interested in finding the optimal power vector $\mathbf{P^t}$ (component-wise) that satisfies the SINR targets, provided that each MN can switch to a different BS. They independently show that, by applying FM (2.9), but allowing each MN to know the interference at each BS and to connect to the one for which the least power is needed to transmit, the algorithm converges to a unique power vector $\mathbf{P^t}$, provided the problem has a solution. It is worth mentioning that, even though $\mathbf{P^t}$ is unique, the assignment BS-MN that leads to $\mathbf{P^t}$ may not be unique (for example, in case there is symmetry in the topology).

These works differ in the following points: Hanly's approach in [44] predetermines the set of BSs that each MN can connect to. This set may be adjusted dynamically throughout the process. In [43], this knowledge is not necessary. Moreover, Yates and Huang in [43] present both synchronous and asynchronous versions of their algorithm, whereas Hanly deals only with the synchronous case. Lastly, Hanly discusses the case where a MN notices rapid oscillations back and forth between two BSs and proposes a small modification of the algorithm to alleviate this phenomenon. A limitation of these works is the absence of a $P_{\max}$ constraint.

Further power control issues such as joint power control, BS assignment, and beamforming, as well as downlink extensions, are discussed in [45].

2.3.3 Power Control and Admission Control

An important disadvantage of all the above mentioned power control schemes is that nodes suffer from fluctuations during the evolution of their powers. In other words, there is no guarantee that when an entity i becomes active, *i.e.*, its SINR is at least γ_i^t, it will remain so in the following iterations of the algorithm. New nodes may desire to enter the network and nodes already in the network might power up, so that some active nodes may not be able to absorb this extra interference. A consequence of this problem is that these power control schemes lead to the following type of error: A new entity is admitted even though it could not safely be admitted. This is the well known dropping probability error (Type I error), which is very annoying for users [46]. Next, we discuss some important works that address this issue and hold for both uplink and downlink.

Bambos and his colleagues [47] are the first that dealt extensively with the joint power control and admission control problem. They divide nodes into two sets that are updated in each transmission round k: the set $\mathbf{A_k}$ of admissible nodes and the set $\mathbf{B_k}$ of inadmissible nodes. For the former, they use a modification of the FM formula as seen in (2.11), introducing a parameter d, where: $d = 1 + e$, with e being a small positive number:

$$P_i(k+1) = \begin{cases} d\gamma_i^t \frac{P_i(k)}{\gamma_i(k)}, i \in \mathbf{A_k}, \\ dP_i(k), i \in \mathbf{B_k}. \end{cases} \tag{2.11}$$

The parameter d allows each active entity i to set its target to $d\gamma_i^t$, so as to provide an e-protection margin for its communication. This scheme has the following nice property for each $i \in \mathbf{A_k}$:

$$\gamma_i(k) \geq \gamma_i^t \Rightarrow \gamma_i(k+1) \geq \gamma_i^t. \tag{2.12}$$

Consequently, $\mathbf{A_k} \subseteq \mathbf{A_{k+1}}$ and $\mathbf{B_k} \supseteq \mathbf{B_{k+1}}$. However, in cases where an entity remains inadmissible for many iterations of the algorithm, chances are that it will remain so in future iterations too. For these cases, it may be better for some nodes to follow a so-called voluntary drop-out policy, *i.e.*, to power off for a while (until channel conditions change) and to retry to power up later on. More specifically, Bambos et al. propose two policies: A time-out drop-out policy and a SINR saturation drop-out policy. The former dictates that if an entity i remains inadmissible for K iterations of the algorithm, then it will try only up to M more times–this number will grow inversely proportionally to the difference between γ_i^t and $\gamma_i(K)$–to achieve its target, before powering off. The latter proposes that if the SINRs of some nodes do not present a significant improvement for K successive rounds of the algorithm, then they flip independent coins to decide whether to power off in the next iteration of the algorithm. Again, the smaller the difference between γ_i^t and $\gamma_i(K)$, the higher the chance for entity i to go on updating its power.

A great advantage of the approach in [47] is that it is fully distributed. However, in case a $P_{\max}$ limitation exists, then some cooperation among nodes is considered necessary, as an active entity should inform the inadmissible ones to power off (forced drop-out policy) when its $P_{\max}$ constraint should be violated in order for it to remain admissible. Moreover, some cooperation is necessary to find the maximum allowable initial power that an entity can transmit without "impacting" the already active nodes. If this does not happen, then an active entity may instantaneously become inactive. Authors eloquently use the motto "once active, always active!" to describe the power update policy for these cases.

A problem of the scheme in [47] is that it may (rarely) lead to Type I errors, as an entity may become admissible with its power diverging to infinity. Moreover, since a voluntary/forced drop-out policy is used, it is possible that an entity is requested to power-off unnecessarily, as it could have been become eventually active (the so-called blocking probability error, or Type II error).

In [46], Andersin, Rosberg, and Zander invent a partially distributed soft and safe (SaS) joint power control and admission control algorithm under a $P_{\max}$ constraint, which is Type I and Type II error free. The key idea of the algorithm is the following: Each time a new entity powers up, all other entities scale their powers uniformly (this demands cooperation among the entities) to overcome the extra interference. If this is possible, then all entities (including the not yet admitted one) apply (2.10) with a view to finding a solution that both demands less power for at least some of the admitted entities and the new entity becomes admissible, with two stopping conditions:

- An admitted entity becomes inadmissible or gets assigned a power higher than $P_{\max}$.
- The new entity becomes admissible, or its power is set higher than $P_{\max}$.

This iterative process converges to the desired solution, though this happens–in general–quite slowly. For this reason, the authors proposed a fast version of the SaS algorithm (F-SaS), where after only one iteration of (2.10), either the new entity becomes active, or it powers off. Though this version is very fast, it is only Type I error free, as Type II errors may arise. However, in general, blocking a new call is less annoying than dropping an ongoing call.

In Table 2.2, we compare the main characteristics of the joint power control and admission control algorithms in [47] and [46]. Apart from our comments in the previous paragraphs, we would like to mention that a disadvantage in [47] is some loss of capacity due to the safety margin that is defined by parameter d. Of course, as d approaches 1, this capacity loss decreases. However, the smaller the d, the more difficult is the admission of new nodes, as active nodes have a lower safety margin to tolerate extra interference. On the

Table 2.2: Joint power control and admission control approaches: a comparison.

Paper	Bambos et al. [47]	Andersin et al. [46]
Type I and II Error Free		✓(only SaS)
Type I Errors	✓(very rare)	
Type II Errors	✓(voluntary Drop-Out)	✓(only F-SaS)
Fast Convergence	✓	✓(only F-SaS)
Loss of Capacity	✓	
Fully Distributed	✓(if No $P_{\max}$)	✓
One Inactive User per Time Update		✓
Synchronous Updates	✓	✓

other hand, algorithms in [46] assume that only one entity desires to power up every time (which is compatible with the assumption for Poisson arrivals made in the paper). Thus, in order to further minimize the probability of two concurrent inadmissible nodes, the authors use only the synchronous version of [41]. Note that synchronous updates are a prerequisite in [47] as well.

As a final note, Gitzenis and Bambos [48] propose a variation for the power update of inadmissible nodes (2.11). By introducing some mini slot time periods, they periodically offer the opportunity to inadmissible nodes to test any desired power in these mini slots. For example, they may even decide to choose the power to be equal to the power they would use if they were to become active in the next iteration of the algorithm. If, during that mini slot, all the active nodes can tolerate this extra interference, then, during the next slot of the algorithm, these nodes deviate from (2.11) and transmit with the power of the previous mini slot period. Clearly, this process will converge faster compared to [47]. Moreover, (partially) asynchronous convergence may be achieved. It remains an open issue whether this scheme will prove even more beneficial if nodes cooperate in order to decide when each one will try to update its power to a higher level than the one that is imposed by (2.11). Of course, this will destroy the fully distributed notion of the algorithm, even in the unconstrained case (*i.e.*, with no $P_{\max}$).

2.3.4 Discrete Power Control

Apart from introducing a $P_{\max}$ constraint, a practical power control scheme should take into account the fact that power is updated only at discrete levels. This is the motivation for discrete power control algorithms [49], [50] that are valid for both uplink and downlink, a subject that has not been developed much all these years.

In [49], Andersin, Rosberg, and Zander use the synchronous and asynchronous versions of [41] and modify each component of the power vector $\mathbf{P}^*$ that arises to the nearest higher or lower discrete power so as to try to satisfy the target of each node. Unless the powers are at exactly the discrete levels, taking the lower discrete power level leads always to a solution where no node is satisfied. On the other hand, by applying the ceiling version, it is proven that convergence to a unique power vector is not guaranteed, as oscillations between power vectors may appear. Smith et al. [51] build upon this work proposing a game-theoretic treatment of this problem.

In [50], Sung and Wong firstly prove that if, for each node i, P_i^t converges to γ_i^t, there exists a quantized power vector that converges to the region $[d^{-1}\gamma_i^t, d\gamma_i^t]$, for any $d > 1$. Then, they propose the power control scheme:

$$P_i(k+1) = \begin{cases} dP_i(k), \text{ if } \gamma_i(k) < d^{-1}\gamma_i^t \\ d^{-1}P_i(k), \text{ if } \gamma_i(k) > d\gamma_i^t \\ P_i(k), \text{otherwise.} \end{cases} \tag{2.13}$$

They prove that this scheme converges in the above region as well.

Moreover, they incorporate an admission control scheme by showing that if, at round k, $\gamma_i(k) \geq d^{-2}\gamma_i^t$, this inequality will also hold in the following transmission rounds. Of course, this extra d margin leads to some loss of capacity for the network.

Comparing the schemes in [49] and [50], we remark the following: The main advantages of [49] are the inclusion of a $P_{\max}$ constraint, as well as the possibility for asynchronous convergence. On the other hand, it is a quite complex algorithm and does not incorporate any admission control mechanism. The algorithm in [50] is simpler and permits an admission control process (sacrificing some capacity), but its performance worsens when $P_{\max}$ is taken into account. As a last note, the performance of discrete power control algorithms depends on the number of power levels. The more power levels there are, the smaller is the loss of capacity, but the slower is the convergence to a power vector. The opposites hold for fewer power levels. However, the distance between two consecutive power levels should not be defined arbitrarily, but it should arise from the type of the cellular network technology that is used.

2.3.5 Classification

We complete this section by providing Table 2.3, which presents the big picture, *i.e.*, we provide some properties and examine which of them are satisfied by the power control algorithms of the works that we discussed. The only exception has to do with [42], as no

Table 2.3: A taxonomy of power control approaches in 2G networks with noise.

Paper	$P_{\max}$	Power Control & Admission Control	Power Control & BS Assignment	Discrete Powers	Asynchronous Version	Approach
Andersin et al. [46]	✓	✓				Partially Distributed
Andersin et al. [49]	✓			✓	✓	Partially Distributed
Bambos [47]	✓	✓				Fully Distributed (when no $P_{\max}$)
Foschini et al. [32]						Fully Distributed
Gitzenis et al. [48]	✓	✓				Partially Distributed
Grandhi et al. [41]	✓				✓	Fully Distributed (& Centralized)
Hanly [44]			✓			Fully Distributed
Mitra [40]					✓	Fully Distributed
Sung et al. [50]		✓		✓		Fully Distributed
Yates et al. [43]			✓		✓	Fully Distributed
Yates [42]	✓	✓	✓		✓	No algorithm proposed

algorithm (but a framework) has been presented. It is interesting to note that the paper by Foschini and Miljanic [32] does not fulfil any of the properties that we have chosen to compare these papers against. However, that paper was the basis for most of the approaches of the papers that were discussed in this section.

2.4 Power Control in 3G/4G Cellular Networks

2.4.1 Introduction

In 3G/4G cellular networks, data applications are the prominent ones. For a data link, in principle, there is no specific acceptable performance level, below which the link is considered useless and above which improved performance is indifferent (as in voice nodes), but a continuous trade-off between achieved performance and the cost to achieve it. Thus, in general, there is never a question of whether a node should power off or not, but rather a question of how to decide on power levels to best optimize various metrics.

A good methodology for modelling and addressing these issues is to consider utility, value, and cost functions. A utility function $U(\cdot)$ expresses the (dis)satisfaction of a link that utilizes system resources. In the case of power control games, the general form of a utility function is $U_i(P_i, \gamma_i) = V_i(P_i, \gamma_i) - C_i(P_i)$, where $V_i(\cdot)$ is a value function that expresses the value that (the owner of) link i perceives and $C_i(\cdot)$ is a cost function that expresses the resources that it has to spend to achieve this value. In the following, we will present some fundamental approaches in this direction. Further material can be found in [45] (mainly in Chapters 5 and 6).

2.4.2 Key Approaches

In [52], Saraydar, Mandayam, and Goodman propose a utility function for each player i that approximates the number of information bits that are successfully received per Joule of energy expended.

$$U_i(P_i, \gamma_i) = \frac{LR}{MP_i} f(\gamma_i), \tag{2.14}$$

where L is the number of information bits per frame, M is the total number of bits in a frame, R expresses the bit rate, P_i is the power of node i and $f(\gamma_i)$ is a function that approximates the probability of correct reception of a frame. They model the problem as a non-cooperative game, where each node tries selfishly to maximize (2.14). The authors show that as long as each node uses the same function f, there is a unique Nash Equilibrium (NE), where the achieved SINR will be the same for each node.

They then modify the utility function to

$$U_i(P_i, \gamma_i) = \frac{LR}{MP_i} f(\gamma_i) - c_i P_i, \tag{2.15}$$

where each node i pays proportional to its power P_i. Parameter c_i is a positive constant.

By applying supermodularity theory, they show that this utility function admits many Nash Equilibria and the NE with the smallest powers can be computed in a (synchronous or asynchronous) distributed way. This NE is more efficient than the one of (2.14), though it leads to different SINRs for the nodes, so it is unfair in that sense. As a final note, they also investigate general (non-linear) pricing functions as a way to find more efficient Nash Equilibria. This idea has two disadvantages: It complicates the problem and it destroys its distributed solution aspect.

Xiao, Shroff, and Chong in [53] choose a sigmoid function with parameters a_i and b_i and apply linear pricing of the transmission power:

$$U_i(P_i, \gamma_i) = \frac{1}{1 + \exp(-a_i(\gamma_i - b_i))} - c_i P_i. \tag{2.16}$$

By adjusting the values of parameters a_i and b_i, utility functions that are suitable for either data nodes (higher SINR target but acceptable to power off for a while), or voice nodes (lower SINR target but not desirable to power off even for a while) may be constructed.

They reformulate the FM formula as

$$P_i(k+1) = \gamma_i^+ \frac{P_i(k)}{\gamma_i(k)}, \tag{2.17}$$

where γ_i^+ is the SINR target that changes from round to round.

By applying the Yates' framework [42], (2.17) converges (synchronously or asynchronously) to a unique power vector from every initial power vector, provided that such a vector exists. The performance of the scheme is improved by applying adaptive pricing, *i.e.*, by taking into account both the channel conditions and the distance between the transmitter and the receiver to decide the pricing coefficient c_i. However, a complete analysis of the optimal linear pricing policy is left as an open issue.

In [54], Leung and Sung propose the concept of opportunistic power control. This means that not only do they decrease SINR targets when channel conditions worsen (as done in [53] too), but they also decrease their transmit powers in this case. An interesting property and significant advantage of this approach is that if some nodes targeting voice applications update their powers using (2.10) and can achieve their γ_i^t targets, they can coexist without falling below their targets.

Finally, Table 2.4 presents a comparison of the schemes that we have discussed. In addition to our previous comments, we mention that the main drawback of [52] is that

Table 2.4: A taxonomy of power control approaches in 3G/4G networks.

Paper	Zero Cost Function	Non-Zero Cost Function	Voice Nodes & Data nodes	Asynchronous Version	$P_{\max}$
Leung et al. [54]		✓	✓		✓
Saraydar et al. [52]	✓	✓		✓	✓
Xiao et al. [53]		✓	✓	✓	

it is only suitable for data nodes. A major limitation of [53] is that no $P_{\max}$ constraint is included in the analysis. Lastly, [54] does not discuss an asynchronous version of the proposed scheme.

2.5 Power Control in Other Types of Wireless Networks

In this section, we shall briefly discuss some power control approaches that focus on various types of wireless networks other than cellular networks using a representative paper for each type.

In [55], Kawadia and Kumar propose various algorithms that focus on either maximizing the network capacity or minimizing the energy consumption of a wireless ad hoc network. They firstly present COMPOW, an algorithm that finds the minimum (common) power that can be used by all nodes of the network so as to maximize the network capacity. This is feasible provided that the distribution of the nodes is homogeneous. If this is not the case, they propose CLUSTERPOW, an algorithm that dynamically creates clusters of nodes that use the same power. They show that this process is optimal in terms of network capacity too. Then, they focus on the energy consumption minimization, by using a variation of the Bellman-Ford Algorithm named MINPOW that leads to a global minimization of the energy spent. Finally, authors present LOADPOW, a scheme that applies power control based on the network load, *i.e.*, nodes increase their power when the load is low and vice versa. We have already seen an application of this idea in [53].

It is worth mentioning that these schemes correspond to a cross-layer design, involving both the physical layer and the MAC sub-layer of the IEEE 802.11 protocol. A common limitation of these ideas is the demand of synchronization among nodes, which is both difficult to achieve and adds an overhead to each method. However, even if the implementation of many of these schemes is questionable (mainly) due to various firmware limitations, these ideas remain interesting.

Table 2.5: A taxonomy of power control approaches in non-cellular wireless networks.

Paper	Max Network Capacity	Min Energy Consumption	Fairness	Congestion/ Load Control	Power Control & Channel Assignment
Kawadia et al. [55]	✓	✓		✓	
Messier et al. [57]		✓			
Morreno et al. [58]			✓	✓	
Nie et al. [56]	✓	✓	✓		✓

In [56], Nie, Comaniciu, and Agrawal deal with power control in the context of cognitive radio networks. A game theoretic model is proposed by using a utility function that takes into account both the interference that an entity receives from other entities and the interference it imposes to other entities that are using the same channel. The key difference from other game theoretic works that we have discussed (e.g., [52], [53]) is that these cognitive devices are able to also adapt their transmission rate. Thus, by changing their modulation scheme, their SINR targets change as well. Extensive simulations that consider power control with and without channel assignment as well as with and without power limitation are presented. They show that the joint power control and channel assignment scheme presents the best performance in terms of *(i)* throughput, *(ii)* energy consumption, and *(iii)* fairness. However, analytical models have not been developed so as to formally prove these findings.

In [57], Messier, Hartwell, and Davies discuss power control in wireless sensor networks. As expected, they focus on minimizing energy consumption, which is reasonable since battery replacement is not always possible in cases where sensors are placed in remote places. They present a cross-layer approach (extending many previous works) that takes into account the link and the physical layers. The goal is to minimize the energy that is spent per symbol transmitted at both the physical layer and at the link layer (due to potential retransmissions of the frames). Further work on reducing the complexity of the scheme is needed to ease its adoption and facilitate its implementation. Moreover, the demand for synchronous nodes is a disadvantage which should be treated carefully. A framework similar to the Yates' seminal paper [42] would be very useful.

Lastly, in [58], Morreno, Mittal, Santi, and Hartenstein apply power control in the context of Vehicular Ad hoc NETworks (VANETs). They present a power control scheme with a view to increasing vehicular traffic safety. Messages that a VANET vehicle may send belong into two categories: *(i)* some periodic messages that transfer standard information and are transmitted by all vehicles and *(ii)* some safety-critical messages that are transmitted only when some emergency arises. The latter have higher priority and are transmitted by higher power (when necessary). Thus, channel saturation for priority messages due to the transmission of periodic messages is avoided. Moreover, their scheme considers fairness in the sense that it maximizes the minimum power used for the transmission of periodic messages by all the nodes of the vehicular network. It is quite interesting that this conception is similar to the key idea of Zander's early works [19], [34], though VANETs do not share many similarities with cellular networks. Fairness is achieved provided there is perfect communication among all the interfering nodes. This is quite unrealistic for a VANET as nodes change their positions rapidly. However, simulations show that the results are close to the theoretical ones. Egea-Lopez et al. [59] have recently presented a scheme that meets similar goals by reducing the number of periodic messages that need to be exchanged.

In Table 2.5, we compare the above mentioned approaches in terms of various characteristics, summarizing our comments in the previous paragraphs.

Finally, it is worth mentioning that whereas power control is implemented in the core of the 3G and 4G technology (a detailed description is provided, e.g., in [45]–mainly in Chapter 10), it has not been so widely adopted in other technologies. Notably, power control schemes that are compliant with IEEE 802.15.4 (a standard for low-rate wireless personal area networks) have been recently proposed, for example, in [60]. This is not so much the case currently for IEEE 802.11 networks, as the hardware and wireless driver support for power control is very limited in many cases. IEEE 802.11h supports transmitter power control, but this standard is not yet supported by the bulk of the current wireless cards [61]. Wide industry adoption of power control for Wi-Fi remains an open issue.

Chapter 3

Power Control and Bargaining in Wireless Networks with Autonomous Nodes

3.1 Introduction and Motivation[1]

In this chapter, we consider a wireless network as described in Section 1.5. We assume that the nodes apply the Foschini-Miljanic (FM) algorithm [32] that we have reviewed in Section 2.3.1. As we have discussed, in this context, nodes are competitors in the sense that each one creates interference to all others, influencing negatively their SINRs. In Table 3.1, we present an equivalent model of FM under the prism of non-cooperative game theory. N wireless nodes apply transmitter power control; the utility function U_i of each player i is the absolute value of the difference of its current SINR γ_i (1.1) minus its SINR target γ_i^t. Indeed, it is easy to show [54] that this game admits a unique Nash Equilibrium (NE) and the iterative scheme that updates the power at round k+1 as:

$$P_i(k+1) = \min\left\{P_{\max}, \gamma_i^t \frac{P_i(k)}{\gamma_i(k)}\right\}, \tag{3.1}$$

converges to this NE. As we have discussed in Chapter 2, (3.1) corresponds to the FM scheme that was subsequently simplified by Bambos [39] and Grandhi, Zander, and Yates [41]. At the NE (3.1), where $P_i(k+1) = P_i(k)$, player i has either achieved its target or it is below its target and transmits with $P_{\max}$.

An important question, which motivates our work in this chapter, is the following: How often does the FM scheme end up at an efficient NE, *i.e.*, How often have all nodes achieved their targets at the NE?

[1] This chapter is based on papers [62], [63].

Table 3.1: Game formulation.

Set of players	Set of nodes $\mathbf{N} = \{1, 2, \ldots, N\}$
Strategy of player i	$P_i \in [0, P_{\max}]$
Utility function for player i	$U_i = -\lvert\gamma_i - \gamma_i^t\rvert$

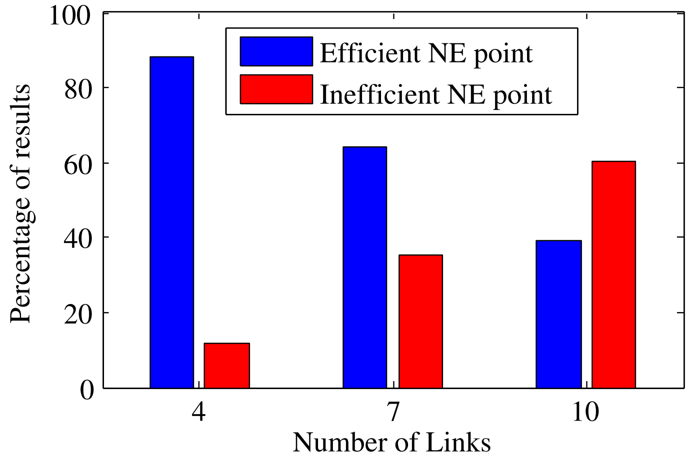


Fig. 3.1: Application of the Foschini-Miljanic (FM) algorithm. The horizontal axis depicts the number of links of the topology and the vertical axis the percentage of scenarios.

To explore this issue, we have simulated a number of small wireless networks consisting of 4, 7, and 10 links (*i.e.*, transmitter-receiver pairs). For each set of links, we run 50,000 scenarios where uniformly distributed links apply the FM algorithm. Fig. 3.1 presents the number of topologies that lead to either *(i)* an efficient NE (meaning that all links reached their targets), or *(ii)* an inefficient NE (meaning that at least one link cannot achieve its target). Simulation parameters are presented in Table 3.2. Even in these small setups where few entities coexist, inefficient NE arise for a significant number of cases (over 10%, over 30%, and over 60%, for 4, 7, and 10 links respectively).

For these cases, some interesting approaches have been proposed in [34] and [53]. In [34], Zander proposes that the $t \geq 1$ weakest entities (*i.e.*, the ones that are farthest from their targets) should power off. We call this approach the Trunc(ated) FM power control algorithm, as N-t entities update their powers according to FM, whereas t entities power off. This is a partially distributed algorithm, as entities need to cooperate to find out the t weakest ones.

In [53], Xiao, Shroff, and Chong formulate a non-cooperative game where entities adapt their targets (which are now soft) to the channel conditions. As a link feels more interference, it decreases its target and may even power off. Pricing of the power is introduced as a way to encourage the nodes to adjust their targets.

Table 3.2: Simulation parameters.

Parameter	Value
# Links of each Topology	4, 7, 10
# Scenarios per Topology	50,000
Simulation Terrain	A square of size 100
Transmitter (Tx) Distribution	Uniform
Receiver (Rx) Distribution	Rx is placed randomly in the interior of a circle of radius 5 from its associated Tx
Path Loss Model	$G = f(d^{-4})$, d: distance between Tx and Rx
Noise Power	10^{-6}
SINR Targets (in dB)	[10,15]
Initial Transmission Powers	$(0, P_{\max}]$
Initial Budget for Tx_i $B_i(0)$	1000

Both these policies, though effective, are not appealing to nodes and therefore quite difficult to implement in practice. Indeed, in modern wireless networks, entities are in general autonomous (e.g., they may belong to different operators that share spectrum) and could not be obliged to power on/off based on the instructions of an external entity.

Motivated by the above remarks, in this chapter, we deal with scenarios that lead to inefficient NE after the application of FM and we make the following contributions:

- We propose the Distributed Bargaining Foschini-Miljanic algorithm (DBFM) that combines FM with a bargaining approach. DBFM is a heuristic method that aims at finding a (N-t)-efficient NE, which we define as a state where N-t nodes achieve their targets.

- We compare DBFM with key related approaches [34], [53] in terms of efficiency (percentage of scenarios that can find (N-t)-efficient NE) and fairness (which subset of nodes achieve their targets at the (N-t)-efficient NE). Simulations show that our scheme outperforms the above approaches under these metrics.

3.2 DBFM: The Distributed Bargaining Foschini-Miljanic Algorithm

We propose the Distributed Bargaining Foschini-Miljanic (DBFM) algorithm for the network that consists of N nodes, which is a heuristic that aims at finding (N-t)-efficient NE, which we define as states where N-t nodes have achieved their targets. DBFM works on top of FM. It takes as input the NE power vector that arises after the application of FM; some nodes have just achieved their γ_i^t targets, whereas those that have failed to achieve their targets transmit with $P_{\max}$.

We now provide a detailed description of how nodes update their powers at a particular round of the scheme.

- Which nodes take part in the negotiations?

  At the NE that arises after the application of FM, nodes are separated in two sets: The set of satisfied nodes $\mathbf{L_a}$ that have achieved their targets and the set of unsatisfied nodes $\mathbf{L_b}$ that are below their targets and transmit at $P_{\max}$. Nodes in the former set are not interested in participating in any sort of negotiation. Nodes in the latter set negotiate pairwise. Negotiations take place through the budget that each node has collected previously from this scheme or other network operations. This budget can be based either on real money or some virtual currency [64] that is used to promote the efficient completion of network operations. It is not critical for our approach to explicitly define the exact form of the budget.

- How does a node decide whether it is going to make or receive an offer?

  In each transmission round, each unsatisfied node i chooses independently whether to belong to the set of Buyers $\mathbf{B}$ or to the set of Sellers $\mathbf{S}$. Then, it broadcasts its status to the network. Each $i \in \mathbf{B}$ makes an offer to a node $j \in \mathbf{S}$.

- How does a Buyer i select its Seller j?

  Provided that there is at least one Seller $j \in \mathbf{S}$, each Buyer i picks up independently a Seller j to negotiate with. It is clear that many Buyers may choose the same Seller to negotiate with.

- How much a Buyer i offers and what does it ask for?

  Buyer i makes a "take it or leave it" offer to Seller j of the form: "I offer you $R_i(j)$ units if you reduce your power $X_i(j)\%$." $X_i(j)$ is the minimum needed power reduction from j so that i achieves its target in the next round. To compute this, i should be able to estimate the exact level of interference from j. $R_i(j)$ is set to

  $$R_i(j) = \max\left\{0, B_i \frac{\gamma_i}{\gamma_i^t} X_i(j)\right\}. \tag{3.2}$$

  Note that $R_i(j)$ depends on three factors: i's budget B_i, i's distance from its target γ_i^t, and the percentage power reduction $X_i(j)$ that it asks for. The closer it is to achieve its target, the bigger the offer that it makes. In case that i cannot achieve its target in the next round by making an offer to j, it makes no offer.

- How does a Seller j evaluate the offers it has received?

  Seller j collects all the offers that it receives and compares each offer $R_i(j)$ that it has received from a Buyer i with the quantity:

$$R_j(i) = \max\left\{0, B_j \frac{\gamma_j}{\gamma_j^t} X_i(j)\right\}. \tag{3.3}$$

  Seller j accepts every offer that fulfils the inequality: $R_j(i) \leq R_i(j)$, *i.e.*, every offer that is greater or equal to the offer that it would have made to Buyer i had it asked for the same percentage reduction. Simulations reveal that this symmetric rule promotes the fairness of our scheme. Let $X_j^{\max}$ be the maximum power reduction that Seller j accepts. Therefore, its power at the next round will be:

$$(1 - X_j^{\max})P(j). \tag{3.4}$$

- How do nodes update their powers at the end of the negotiations?

  Let $\mathbf{M} = \{1, 2, \ldots, M\}$ be the set of Sellers that have a successful negotiation. Each $m \in M$ transmits at $(1 - X_m^{\max})P(m)$ and will receive a smaller QoS in the next transmission round. The remaining N-m nodes apply the FM scheme. In case that $m = t$, an (N-t)-efficient state has been achieved and the algorithm stops.

We conclude this section by presenting Algorithm 1 that formalizes the previous discussion.

Algorithm 1 DBFM: Distributed Bargaining Foschini-Miljanic Algorithm.

$\mathbf{N}$: Set of nodes, $\mathbf{L_b}$: Set of nodes that are below their SINR targets, $\mathbf{L_a}$: Set of nodes that have achieved their SINR targets, $\mathbf{S}$: Set of Sellers, $\mathbf{B}$: Set of Buyers, k: transmission round.

for $k = 1 \to$ MAX_NUMBER_OF_ITERATIONS **do**
    Each $a \in \mathbf{L_a}$ applies FM.
    Each $b \in \mathbf{L_b}$ independently decides whether it is a Seller or a Buyer and broadcasts its status to the network.
    if $S \geq 1 \curlywedge B \geq 1$ **then**
        Each $i \in \mathbf{B}$ selects at random one $j \in \mathbf{S}$ to negotiate with.
        i offers $R_i(j)$ units to j using (3.2).
        Each $j \in \mathbf{S}$ collects all the offers that it receives and evaluates them using (3.3).
        if j has accepted at least one offer **then**
            j updates its power using (3.4).

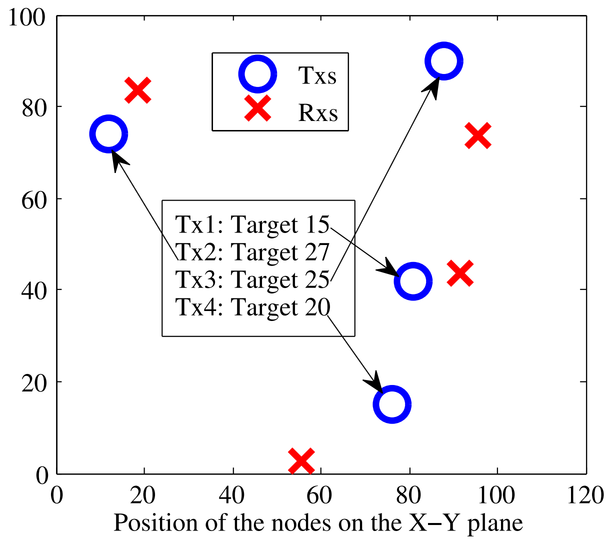


Fig. 3.2: A small wireless network consisting of 4 links. Each transmitter node (Tx) wants to communicate with its associated receiver node (Rx) causing interference to all other links.

Algorithm 1 DBFM (continued)

```
        else
            it applies FM.
        end if
        Each i ∈ B applies FM.
        if L_a ≤ N − t then
            break;
        end if
    end if
end for
```

3.3 DBFM: An Example

In this section, we illustrate the functionality of DBFM with an example. Fig. 3.2 presents a small wireless network consisting of N=4 interfering links. This setup corresponds to a wireless network as described in Section 1.5.

Fig. 3.3a presents the SINR evolution after the application of FM. After four iterations, FM finds out the unique NE where both Tx_2 and Tx_4 are below their targets (targets are presented as dashed lines on the diagram). In this example, we look for a (N-1)-efficient state, where 3 (out of 4) nodes achieve their targets.

We then apply DBFM, which takes as input the NE state of FM (Fig. 3.3b). This is the reason why the SINR of each link at the 1st round of DBFM coincides with the SINR value at the last round of FM.

Table 3.3: Negotiations among nodes.

Round	Buyer	Corresponding Seller	Buyer's Offer	Buyer's Request for % Power Reduction	Seller's Decision
1	Tx_4	Tx_2	70.5	330	NO
2	Tx_4	Tx_2	77.3	380	NO
3	Tx_4	Tx_2	82.6	450	NO
4	Tx_2	Tx_4	74	470	YES

Table 3.3 presents an example scenario that arises after the application of DBFM in that topology. It shows the negotiations among the unsatisfied nodes during each round of DBFM. The last column depicts the outcome of the negotiation. NO means that the Buyer's offer is not accepted. YES means that the Buyer's offer is accepted.

In the first 3 rounds, Tx_4 makes an offer to Tx_2 that is rejected. After a rejected offer, Tx_4 voluntarily reduces a bit its power (10%) to avoid re-offering the same amount (Fig. 3.3d). This voluntarily reduction of the power is not, in general, necessary. This is the reason why we have not included it in Algorithm 1. However, in cases where are 2 nodes that negotiate with each other, it is of the Buyer's benefit to do that to be able to make a bigger offer.

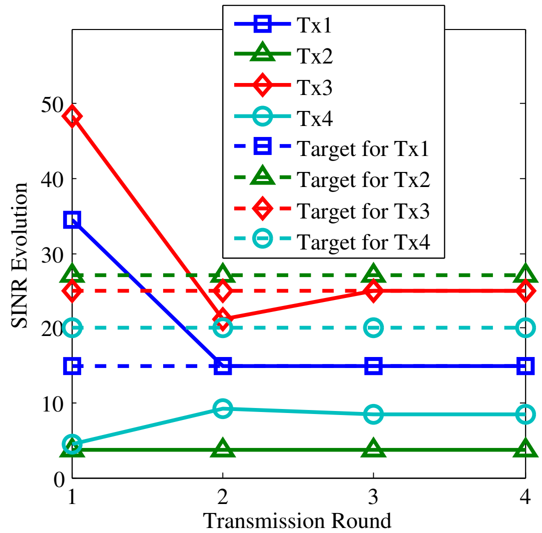
In the 4^{th} round (Fig. 3.3b, Fig. 3.3d), Tx_2 makes an offer to Tx_4 that is accepted. Tx_4 reduces its power to the level so that Tx_2 achieves its target at the next iteration. Therefore, the algorithm stops and a (N-1)-efficient state arises.

3.4 DBFM vs. Trunc FM

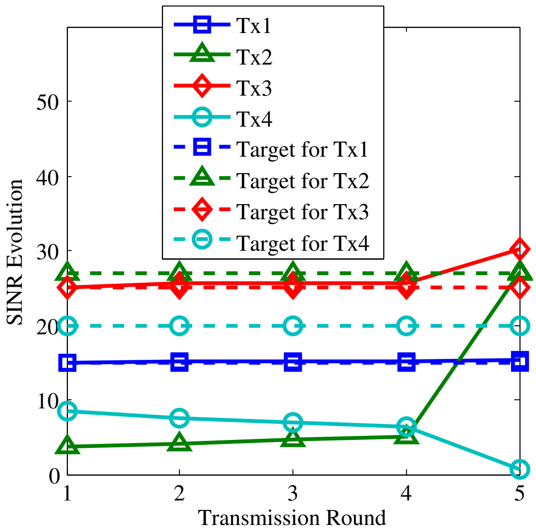
We then compare DBFM with the Trunc(ated) FM power control algorithm. In Fig. 3.3c, Tx_2 is the one who powers off, since it is farthest from its target at the NE. Then, the SINRs of all other nodes are improved (to notice this, just compare the SINR values at the 1^{st} round of Trunc FM and the 5^{th} round of FM). However, after two more iterations, Trunc FM stops. Indeed, as Fig. 3.3e shows, the powers at 2^{nd} and 3^{rd} round remain invariable. However, this state is not (N-1)-efficient, as, although Tx_2 powers off, Tx_4 remains below its target.

This small example illustrates how powerful the integration of power control and bargaining can be. Though Trunc FM has forced Tx_2 to power off, this is not sufficient so that Tx_4 achieves its target. On the other hand, with our scheme, we give the opportunity to Tx_2 to achieve its target.

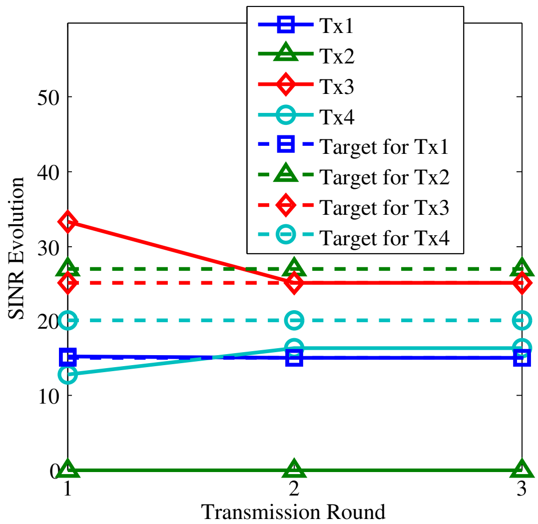
We finally compare the number of (N-1)-efficient states that DBFM and Trunc FM find. We use the simulation parameters from Table 3.2. As shown in Fig. 3.4, DBFM slightly



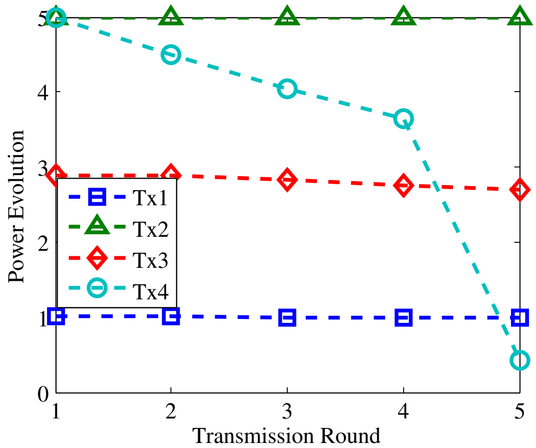
(a) FM



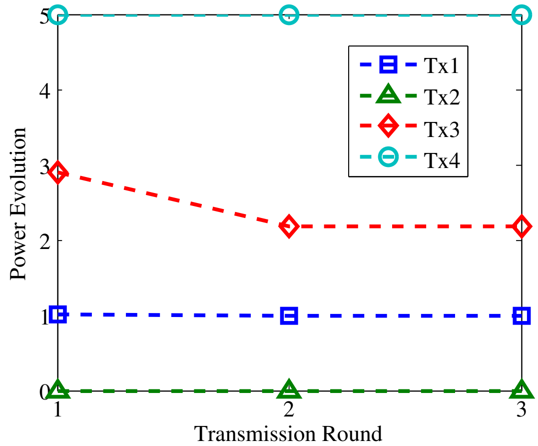
(b) DBFM SINR



(c) Trunc FM SINR



(d) DBFM Power



(e) Trunc FM Power

Fig. 3.3: SINR and power evolution of nodes after the application of FM, DBFM, and Trunc FM. Horizontal dashed lines correspond to the targets of the nodes.

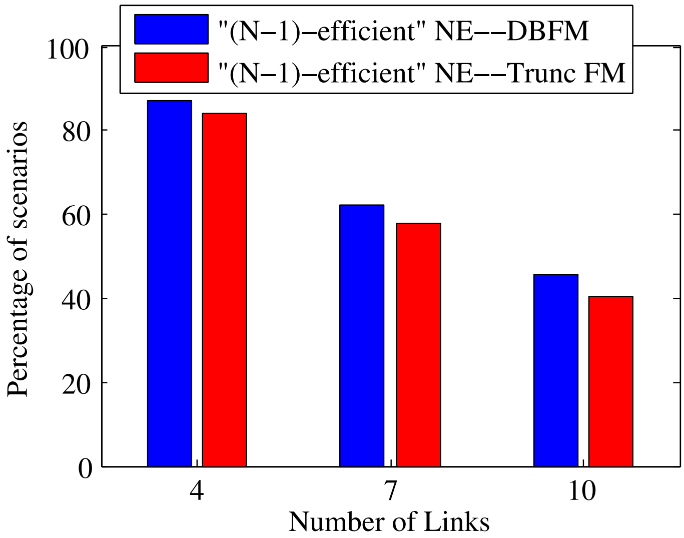


Fig. 3.4: Percentage of (N-1)-efficient states after the application of both DBFM and Trunc FM for 1000 scenarios where FM did not lead to a NE with all nodes achieving their targets.

outperforms Trunc FM (and this becomes clearer as the number of entities increases), even if it does not force an entity to power off. Therefore, our scheme is a preferable choice both in theory and in practice than Trunc FM.

3.5 DBFM vs. Utility-Based Power Control

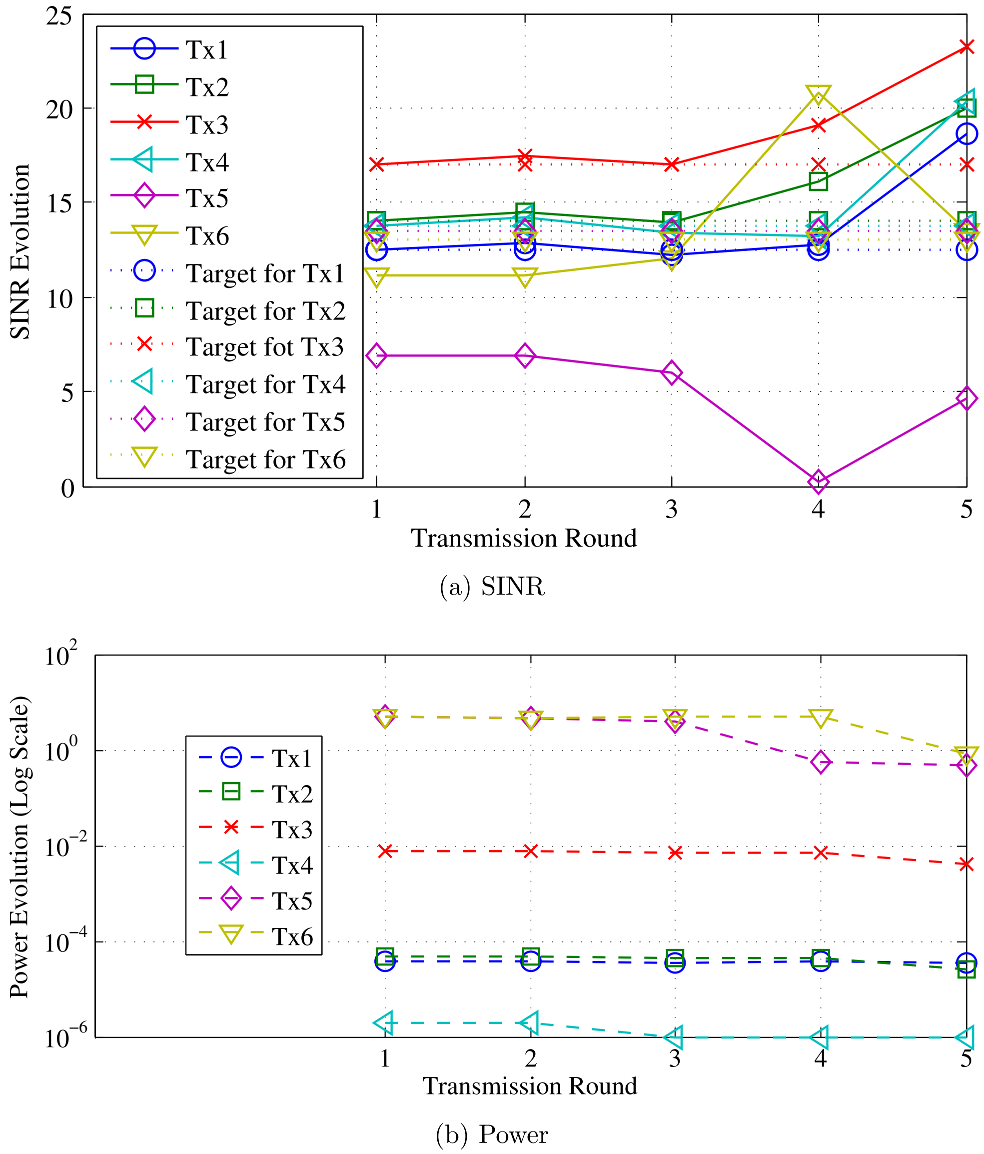
In this section, we compare DBFM with the utility-based power control (UBPC) scheme [53] for the same topology and path loss model that the authors of [53] have used in their simulations. There are six transmitter-receiver pairs, where each one should satisfy the respective SINR targets (units are in dB): 12.5, 14, 17, 13.75, 13.5, and 13. As previously, we look for a (N-1)-efficient state.

Table 3.4 presents the negotiations for DBFM. N/A means that there is no available Seller. As discussed previously, when the Buyer's offer is rejected, the Buyer reduces its power by 10%. Fig. 3.5a shows the SINR evolution and Fig. 3.5b the power evolution in logarithmic scale. We notice that, after the 2$^{\text{nd}}$ round, Tx_1 and Tx_4 fall below their targets, though they had achieved their targets at the last state of FM. This happens as FM does not provide an "active node protection" mechanism as that of [39], so that a node that achieves its target would not necessarily retain it during the next iterations of the algorithm.

Another observation concerns the SINR evolution after the 3$^{\text{rd}}$ round. We can see that Tx_2 and Tx_3 overcome their targets without having offered any reward. In addition, the SINR of Tx_6 is two times its target, even though Tx_6 has asked for the minimum reduction needed (19.38%) to simply reach its target. This happens because Tx_1 asked for a bigger reduction (86.74%) and since both offers got accepted, the SINR of Tx_6 is greatly increased.

Table 3.4: Negotiations among nodes.

Round	Buyer	Corresponding Seller	Buyer's Offer	Buyer's Request for % Power Reduction	Seller's Decision
1	Tx_5 Tx_6	N/A N/A	NO OFFER NO OFFER	NO OFFER NO OFFER	N/A N/A
2	Tx_5	Tx_6	171.64	77.99	NO
3	Tx_1 Tx_4 Tx_6	Tx_5 Tx_5 Tx_5	819.37 NO OFFER 156.24	86.74 NO OFFER 19.38	YES NO YES
4	Tx_5	Tx_4	NO OFFER	NO OFFER	NO



(a) SINR

(b) Power

Fig. 3.5: SINR and power evolution after the application of DBFM.

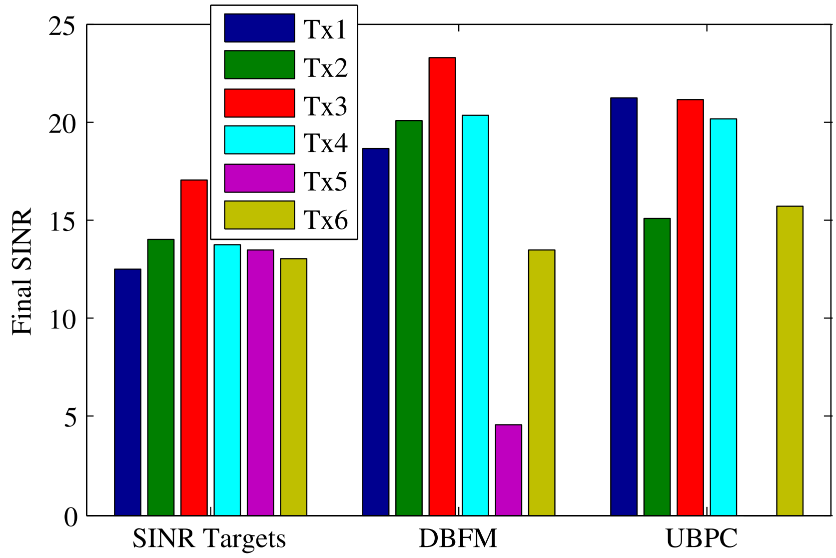


Fig. 3.6: Comparison of the SINR that arises after the application of DBFM and UBPC. In both cases, 5 out of 6 nodes achieve their SINR targets. DBFM leads to a better SINR for 4 out of 6 nodes.

This is due to the fact that each successful bargaining causes a positive side effect even to nodes that do not take part in any negotiation, since each of them can increase its SINR as well. However, this positive side effect is, in general, not sufficient for an unsatisfied node to achieve its target.

In Fig. 3.6, we compare the performance of UBPC vs. DBFM. We have used the parameters for UBPC that have proposed the authors in [53]. Both algorithms lead to an (N-1)-efficient solution; DBFM outperforms UBPC as 4 out of the 6 nodes achieve higher SINR, even though DBFM does not enforce a node to power off; the reason is that negotiations among nodes provide the opportunity to lead to more efficient operating points.

3.6 On the Fairness of DBFM

Up to now, we have studied the results of the proposed schemes focusing on a single transmission round. However, nodes are expected to coexist in the same topology for longer intervals. This implies that a power control scheme should be fair in the following sense: if the same set of entities with the same targets apply the proposed scheme continuously, the set of entities that satisfy their targets should vary over time. Unfortunately, Trunc FM and UBPC are by design unfair, as they always penalize the weakest node, which will never have the opportunity (not even to try) to transmit.

We propose that during the $(m+1)^{\text{th}}$ transmission round, nodes reset all their parameters to the last state of FM except their budgets, which are the ones that arise after the

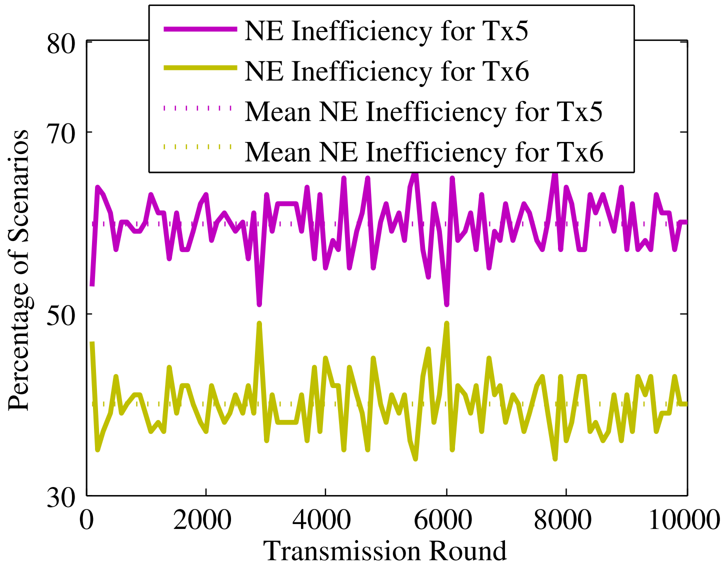


Fig. 3.7: We apply DBFM for the same set of nodes, by resetting their parameters to the last state of FM. The budget at the $(m+1)^{\text{th}}$ round is the one that arises at the end of the m^{th} round. For every period of 100 transmission rounds, we count how many times Tx_5 and Tx_6 do not achieve their targets.

application of DBFM at the m^{th} transmission round. So, the rewards that the unsatisfied nodes may have collected during negotiations of previous rounds could be used to increase their chances to make an offer that will get accepted during a following round.

Fig. 3.7 presents the results of DBFM for the topology of the previous section for 10000 transmission rounds. The initial budget is set to 1000 units for all nodes. Every period of 100 transmission rounds, we count the number of times that each node fails to achieve their target. In our example, these are Tx_5 and Tx_6. The numbers of their respective transmissions have an an average ratio 3:2 per period, *i.e.*, 60 out of 100 transmission rounds Tx_5 powers off, whereas the remaining 40 Tx_6 powers off. Most importantly, simulations reveal that the rotation between the two transmissions are frequent, so that our scheme promotes fairness even in short time scales. This is an important advantage of DBFM, as all nodes regularly and frequently get the opportunity to transmit their data.

This behaviour is due to the dynamically adjusting mechanism that nodes follow when they either make or evaluate an offer. Each Buyer i computes its offer in terms of percentage of its current budget, not as an absolute value of the form "I offer up to X units to ask for a reduction up to $Y\%$". Each Seller j follows the same strategy too. This is a fair system as: *(i)* Each Buyer offers the same percentage of its budget when it asks for the same percentage reduction. *(ii)* Each Seller rejects/accepts an offer based on the reward that it would have offered had it asked for the same power reduction. Observe that the exclusive use of percentages means that the operation of the scheme does not depend on the absolute values of the initial budgets, but only their relative sizes.

3.7 Conclusions

In modern wireless heterogeneous networks, distributed schemes for efficient spectrum management are a prerequisite for their successful deployment. Our work provides a solution to a problem that arises very often in such networks, *i.e.*, how to increase the number of entities that can achieve their QoS targets. Through bargaining, nodes create incentives to other nodes to reduce their powers, leading to operating points that are more preferable than the NE point without bargaining. Simulations show that our distributed approach is efficient and fair, promoting the smooth coexistence of the nodes.

Chapter 4

Non-Cooperative Power Control in Two-Tier Small Cell Networks

4.1 Introduction and Motivation[1]

As discussed in Chapter 1, the demand for mobile data is currently increasing with a tremendous rate, with about 80% of the traffic generated indoors (mostly at home or at the office) [66]. A major challenge for mobile operators is to continue to provide excellent data experience indoors given this significant growth of data traffic. However, a prerequisite for excellent indoor data traffic is excellent signal strength. New wireless cellular standards, such as 3GPP High Speed Packet Access (HSPA) and Long Term Evolution (LTE) achieve considerable improvements in system capacity and throughput, but at the cost of high operational expenses and capital expenditures. A way to solve this problem is to deploy, in addition to standard cells, termed *macrocells* in our context, a large number of smaller and cheaper cells which are called *small cells* and connect to the mobile operator network using residential DSL or cable broadband connections [6]. Small cells are expected to be a key feature of 5G networks, where all cells will be self-organizing [4].

Indoor users that are connected to small cells experience superior indoor reception and achieve better data rates than the users that are connected to macrocells. Often, this is achieved with low transmission power, so that battery life prolongation is also achieved. Such networks, comprised of a conventional macrocell network overlaid with a number of small cell base stations are referred to as *two-tier small cell networks.*

One of the biggest challenges for the successful deployment of these networks is mitigating the interference that small cell nodes cause to macrocell nodes (and vice versa) when they share the same frequency bands (which is the typical case) [67]. If the level of

[1]This chapter is based on paper [65].

interference is not controlled, the deployment of two-tier small cell networks is problematic. Observe that cellular networks have been dimensioned without taking into account the future existence of small cells, and therefore it is imperative that their mobile nodes be protected. Consequently, the adoption of radio resource management techniques is of crucial importance in alleviating the problems of the additional interference that arises in these networks.

Moreover, in such networks, where both macrocells and small cells are present, entities have heterogeneous targets and needs. Providing schemes that, depending on the entity, will focus either on voice or on data services remains an important open topic. In our work, we assume that macrocell (traditional) nodes are mostly interested in making voice calls. On the other hand, small cell nodes focus on data services.

The goal of this chapter is to study the above challenging scenario under the prism of non-cooperative game theory. Contrary to typical formulations that use a single utility function, we capture the different behaviours of the nodes by defining different utility functions. Our contributions are the following:

- For the above defined game, we prove the existence and derive conditions for the uniqueness of a Nash Equilibrium (NE).
- We propose a distributed power control scheme that, based on the best response dynamics method [68], converges to the unique NE.
- We show through simulations that, with our scheme, nodes can efficiently coexist, achieving their performance targets in the vast majority of simulated scenarios.

4.2 System Model

We study a CDMA network that consists of N_1 macrocell mobile nodes (MNs) and N_2 small cell mobile nodes (SCMNs) that coexist in the same area as described in Section 1.5. We focus on the uplink and we assume a closed-access model [6]. This means that each small cell base station (SCBS) may associate only with predefined SCMNs and no MNs can connect to it. In Table 4.1, we present the non-cooperative game formulation of this setup.

In this game G, each MN_i updates its transmission power P_i that belongs to $[0, P_{\max}]$ aiming at maximizing its utility function which is a logarithmic function of the SINR.We restate in (4.1) the SINR definition having explicitly included the spread factor of the CDMA network, denoted by L. We also define as $R_i = \sum_{j\neq i}^{N} G_{ji}P_j + n$ the total interference plus noise that node i receives:

$$\gamma_i \triangleq \mathrm{SINR}_i = L\frac{G_{ii}P_i}{\sum_{j\neq i}^{N} G_{ji}P_j + n} = L\frac{G_{ii}P_i}{R_i}. \tag{4.1}$$

Table 4.1: Game formulation.

Set of players	Set of MNs $\mathbf{N}_1 = \{1, 2, \ldots, N_1\}$	Set of SCMNs $\mathbf{N}_2 = \{1, 2, \ldots, N_2\}$
Strategy of player i	$P_i \in [0, P_{\max}]$	$P_i \in [0, \mathrm{SCP}_{\max}]$
Utility function for player i	$U_i = B_i \log(1 + \gamma_i)$, where $\gamma_i \leq \gamma_i^t$	$U_i = B_i \log(1 + \gamma_i) - c_i P_i$

The utility function we use (see Table 4.1) can be interpreted as being proportional to the Shannon capacity and is weighted by a positive player-specific parameter B_i that corresponds to the player's desire for SINR. Moreover, there is one constraint: The SINR of player i should belong to the interval $(0, \gamma_i^t]$.

On the other hand, each SCMN_i updates its power P_i such that it belongs to a different interval $[0, \mathrm{SCP}_{\max}]$ and uses a different utility function. Clearly, according to the current state-of-the-art, $\mathrm{SCP}_{\max} < P_{\max}$. Apart from the value part (which is the same with the one of a macrocell node), there is also a cost part, which is a linear function of P_i and reflects a price that player i has to pay for using a specific amount of power. This utility function is inspired by the one proposed in [33].

The reason that we choose different objective functions for each category of players is the following: Macrocell nodes have a higher priority to be served by the mobile operators, as they will be mostly used for inelastic, voice traffic. For this reason, they are not interested in having an SINR higher than γ_i^t. They can use any power up to $P_{\max}$ (without paying for their choice) to overcome the extra interference that is caused by the small cell nodes. On the other hand, small cells are deployed by indoor users for their own interest. Consequently, a pricing policy is applied to discourage them from creating high interference to the macrocell nodes. However, as small cells have generally higher demands for QoS, there is no maximum SINR constraint for them.

As a final comment, we point out that the idea of using different objective functions for small cell and macrocell nodes has already been proposed in [69]. However, the approach there is based directly on the SINR. The authors demand that the SINR of each node i belongs to an interval $[\mathrm{minSINR}_i, \mathrm{maxSINR}_i]$, and in case that the targets of macrocell nodes cannot be achieved, small cell nodes are obliged to adjust their targets to the interval $[k \cdot \mathrm{minSINR}_i, k \cdot \mathrm{maxSINR}_i]$, where $0 < k < 1$. Other power control approaches in small cells are reviewed in [70].

Finally, after our work in [65] that is summarized in this chapter, there have been published two works that share a similar vantage point: [71], [72]. In [71], Tsiropoulou et al. present a non-cooperative power control scheme focusing on the uplink. They also propose different utility functions to model the preferences of the heterogeneous users. However, contrary to our work, they build upon [52]. They adopt similar utility functions (which are somehow artificial, as we have discussed in Chapter 2) so as to formulate a supermodular game. In [72], Huang et al. focus on the downlink and adopt a variation of the utility function that we have used for the small cells nodes. Instead of using as a pricing function a linear function of the transmission power, they apply a linear function of the sum of the interference that each SCBS causes to the neighbouring nodes. They show that the scheme admits a NE, however conditions on the uniqueness are missing.

4.3 Existence of a NE in the Two-Tier Small Cell Network Game

To prove that the game G has at least one NE, we use the following theorem by Debreu-Fan-Glicksberg (1952) [73]:

Theorem 1. *Let G be a strategic non-cooperative game. Suppose that $\forall i \in \mathbf{N} = \{1, 2, ..., N\}$:*

- *The strategy set S_i is compact and convex.*
- *The utility $U_i(\mathbf{s})$, where $\mathbf{s} = [s_1, s_2, \ldots, s_N]^T$, is continuous in $\mathbf{s}$ and quasi-concave in s_i.*

Then the game G has at least one NE.

Theorem 2 (Existence of a NE)**.** *The two-tier small cell network game G that was defined in the previous section has at least one NE.*

Proof. We distinguish two cases. In case 1, let player i be a SCMN. Each SCMN$_i$ has a strategy set $P_i \in [0, \mathrm{SCP}_{\max}]$, which is compact and convex. The utility function $U_i(P_i, \mathbf{P}_{-i}) = B_i \log(1 + \gamma_i) - c_i P_i$, is continuous in $\mathbf{P} = [P_1, P_2, \ldots, P_N]^T$. It is also twice differentiable so we can take the second order partial derivative with respect to P_i.

$$\frac{\partial^2 U_i}{\partial P_i^2} = -B_i \left(\frac{G_{ii}}{R_i}\right)^2 \frac{\left(1 + \frac{G_{ii} P_i}{R_i}\right)^2}{1 + \frac{G_{ii}}{R_i}}. \tag{4.2}$$

As the second order partial derivative with respect to P_i is negative, the function $U_i(P_i, \mathbf{P}_{-i})$ is concave in P_i, hence quasi-concave [74]. Therefore, we prove that both conditions of Theorem 1 hold for each SCMN$_i$.

In case 2, let player i be a MN. Similarly, the strategy set $[0, P_{\max}]$ is a compact and convex set. The utility function $U_i(P_i, \mathbf{P}_{-i}) = B_i \log(1 + \gamma_i)$ is continuous in $\mathbf{P}$ and concave in P_i (it coincides with (4.2)). So, all conditions of Theorem 1 hold for each MN too.

Consequently, our game G has at least one NE. □

Table 4.2: Application of best responses in a two-players game. Numbers in cells correspond to the utility of each player.

Player$_1$ \ Player$_2$	Bach	Stravinsky
Bach	(2,2)	(0,0)
Stravinsky	(0,0)	(1,1)

4.4 Best Response Dynamics Schemes

Given the fact that we know that a game has a NE, how can we devise an algorithm that converges to a NE? We shall present the fundamentals of best response dynamics schemes, which may lead to a NE.

Definition 3. *Let G be a strategic non-cooperative game. The best response strategy of player i is the one that maximizes its utility, taking other players' strategies as given.*

An equivalent definition of the NE incorporates the notion of best response:

Definition 4. $\mathbf{s} = [s_1, s_2, \ldots, s_N]^T$ *is a NE of a game G with N players iff every player's strategy is a best response to the other players' strategies.*

The idea of best response is useful when we are trying to find an approach to reach a NE of a game. A best response dynamics scheme consists of a sequence of rounds, where in each round after the first, each player i chooses the best response to the other players' strategies in the previous round. In the first round, the choice of each player is the best response based on its arbitrary belief about what the other players will choose.

In some games, the sequence of strategies generated by best response dynamics converges to a NE, regardless of the players' initial strategies. However, this does not hold in general. A nice counter-example is presented in Table 4.2 [30]. Let us suppose that, at round 1, Player$_1$ believes that Player$_2$ will choose Bach, whereas Player$_2$ believes that Player$_1$ will choose Stravinsky. So, Player$_1$ will choose Bach as best response to that belief and Player$_2$ will choose Stravinsky correspondingly. So, at round 1, they will play (Bach, Stravinsky) and the utilities will be $(0, 0)$. At round 2, the best responses to round 1 will lead to (Stravinsky, Bach) and the utilities will be $(0, 0)$. So, the choices will infinitely switch from (Bach, Stravinsky) to (Stravinsky, Bach) and vice versa. Players will never reach one of the two NE of the game, *i.e.*, (Bach, Bach), (Stravinsky, Stravinsky).

4.5 Power Control under Best Response Dynamics

Although the adoption of the best response dynamics scheme is neither a necessary nor a sufficient condition for reaching a NE, it has been used as the basis for distributed power control schemes in many cases [73], [75]. We shall adopt it here as well, exploring the conditions that guarantee its convergence.

We can see our game G as a collection of N parallel optimization problems, where each (SC)MN aims at maximizing its own utility function U_i (equivalently, minimizing $-U_i$) with no interest for the others. We shall pose these optimization problems and solve them with the use of Karush-Kuhn-Taker (KKT) conditions [74].

The minimization problem of MN_i is defined as follows:

$$\min_{P_i} -U_i(P_i, \mathbf{P}_{-i}) = \min\{-B_i \log(1+\gamma_i)\},$$
$$\text{subject to: } 0 \le P_i \le P_{\max} \text{ and } \gamma_i \le \gamma_i^t.$$

The constraints can be rewritten as:

$$-P_i \le 0, \quad P_i \le P_{\max}, \quad L\frac{G_{ii}P_i}{R_i} \le \gamma_i^t.$$

The KKT conditions are:

$$-\lambda_1 P_i = 0, \quad \lambda_2(P_i - P_{\max}) = 0,$$
$$-\lambda_3\left(L\frac{G_{ii}P_i}{R_i} - \gamma_i^t\right) = 0, \quad -B_i L\frac{G_{ii}}{R_i + G_{ii}P_i} - \lambda_1 + \lambda_2 + \lambda_3 L\frac{G_{ii}}{R_i} = 0,$$
$$\lambda_i \ge 0, \quad i = \{1, 2, 3\}.$$

The objective function and the inequality constraints functions are differentiable convex functions. Therefore, the KKT conditions are necessary and sufficient conditions for having primal and dual optimality [74]. By solving the system of the KKT conditions, we get the optimal power $P_i^\star$:

$$P_i^\star = \min\left\{P_{\max}, \gamma_i^t\frac{R_i}{LG_{ii}}\right\}. \tag{4.3}$$

Therefore, we arrive at the well-known Simplified Foschini-Miljanic formula with a $P_{\max}$ constraint [41]. However, the key difference is that, contrary to [41], where each node's utility value is either 0 (when the γ_i^t target is not achieved) or 1 (when the target is achieved), each node gets a non-zero value even if it has not achieved its target.

The minimization problem of SCMN_i is defined as follows:

$$\min_{P_i} -U_i(P_i, \mathbf{P}_{-i}) = \min\{c_i P_i - B_i \log(1+\gamma_i)\},$$
$$\text{subject to: } 0 \le P_i \le \mathrm{SCP}_{\max}.$$

The constraints can be rewritten as:

$$-P_i \leq 0, \quad P_i \leq \mathrm{SCP}_{\max}, \quad L\frac{G_{ii}P_i}{R_i} \leq \gamma_i^t.$$

The KKT conditions are:

$$\begin{aligned} -\lambda_1 P_i = 0, \quad \lambda_2(P_i - \mathrm{SCP}_{\max}) = 0, \\ -\lambda_3\left(L\frac{G_{ii}P_i}{R_i} - \gamma_i^t\right) = 0, \quad -B_i L\frac{G_{ii}}{R_i + G_{ii}P_i} - \lambda_1 + \lambda_2 + \lambda_3 L\frac{G_{ii}}{R_i} = 0, \\ \lambda_i \geq 0, \quad i = \{1,2,3\}. \end{aligned}$$

The objective function and the inequality constraints functions are differentiable convex functions. By solving the system of the above equations, we can compute the optimal power $P_i^\star$:

$$P_i^\star = \max\left\{0, \min\left\{\frac{B_i}{c_i} - \frac{R_i}{LG_{ii}}, \mathrm{SCP}_{\max}\right\}\right\}. \tag{4.4}$$

We then present the pseudocode of Algorithm 2 which is based on an iterative application of the equations (4.3) and (4.4).

Algorithm 2 Power control under best response dynamics for a two-tier small cell network

for $k = 1 \rightarrow$ MAX_NUMBER_OF_ITERATIONS **do**
  each (SC)MN$_i$ passes to its associated (SC)BS$_i$ the level of the total received power $R_i = \sum_{j \neq i}^{N} G_{ji}P_j + n$.
  each (SC)MN$_i$ computes the quantity $\gamma_i = L\frac{G_{ii}P_i(k)}{R_i(k)}$.
  if i is a macrocell node **then**
    it updates its power at round k+1 according to (4.3), as follows:

$$P_i(k+1) = \min\left\{P_{\max}, \gamma_i^t \frac{R_i(k)}{LG_{ii}}\right\}. \tag{4.5}$$

  end if
  if i is a small cell node **then**
    it updates its power according to (4.4), as follows:

$$P_i(k+1) = \max\left\{0, \min\left\{\frac{B_i}{c_i} - \frac{R_i(k)}{LG_{ii}}, \mathrm{SCP}_{\max}\right\}\right\}. \tag{4.6}$$

  end if
  if $\forall i \in \mathbf{N} : \|P_i(k+1) - P_i(k)\| \leq \epsilon$ **then** $\triangleright$ ϵ : a small positive quantity.
    break;
  end if
end for

Observe from Algorithm 2, that each (SC)MN$_i$ needs to know the following elements to update its power:

1. its power at the previous transmission round k,
2. the values of the parameters L, G_{ii},
3. the total interference that it has received at the previous transmission round,
4. (if it is a MN) its target γ_i^t,
5. (if it is a SCMN) the values of the parameters B_i and c_i.

Elements 1, 2, 4, and 5 are known to (SC)MN$_i$; element 3 can be passed by the associated receiver of node i. Therefore, Algorithm 2 is a fully distributed scheme. We also mention that Algorithm 2 is a synchronous scheme, in the sense that (SC)MNs should update their powers concurrently. Note that it works even with asynchronous updates, provided that each (SC)MN measures its level of interference at round k and updates its power in the semi-open time interval $[k, k+1)$.

4.6 Uniqueness of the NE for Two-Tier Small Cell Network

In this section, we prove that Algorithm 2 converges to a NE and this is the unique NE of the game. Mathematically, the uniqueness of a NE is equivalent to proving the existence of a unique *fixed point*. Given a function $f(x)$, c is a fixed point of the function $f(x)$ if and only if $f(c) = c$. We restate the following notions from distributed optimization [76], which will be useful in the rest of this section.

Definition 5. *Let $M(\cdot)$: $X \to X$ be a mapping and $\mathbf{x}^\star \in X$ be a fixed point. M is a pseudo-contraction mapping with respect to some norm $\|\cdot\|$ if there exists $k \in [0, 1)$ so that*

$$\|M(\mathbf{x}) - \mathbf{x}^\star\| \leq k\|\mathbf{x} - \mathbf{x}^\star\|, \quad \forall \mathbf{x} \in X.$$

The difference from a contraction mapping is that, in a pseudo-contraction mapping, $\mathbf{x}^\star$ is fixed.

Theorem 3. *Let $X \subset \Re^n$ and the mapping $M(\cdot) : X \to X$ be a pseudo-contraction with a fixed point $\mathbf{x}^\star \in X$, i.e., $M(\mathbf{x}^\star) = \mathbf{x}^\star$. Then M has no other fixed points and the sequence $\{\mathbf{x}(k)\}$ generated by $\mathbf{x}(k+1) = M(\mathbf{x}(k))$ converges to $\mathbf{x}^\star$.*

Let $T_i(k) = G_{ii}P_i(k)$ be the received power from the transmission of (SC)MN$_i$ at time k. Equations (4.5) and (4.6) can be rewritten as:

$$\text{MN received power: } T_i(k+1) = \min\left\{T_{i,\max}, \gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L}\right\}. \tag{4.7}$$

$$\text{SCMN received power: } T_i(k+1) = \max\left\{0, \min\left\{\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}, \mathrm{SCT_{i,max}}\right\}\right\}. \tag{4.8}$$

Similarly, the received power level at the NE $T_i^\star$ can be rewritten as:

$$\text{MN NE received power: } T_i^\star = \min\left\{T_{i,\max}, \gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L}\right\}. \tag{4.9}$$

$$\text{SCMN NE received power: } T_i^\star = \max\left\{0, \min\left\{\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}, \mathrm{SCT_{i,max}}\right\}\right\}. \tag{4.10}$$

Let also

$$\Delta T_i(k) = T_i(k) - T_i^\star \tag{4.11}$$

be the distance between the received power from the transmission of (SC)MN$_i$ at time k and the received power level at the NE. We state the following theorem:

Theorem 4. *Let* $\mathbf{N} = \{1, 2, ..., N\}$ *be the set of players in the two-tier small cell network game. The following inequalities hold* $\forall i \in \mathbf{N}$*:*

- *If i is a MN, then:*

$$|\Delta T_i(k+1)| \leq \left|\gamma_i^t \frac{1}{L} \sum_{j\neq i}^{N} \Delta T_j(k)\right|.$$

- *If i is a SCMN, then:*

$$|\Delta T_i(k+1)| \leq \left|\frac{1}{L} \sum_{j\neq i}^{N} \Delta T_j(k)\right|.$$

Proof. The proof is based on the examination of all possible combinations for the form of the pair $(T_i(k+1), T_i^\star)$. For each combination, we use properties of the absolute value.

- Let i be a MN. We distinguish 4 cases:

  Case 1:

$$T_i(k+1) = \gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L}, \quad T_i^\star = \gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L}.$$

Then:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = \left|\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L} - \gamma_i^t \frac{n}{L} - \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L}\right| =$$
$$= \left|\gamma_i^t \frac{\sum_{j\neq i}(T_j(k) - T_j^\star)}{L}\right| = \gamma_i^t \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.$$

Case 2:

$$T_i(k+1) = T_{i,\max}, \quad T_i^\star = \gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L}.$$

From (4.7) and (4.9) we get:

$$\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L} > T_{i,\max}, \quad 0 < \gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L} \leq T_{i,\max}.$$

So:

$$\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L} - \left(\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L}\right) \geq T_{i,\max} - \left(\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L}\right). \quad (4.12)$$

By using (4.12) we get:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = \left|T_{i,\max} - \gamma_i^t \frac{n}{L} - \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L}\right| \leq$$
$$\leq^{(4.12)} \left|\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L} - \gamma_i^t \frac{n}{L} - \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L}\right| =^{\text{Case1}} \gamma_i^t \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.$$

Case 3:

$$T_i(k+1) = \gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L}, \quad T_i^\star = T_{i,\max}.$$

From (4.7) and (4.9) we get:

$$\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L} > T_{i,\max}, \quad 0 < \gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L} \leq T_{i,\max}.$$

So:

$$\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j T_j^\star}{L} - \left(\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L}\right) \geq T_{i,\max} - \left(\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L}\right). \quad (4.13)$$

By using (4.13) we get:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = \left|\gamma_i^t \frac{n}{L} - \gamma_i^t \frac{\sum_{j\neq i} T_j(k) - T_{i,\max}}{L}\right| \leq$$
$$\leq^{(4.13)} \left|\gamma_i^t \frac{n}{L} + \gamma_i^t \frac{\sum_{j\neq i} T_j(k)}{L} - \gamma_i^t \frac{n}{L} - \gamma_i^t \frac{\sum_{j\neq i} T_j^\star}{L}\right| =^{\text{Case1}} \gamma_i^t \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.$$

Case 4:

$$T_i(k+1) = T_{i,\max}, \quad T_i^\star = T_{i,\max}.$$

Then:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = |T_{i,\max} - T_{i,\max}| = 0.$$

After examining all possible cases, we find that:

$$|\Delta T_i(k+1)| \le \left| \gamma_i^t \frac{1}{L} \sum_{j\neq i}^{N} \Delta T_j(k) \right|.$$

- Let i be a SCMN. We distinguish 9 cases:

Case 1:

$$T_i(k+1) = \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}, \quad T_i^\star = \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}.$$

Then:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = \left| \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \frac{G_{ii}B_i}{c_i} + \frac{n}{L} + \frac{\sum_{j\neq i} T_j^\star}{L} \right| =$$

$$= \left| \frac{\sum_{j\neq i}(T_j(k) - T_j^\star)}{L} \right| = \frac{1}{L} \left| \sum_{j\neq i} (\Delta T_j(k)) \right|.$$

Case 2:

$$T_i(k+1) = 0, \quad T_i^\star = \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}.$$

From (4.8) and (4.10) we get:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} < 0, \quad \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} \ge 0.$$

So:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} \right) > \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}. \tag{4.14}$$

By using (4.14) we get:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = \left| 0 - \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} \right| \le$$

$$\le^{(4.14)} \left| \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} \right) \right| =$$

$$=^{\text{Case1}} \frac{1}{L} \left| \sum_{j\neq i} (\Delta T_j(k)) \right|.$$

Case 3:

$$T_i(k+1) = SCT_{i,\max}, \quad T_i^\star = \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}.$$

From (4.8) and (4.10) we get:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} > \mathrm{SCT}_{\mathrm{i,max}}, \quad 0 \leq \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} \leq \mathrm{SCT}_{\mathrm{i,max}}.$$

So:

$$\begin{aligned} \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right) \geq \\ \geq \mathrm{SCT}_{\mathrm{i,max}} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right). \end{aligned} \tag{4.15}$$

By using (4.15) we get:

$$\begin{aligned} |\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = \left|\mathrm{SCT}_{\mathrm{i,max}} - \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right| \leq \\ \leq^{(4.15)} \left|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}\right)\right| = \\ =^{\mathrm{Case1}} \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|. \end{aligned}$$

Case 4:

$$T_i(k+1) = \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}, \quad T_i^\star = 0.$$

From (4.8) and (4.10) we get:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} \geq 0, \quad \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} < 0.$$

So:

$$0 \leq \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} < \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right). \tag{4.16}$$

By using (4.16) we get:

$$\begin{aligned} |\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = \left|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - 0\right| < \\ <^{(4.16)} \left|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}\right)\right| = \\ =^{\mathrm{Case1}} \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|. \end{aligned}$$

Case 5:

$$T_i(k+1) = 0, \quad T_i^\star = 0.$$

Then:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = |0-0| = 0.$$

Case 6:

$$T_i(k+1) = \mathrm{SCT_{i,max}}, \quad T_i^\star = 0.$$

From (4.8) and (4.10) we get:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} > \mathrm{SCT_{i,max}}, \quad \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} < 0.$$

So:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right) > \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}. \quad (4.17)$$

By using (4.17) we get:

$$\begin{aligned}|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = |\mathrm{SCT_{i,max}} - 0| < \left|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}\right| < \\ <^{(4.17)} \left|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right)\right| = \\ =^{\mathrm{Case1}} \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.\end{aligned}$$

Case 7:

$$T_i(k+1) = \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}, \quad T_i^\star = \mathrm{SCT_{i,max}}.$$

From (4.8) and (4.10) we get:

$$0 \leq \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} \leq \mathrm{SCT_{i,max}}, \quad \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} > \mathrm{SCT_{i,max}}.$$

So:

$$\begin{aligned}\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right) \geq \\ \geq \mathrm{SCT_{i,max}} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right).\end{aligned} \quad (4.18)$$

By using (4.18) we get:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = \left|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \mathrm{SCT_{i,max}}\right| \leq$$
$$\leq^{(4.18)} \left|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right)\right| =$$
$$=^{\mathrm{Case1}} \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.$$

Case 8:

$$T_i(k+1) = 0, \quad T_i^\star = \mathrm{SCT_{i,max}}.$$

From (4.8) and (4.10) we get:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} < 0, \quad \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} > \mathrm{SCT_{i,max}}.$$

So:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}\right) > \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}. \quad (4.19)$$

By using (4.19) we get:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = |0 - \mathrm{SCT_{i,max}}| < \left|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right| <$$
$$<^{(4.19)} \left|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right)\right| =$$
$$=^{\mathrm{Case1}} \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.$$

Case 9:

$$T_i(k+1) = \mathrm{SCT_{i,max}}, \quad T_i^\star = \mathrm{SCT_{i,max}}.$$

Then:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = |\mathrm{SCT_{i,max}} - \mathrm{SCT_{i,max}}| = 0.$$

After examining all possible cases, we find that:

$$|\Delta T_i(k+1)| \leq \left|\frac{1}{L}\sum_{j\neq i}^{N} \Delta T_j(k)\right|.$$

□

Theorem 5 (uniqueness of the NE for the two-tier small cell network game)**.** *Let L be the spread factor of the system and $\gamma_{\max} = \max_{i \in \mathbf{N}} \gamma_i^t$. If $N < \max\left\{\frac{L}{\gamma_{\max}} + 1, L+1\right\}$, then:*

- *The two-tier small cell network game has a unique NE.*
- *The power control scheme under best response dynamics of Algorithm 2 for SCMNs and MNs converges to this NE.*

Proof. We introduce the N-size vector $\mathbf{\Delta T}$ that contains all the parameters ΔT_i and we take the maximum norm of that vector. By using Theorem 3 and Theorem 4, we can prove the existence of a pseudo-contraction under the above condition and the convergence of Algorithm 2 to a unique fixed point (*i.e.*, a NE). As the best response dynamics scheme is a pseudo-contraction in the entire strategy space, this is the unique NE of the two-tier small cell network game [77].

In more detail, let

$$\mathbf{\Delta T} = [\Delta T_{\mathrm{MN\_1}}, \Delta T_{\mathrm{MN\_2}}, \cdots, \Delta T_{\mathrm{MN\_N1}}, \Delta T_{\mathrm{SCMN\_1}}, \Delta T_{\mathrm{SCMN\_2}}, \cdots, \Delta T_{\mathrm{SCMN\_N2}}]^T$$

be a N-size vector. Its maximum norm $\|\mathbf{\Delta T}\|_\infty$ is defined as:

$$\|\mathbf{\Delta T}\|_\infty = \max\left\{|\Delta T_{\mathrm{MN\_1}}|, |\Delta T_{\mathrm{MN\_2}}|, \cdots, |\Delta T_{\mathrm{MN\_N1}}|, |\Delta T_{\mathrm{SCMN\_1}}|, \cdots, |\Delta T_{\mathrm{SCMN\_N2}}|\right\}.$$

Then, by using Theorem 4, we get:

$$\|\mathbf{\Delta T}(k+1)\|_\infty = \max\{|\Delta T_{\mathrm{MN\_1}}(k+1)|, |\Delta T_{\mathrm{MN\_2}}(k+1)|, \cdots,$$
$$|\Delta T_{\mathrm{MN\_N1}}(k+1)|, |\Delta T_{\mathrm{SCMN\_1}}(k+1)|, |\Delta T_{\mathrm{SCMN\_2}}(k+1)|, \cdots, |\Delta T_{\mathrm{SCMN\_N2}}(k+1)|\}^T.$$

We distinguish two cases:

- Case 1: The maximum norm $\|\mathbf{\Delta T}(k+1)\|_\infty$ belongs to a SCMN. Then:

$$\|\mathbf{\Delta T}(k+1)\|_\infty = \max_i\{|\Delta T_i(k+1)|\} \leq \max_i\left\{\left|\frac{1}{L}\sum_{j\neq i}^{N} \Delta T_j(k)\right|\right\} =$$
$$= \frac{1}{L}\max_i\left\{\left|\sum_{j\neq i}^{N} \Delta T_j(k)\right|\right\} \leq \frac{1}{L}\max_i\left\{\sum_{j\neq i}^{N} |\Delta T_j(k)|\right\} \leq$$
$$\frac{1}{L}(N-1)\max_j\{|\Delta T_j(k)|\}.$$

So:

$$\|\mathbf{\Delta T}(k+1)\|_\infty \leq \frac{N-1}{L}\|\mathbf{\Delta T}(k)\|_\infty. \tag{4.20}$$

- Case 2: The maximum norm $\|\mathbf{\Delta T}(k+1)\|_\infty$ belongs to a MN. Then:

$$\|\mathbf{\Delta T}(k+1)\|_\infty = \max_i\{|\Delta T_i(k+1)|\} \leq \max_i \left\{ \left| \frac{1}{L}\gamma_i^t \sum_{j\neq i}^{N} \Delta T_j(k) \right| \right\} =$$
$$= \frac{1}{L}\max_i\{\gamma_i^t\} \max_i \left\{ \left| \sum_{j\neq i}^{N} \Delta T_j(k) \right| \right\} \leq \frac{1}{L}\gamma_{\max} \max_i \left\{ \sum_{j\neq i}^{N} |\Delta T_j(k)| \right\} \leq$$
$$\leq \frac{1}{L}\gamma_{\max}(N-1)\max_j\left\{|\Delta T_j(k)|\right\}.$$

So:

$$\|\mathbf{\Delta T}(k+1)\|_\infty \leq \frac{N-1}{L}\gamma_{\max}\|\mathbf{\Delta T}(k)\|_\infty. \tag{4.21}$$

From (4.20) and (4.21) we get:

$$\|\mathbf{\Delta T}(k+1)\|_\infty \leq \max\left\{\frac{N-1}{L}\gamma_{\max}, \frac{N-1}{L}\right\}\|\mathbf{\Delta T}(k)\|_\infty \Leftrightarrow$$
$$\|\mathbf{T}(k+1)-\mathbf{T}^\star\|_\infty \leq \max\left\{\frac{N-1}{L}\gamma_{\max}, \frac{N-1}{L}\right\}\|\mathbf{T}(k)-\mathbf{T}^\star\|_\infty.$$

From Definition 5, this is a pseudo-contraction mapping IFF:

$$\max\left\{\frac{N-1}{L}\gamma_{\max}, \frac{N-1}{L}\right\} < 1 \Leftrightarrow N < \max\left\{\frac{L}{\gamma_{\max}}+1, L+1\right\}.$$

Consequently, from Theorem 3, the power control game under best response dynamics for SCMNs and MNs converges to a unique NE.

Moreover, as the best response dynamics scheme is a pseudo-contraction in the entire strategy space, this is the unique NE of the two-tier small cell network game [77]. □

4.7 Performance Evaluation

We have simulated our scheme for topologies that consist of one BS that is placed at the origin $(0,0)$ and is associated with two MNs $(\mathrm{MN}_1, \mathrm{MN}_2)$, as well as two SCBSs $(\mathrm{SCBS}_1, \mathrm{SCSB}_2)$, each one having two SCMNs: SCMN_1 and SCMN_2 that are associated with SCBS_1, and SCMN_3 and SCMN_4 that are associated with SCBS_2.

We focus on the uplink and we examine the utility values and the SINR for each (SC)MN at the NE. All system parameters are available in Table 4.3 and are based on an extensive study conducted by the Small Cell forum [5]. For the computation of the received power P_r, we use the following formula:

$$P_r(\mathrm{dBm}) = P_t + G_t + G_r - \Lambda_t - \Lambda_r - \mathrm{PL},$$

where G denotes the antenna gain, Λ denotes the loss, PL denotes the path loss, subscript t refers to the transmitter, and subscript r refers to the receiver.

Table 4.3: Simulation parameters.

Base Station Antenna Gain	17 dBi
Base Station Loss	3 dB
Small Cell Base Station Antenna Gain	0 dBi
Small Cell Base Station Loss	1 dB
(Small Cell) Mobile Node Antenna Gain	0 dBi
(Small Cell) Mobile Node Loss	3 dB
(Small Cell) Mobile Node Height	1.5 m
Small Cell Base Station Height	1.5 m
Base Station Height	30 m
Max. Power Mobile Node P_{max}	40 dBm
Max. Power Small Cell Mobile Node SCP_{max}	21 dBm
Frequency	850 MHz
CDMA Spread Factor	128
Initial MN SINR Target	8 dB
Update of the MN SINR Target Δ_{SINR}	0.5 dB
Update of the Position of (SC)MN $\Delta_{\text{(SC)MN}}$	2 m
Update of the Position of SCBS Δ_{SCBS}	2 m
Indoor-to-Indoor Path Loss Model: ITU P.1238	
Indoor-to-Outdoor Path Loss Model: Okumura-Hata for large cities	

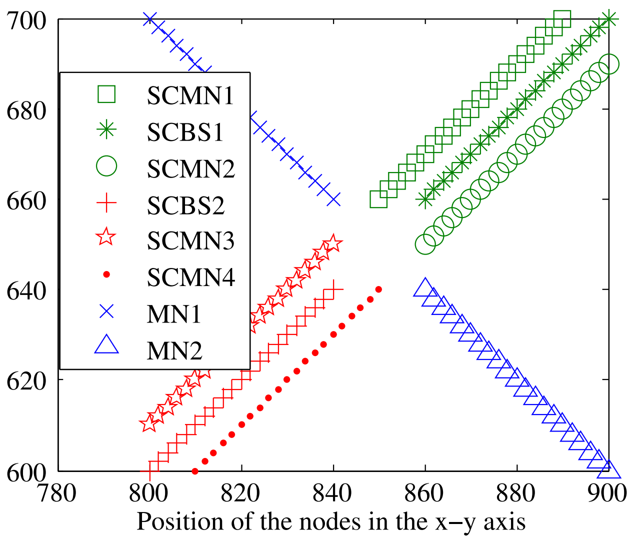


Fig. 4.1: Evolution of the positions of the nodes. SCMN_1, SCMN_2, and SCBS_1 are moving towards the north-east. SCMN_3, SCMN_4, and SCBS_2 are moving towards the south-west. MN_1 is moving towards the south-east and MN_2 is moving towards the north-west.

We distinguish two cases for the path loss model. For the indoor-to-indoor communication where (SC)MNs communicate with the SCBS, we use the ITU P.1238 model [5]:

$$PL(\mathrm{dB}) = 20\log_{10}(f) + V\log_{10} d + L_f(z) - 28.$$

According to [5], $V = 28$ is a suitable value and $L_f(z) = 0$, as we consider that all nodes are placed on the same floor. By replacing the values from Table 4.3, we get the Path Loss formula as a function of the distance d (in meters) between the (SC)MN and the SCBS:

$$PL(\mathrm{dB}) = 30.59 + 28\log_{10} d.$$

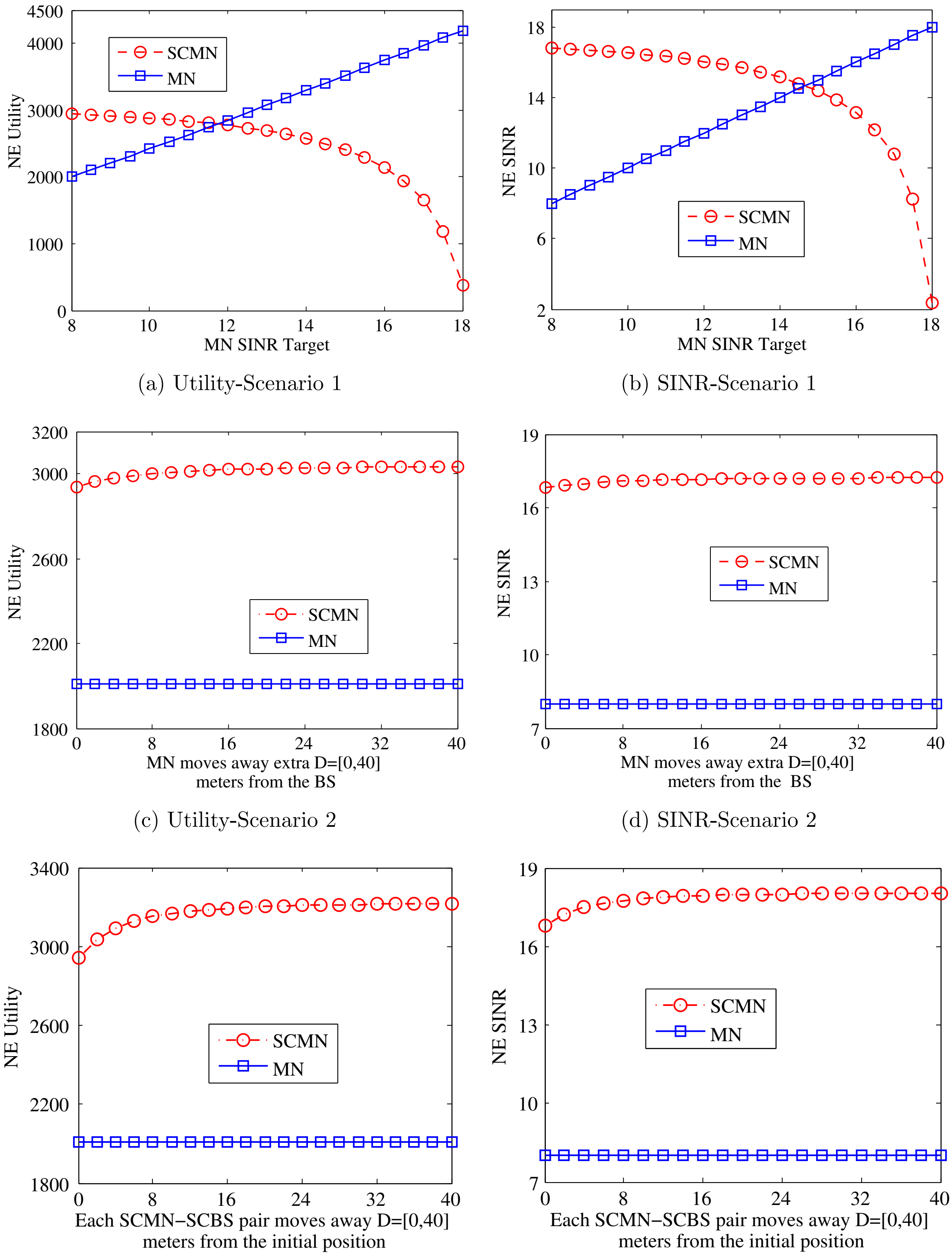
For the indoor-to-outdoor communication where (SC)MNs communicate with the BS, we use the Okumura-Hata model for large cities [5]. By replacing the values from Table 4.3, we get the Path Loss formula as a function of the distance d (in km) between the (SC)MN and the BS:

$$PL(\mathrm{dB}) = 125.76 + 35.22\log_{10} d, \quad d > 1\mathrm{km}.$$

We have studied 6 scenarios and simulated 20 simulation rounds per scenario. In each scenario, we gradually update some of the following parameters of the topology: the positions of the MNs, the positions of the SCMNs, the positions of the SCBSs, and MN targets. Simulation round 1 corresponds to the initial topology. Simulation round 20 corresponds to the topology in which the values of the parameters that are updated in that particular scenario differ the most from the ones of the initial topology (Fig. 4.1).We present the utility value and the SINR value at the NE for each round for each MN and SCMN. As we have studied symmetric topologies, all (SC)MNs end up at the same utility value (and the corresponding SINR is the same).

Scenario 1 (Fig. 4.2a, Fig. 4.2b) corresponds to the case that the positions of all entities are fixed. In each new simulation round, the target of each MN increases with a step equal to Δ_{SINR}. As expected, the utility value/SINR at the NE of the MN is increasing as the target increases. In addition, so as to achieve a higher utility value/SINR, each MN uses higher power. As the positions of all entities are fixed, the interference that each SCMN receives is increasing. So, the utility value/SINR at the NE is decreasing. However, it is worth noting that apart from the last simulation, the SINR achieved per SCMN is over 8 dB, which is sufficient for smooth voice communication [5]. This means that even if MNs choose a high target (up to 17.5 dB), SCBSs will be able to serve their SCMNs efficiently at least for voice.

Scenario 2 (Fig. 4.2c, Fig. 4.2d) corresponds to the case where, in each new simulation round, MN_1 updates its position to $(\mathrm{MN}_1.\mathrm{x} + \Delta_{\mathrm{MN}}, \mathrm{MN}_1.\mathrm{y} - \Delta_{\mathrm{MN}})$ and MN_2 sets its



(a) Utility-Scenario 1

(b) SINR-Scenario 1

(c) Utility-Scenario 2

(d) SINR-Scenario 2

(e) Utility-Scenario 3

(f) SINR–Scenario 3

Fig. 4.2: (SC)MN evolution of the NE utility value/SINR under Scenarios 1, 2, 3.

new position to $(\mathrm{MN}_2.\mathrm{x} - \Delta_{\mathrm{MN}}, \mathrm{MN}_2.\mathrm{y} + \Delta_{\mathrm{MN}})$. All other parameters are fixed. We can see that at the NE of each simulation round, the MN utility value/SINR is invariable. This is justified as the MN is able to achieve its target even if it moves away from the BS. As far as the SCMN, utility value/SINR at the NE presents a small increase as the MN moves away. This increase is expected as the SCBS receives a bit less interference from the MNs. In any case, it is worth mentioning that the SINR of each SCMN at the NE is always more than 17 dB, which is sufficient for data communications.

Scenario 3 (Fig. 4.2e, Fig. 4.2f) corresponds to the case where, in each new simulation round, each pair (SCBS, SCMN) updates jointly its position. All other parameters are fixed. Each MN always manages to achieve its target. From round to round this happens easier as the interference from the SCMNs lowers. Concerning the SCMNs, the farthest we place them from the MNs, the more utility value (SINR) they achieve at the NE. As in Scenario 2, the SINR of each SCMN at the NE is always more than 17 dB.

Scenario 4 (Fig. 4.3a, Fig. 4.3b) corresponds to the case that, in each new simulation round, each SCMN gradually moves away from its associated SCBS. All other parameters are fixed. Up to round 4, the utility value/SINR at the NE of each SCMN is increasing. This means that each SCMN is able to increase its power so as to augment its utility/SINR. From round 5 and on, the utility value/SINR at the NE is decreasing. This happens as each SCBS gradually receives less power from each SCMN (which transmits at $\mathrm{SCP}_{\mathrm{max}}$ but the distance SCMN-SCBS increases). However, apart from the last two rounds, the SINR achieved per SCMN is over 8 dB, which is sufficient for smooth voice communication. Concerning the MNs, they keep the same level of utility value/SINR at the NE.

Scenario 5 (Fig. 4.3c, Fig. 4.3d) corresponds to the case that, in each new simulation round, both the SCMNs and the MNs gradually move away from their associated SCBS/BS respectively. All other parameters are fixed. These changes have no influence in the MN utility value/SINR at the NE. Concerning the SCMNs, up to round 4, the utility value/SINR at the NE follows the same trend with Scenario 4. From round 5 and on, we notice a rather small decrease in the utility value/SINR. Though, as in Scenario 4, each SCBS gradually receives less power from each SCMN, it also receives less interference, since the MNs are moving away from both the BS and the SCBSs. This restricts the utility value/SINR loss at the NE.

Scenario 6 (Fig. 4.3e, Fig. 4.3f) corresponds to the case that the positions of all entities (SCBSs, SCMNs, MNs) are changing from round to round. The results are similar with Scenario 3. The MN utility values/SINR are not influenced by these changes, whereas each SCMN achieves a small increase in the utility value/SINR.

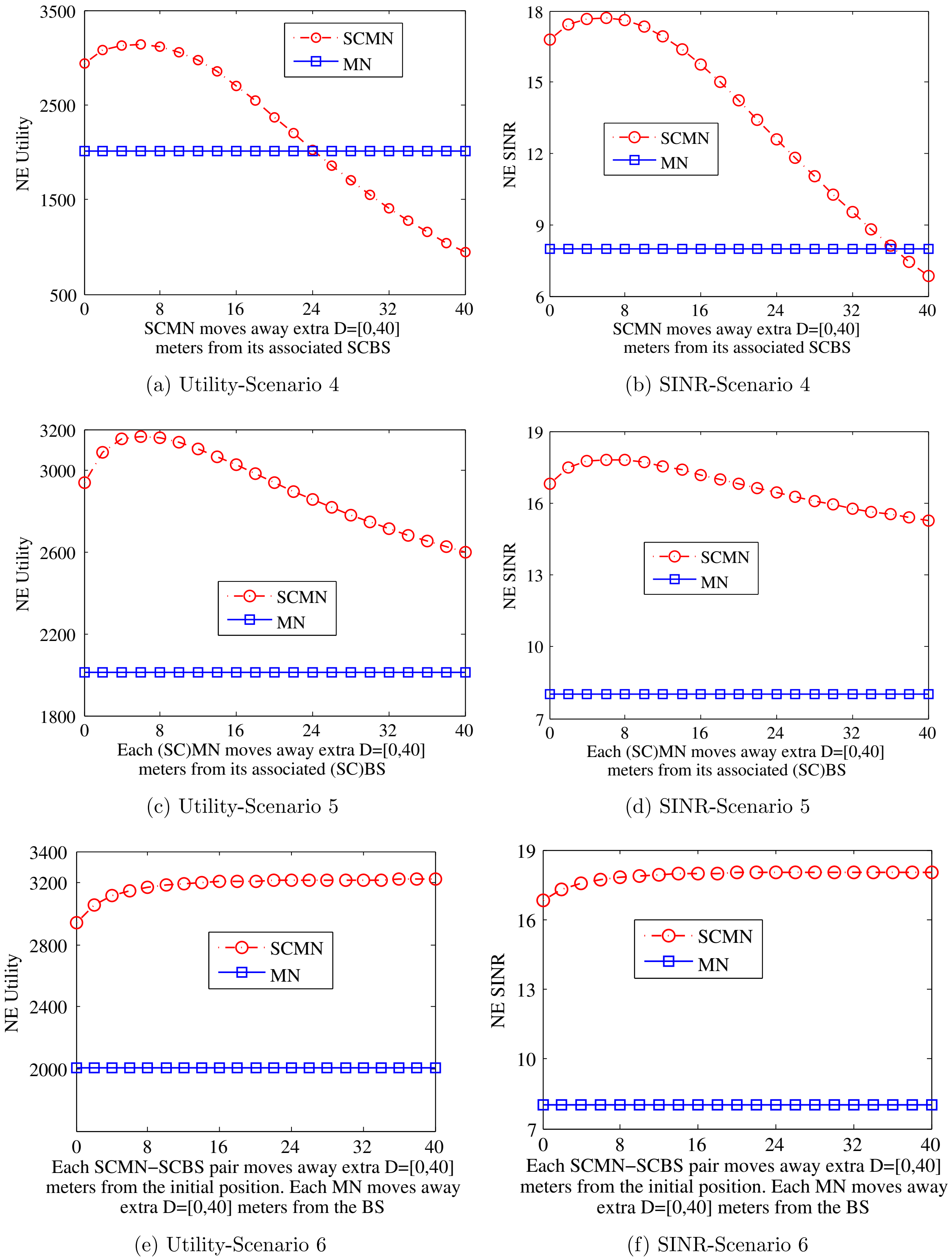


Fig. 4.3: (SC)MN evolution of the NE utility value/SINR under Scenarios 4, 5, 6.

4.8 Conclusions

In this work, we present a distributed power control scheme under best response dynamics that promotes the smooth coexistence of nodes that share the same portion of the radio spectrum in a two-tier small cell network. We argue that in this type of network MNs focus mostly on voice communications, whereas SCMNs focus on data communications. Based on that, we define a non-cooperative power control game where each (SC)MN aims at maximizing its own objective function. We derive conditions that guarantee the existence of a unique NE of our game, we determine the corresponding powers at the NE and we provide a distributed scheme that converges to them. Extensive simulations that are based on realistic assumptions examine the evolution of the NE utility values/SINRs of (SC)MNs. In all cases, MNs achieve sufficient SINR for voice communication. In almost all cases, SCMNs achieved more than sufficient SINR for data communications. The above results clearly indicate that the application of power control by distinguishing the utility functions for each category of players based on their QoS requirements leads almost always to a smooth coexistence in a two-tier small cell network.

Chapter 5

Channel Access Competition in Device-to-Device Networks

5.1 Introduction and Motivation[1]

As discussed in Chapter 1, mobile data traffic, especially mobile video traffic, has dramatically increased in recent years with the emergence of smartphones and tablets. A major issue in future cellular networks is to make high bit rates available to a larger portion of the cell, especially to users in exposed positions in-between several base stations. We already discussed in Chapter 4 how small cells can be used to meet this challenge. Apart from them, in future 5G wireless networks, devices that use the same channel and are close to each other are expected to be able to communicate directly, without needing to use a Base Station or Access Point. This concept, called Device-to-Device (D2D) communication [80], is receiving increasing attention since it can facilitate various applications: peer-to-peer file sharing, video dissemination, cellular offloading, etc. Traffic can be offloaded from the core networks, better service is provided to users, and both cellular coverage and energy efficiency are improved.

In such networks, devices can naturally form dense ad hoc wireless networks that have various applications. For example, Nishiyama et al. present in May 2014 trials in Japan where smartphones exchanged messages in a densely populated area without using the cellular network infrastructure [8]. Since these devices belong to different users, they are selfishly competing for channel access, meaning that each one is interested in sending its own data, without regard for the interference it is causing to other users.

There have been various multiple access methods proposed that allow multiple nodes to share a common channel when they transmit. Such multiple access schemes can be

[1]This chapter is based on papers [78], [79].

classified as either contention-free channel access (e.g., FDMA) or contention-based random access methods (e.g., Aloha). In a multiple access scheme, nodes can either compete or cooperate so that either an individual or a group objective can be achieved. For this reason, the framework of game theory has recently become a very useful mathematical tool for modelling and analysing multiple access schemes in wireless networks [81].

In our setting, solving the problem of multiple access through a centralized scheme imposes a significant communication and computation overhead that increases significantly with the network size. In contrast, efficient distributed algorithms can be designed based on non-cooperative game theory that neither are computationally expensive, nor increase network overhead.

We model such D2D networks as graphs, focusing on linear networks and tree networks. We assume that nodes want to transmit their packets only to other nodes that are 1-hop away, *i.e.*, their immediate neighbours. Many interesting scenarios fall into this category. For example, there have been proposed important real-life applications where nodes naturally form a linear ad hoc network: monitoring some critical infrastructures and geographic areas by using wireless sensor networks [82] as well as vehicle-to-vehicle networks for road safety communications [83].

In the context of D2D networks, there are scenarios that a tree topology arises naturally. Consider a cell phone that is connected to an access point and some other devices to be connected to the cell phone. These devices may be far from each other or tune their power to connect only to the cell phone, making direct transmission and loops impossible. Finally, the access point to which the cell phone connects might be connected to other cell phones in a similar way, leading to a 3-level tree topology. If the access point is not fixed, but also a mobile device, it could be part of an ad hoc network, possibly with a tree topology, leading to deeper trees.

The goal of this chapter is to study these types of D2D networks using a special class of non-cooperative games called graphical games [84]. Contrary to the general case of a non-cooperative game where the payoff of a node depends on the strategies of the other players in an arbitrary manner, in the case of graphical games this dependence is structured. In the particular case of our game, all nodes are placed on an undirected graph, and the payoff of the nodes depends only on the strategies of (some of) their near neighbours on this graph, specifically those that are up to 2-hops away. Our contributions are the followings:

- We show that a Nash Equilibrium (NE) exists under any tree and linear network that is also a Pareto optimal point. In fact, at a NE, an efficient scheduling of the transmissions is achieved, in the sense that there are no collisions, *i.e.*, a node either transmits successfully, or stays quiet.

- We present Scheme 1, a simple distributed scheme that iteratively converges to a NE in any tree and linear network.

- For linear D2D networks, we analyse the structural properties of a strategy vector that is a NE. Based on this analysis, we propose Scheme 2, a sophisticated distributed scheme that is guaranteed to monotonically converge to a NE using these properties.

- We study the performance of these schemes in terms of the speed of convergence to the NE and the number of successful transmissions at a NE through extensive simulations. Finally, we compare our schemes with the idea of simply aiming at finding a maximal transmission schedule, which is the standard goal of transmission scheduling algorithms [85], [86], highlighting the differences from our work.

5.2 Related Work

Various channel access schemes under the prism of non-cooperative game theory have been studied during the last decade. Mackenzie et al. [87] were the first to propose the modelling of Slotted Aloha as a non-cooperative game and analyse the NE of the game. In [88], Altman et al. extend this work by relaxing the assumption that each node has a packet to send at each time slot. Moreover, they also consider a team optimization approach (though without applying coalitional game theory). In [89], Wang et al. use pricing in the utility function to motivate the nodes to cooperate. With this mechanism, the throughput of the centralized Slotted Aloha can be achieved in a distributed network in which selfish users access the network attempting to maximize their own utility. However, in contrast to our work, these approaches consider a fully connected wireless network, where all nodes interfere with each other and consequently only one among the N nodes of the network is able to transmit successfully at each slot. In [90], Cui et al. consider a single-cell wireless local area network providing a general game-theoretic framework for designing contention-based medium access control protocols. Various utility functions are proposed and conditions for the existence and the uniqueness of a NE are derived. Simulation results show that the framework can achieve superior performance over the standard IEEE 802.11 Distributed Coordination Function. Again, the assumption is that every wireless node can hear every other node in the network.

Graphical games have already been applied a few times in wireless networks. In [91], Li and Han study channel selection for cognitive radio networks. Each secondary user chooses a channel to transmit, assuming that only its neighbouring nodes that have chosen the same channel cause non-negligible interference to it. The target is the minimization of the total regret. The no-regret approach is used in conjunction with other learning techniques to find a NE of the game. In [92], Hu et al. study the same problem using graphical games (even though they call them "local interaction games") and propose two approaches: *(i)* the minimization of the number of competing neighbours (aiming at network collision minimization) and *(ii)* an approach based on an altruistic payoff that includes also the payoff of its neighbours (aiming at network throughput maximization). Contrary to our work, these communication targets correspond to nodes that belong to the same operator.

There is a substantial body of work on the topic of transmission scheduling in wireless ad hoc networks. A work close to ours is the Five-Phase Reservation Protocol (FPRP) [85] that is used for distributed scheduling. Similar to our approach, the scheme is based on local interactions among the neighbouring nodes that examine whether they can have a successful transmission (and inform their neighbours when they achieve it). However, contrary to our schemes, FPRP is used only for multicast transmissions and the target is simply to schedule the transmissions to find a maximal transmission vector and not to find a NE of the game. This is not necessarily a NE, as we will show. Therefore, a maximal transmission vector is not always a unanimously desirable outcome. Given the fact that nodes are selfish (this assumption holds in FPRP even if nodes do not follow a game-theoretic approach), FPRP may produce a strategy vector where there will be at least one node that could have had a successful transmission but is forced to stay quiet. Finally, in rare cases, the FPRP algorithm leads to a transmission vector where a node both transmits successfully and receives packets that cannot decode. In our approach, this will never happen at a NE. Another distributed scheduling algorithm that aims at eliminating collisions is presented in [86]. Each time slot is divided in six mini-slots and the first five of them are used by neighbouring nodes that exchange control messages aiming at reserving the channel. If the channel is guaranteed to be idle, a transmission occurs. The approach considers both multicast and unicast transmissions. Simulations show that the performance of this scheme is similar to FPRP.

5.3 Tree D2D Networks

5.3.1 System Model

We consider a single channel wireless network that consists of $N \geq 2$ wireless nodes, indexed by $\{1, 2, \ldots, N\}$. These nodes form an undirected graph $G = (V, E)$, where a vertex $v \in V$ corresponds to a wireless node and an edge $e \in E$ corresponds to a communication link that connects a pair of nodes $\{u, v\}$. We consider any type of tree D2D networks. For

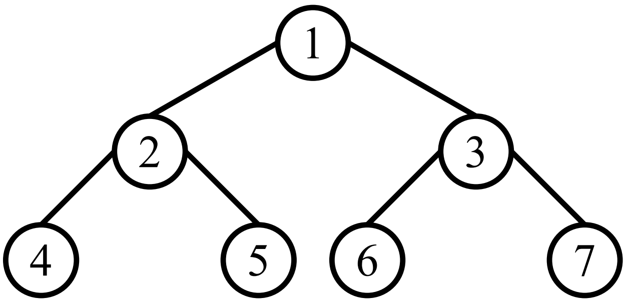


Fig. 5.1: A wireless D2D network that consists of 7 nodes. Each node can send a packet to one of its 1-hop neighbours.

illustration purposes, we present a particular example of a perfect tree network in Fig. 5.1. We denote the set of nodes that are 1-hop away from i with $\mathbf{D}_i$ and the number of nodes of this set with $|\mathbf{D}_i|$.

We assume that time is divided in slots, transmissions can start only at the beginning of a time slot, and that each packet needs exactly one slot to be transmitted. In addition, all queues are always full. We consider the unicast case, where a node i aims at transmitting a packet to exactly one of its neighbours, but has packets in its queues for all neighbours. Under our model, a node is not interested in transmitting at a particular node, but simply wants to transmit a packet to any of its 1-hop neighbouring nodes.

Each node i has $|\mathbf{D}_i|+1$ options at each time slot: *(i)* To send a packet to a neighbour $d_i \in \mathbf{D}_i$. We denote that option with T_i. *(ii)* To not transmit a packet (*i.e.*, to wait). We denote that option with W. We mention at this point that, when i transmits to d_i, all other 1-hop neighbours of i also receive the packet, but this packet is "noise" for them, as it is not intended to them.

As a collision model, we assume that a collision occurs under the following circumstances (similar to the collision models typically assumed in the study of Slotted Aloha [93]): *(i)* When a node receives packets simultaneously from at least two nodes, in which case all such packets collide. *(ii)* When node i transmits a packet to node j and node j also transmits. In this case, the transmission of i fails.

5.3.2 Graphical Game Model

To model the given graph setting as a non-cooperative game, we need to specify 3 elements: The players of the game and, for each player, its strategy, as well as its payoff. Concerning the players, these are the N nodes of the graph that correspond to the wireless nodes. The strategy of a player i is one of the following: Either to transmit to one of its $|\mathbf{D}_i|$ 1-hop neighbours or to wait.

Table 5.1: Payoff models. Tx corresponds to transmission, Rx corresponds to reception.

Model 1		Model 2	
Status	Payoff	Status	Payoff
Successful Tx	*1-c*	Wait and Successful Rx	*1-e*
Wait	*0*	Successful Tx	*1-c*
Failed Tx	*-c*	Wait and No Rx	*0*
		Wait and Failed Rx	*-e*
		Failed Tx	*-c*

Concerning the payoff of each player i, we should take into account the collision model of the previous section. We study two payoff models that are summarized in Table 5.1 (the strategies are presented in decreasing order of payoff): In model 1 (inspired by [87]), the motivation is that a successful transmission is preferable to waiting, which is also preferable to a failed transmission. Note also that a receiver gets zero payoff no matter whether it receives successfully a packet or not. If a transmission is successful, a node receives a payoff $1 - c$, where 1 corresponds to the throughput from the transmission of the packet and $c \in (0, 1)$ is a constant that corresponds to the cost of transmission. If a transmission collides with another transmission, the payoff is just $0 - c = -c$. If a node chooses to wait, its payoff is $0 - 0 = 0$, as its throughput is zero and its cost of transmission is also zero.

Under model 2, the receiver can get a non-zero payoff too. We explicitly make the standard assumption that a node that transmits cannot receive, so we examine three cases for a node that waits: If it has a successful reception, it receives a payoff $1 - e$, where 1 corresponds to the net benefit from the reception of the packet and $e \in (0, c)$ is a constant that corresponds to the cost of decoding the packet. Note that we assume that the decoding cost e is smaller than the cost of transmission c. If it cannot receive successfully a packet that is addressed to it, its payoff is $0 - e = -e$. If no packet is addressed to it, its payoff is $0 - 0 = 0$.

Depending on the application, payoff model 1 may be more preferable than payoff model 2 and vice versa. For example, if nodes are also interested in forwarding the packets that they receive, then payoff model 2 should be adopted.

For a general game with N players, in which each player has m possible strategies, the size of a normal form representation of the game would be $O(m^N)$ [30], since the payoff of a player that chooses a particular strategy depends on its strategy and the strategy of the remaining $N - 1$ players. Such a large representation would be needed if our network was fully connected. However, in our setup, the payoff of a player depends only on its strategy and the strategy of some of its neighbours. This corresponds to a special type of non-cooperative games that are called graphical games [84].

To identify the subset of neighbours that influence the payoff of a player i, we need to produce the square G^2 of the graph G, which is a graph that has the same set of nodes, but in which nodes i and j are neighbours when their distance in the graph G is at most two edges. In G^2, we compute the maximal degree d. If G is a tree and node i wants to transmit to node j, the payoff that it will receive depends on the strategy of j, as well as the strategy of all 1-hop neighbours of j, denoted by $|D_j|$. Consequently, the payoff is a function of $|D_j| + 1$ nodes, *i.e.*, the number of the 1-hop neighbours of i in G^2. Therefore, the size of a graphical form representation would be $O(m^{|D_{\max}|+1})$, where $|D_{\max}|$ is the maximal degree. If $|D_{\max}| \ll N$ (which is the typical case), the size of the graphical representation of the game is much smaller than the one in a normal form game.

5.3.3 Nash Equilibria

Having transformed this setup into an equivalent graphical game, we should address the fundamental question of the existence of a Nash Equilibrium (NE) in this game. As a first remark, we mention that, at any NE, the corresponding strategy vector $\mathbf{s} = (s_1, s_2, \ldots, s_N)$ should be collision-free. This is true since if a NE included collisions, then the nodes whose transmissions collided could improve their payoffs by simply deciding to wait.

We then explain the difference of a NE under payoff model 1 from the notion of the maximal strategy (transmission) vector that plays a central role in transmission scheduling [85], [86]. Using similar terminology with [94], we call a strategy vector feasible if all nodes in the strategy vector either wait or have a successful transmission. A strategy vector is called a maximal strategy vector if adding an extra transmission will result in an infeasible strategy vector, meaning that a collision occurs. All subsets of a maximal strategy vector are also feasible strategy vectors.

Though a NE under payoff model 1 fulfils the definition of a maximal strategy vector, a maximal strategy vector is not necessarily a NE under payoff model 1. To show this, consider Fig. 5.2. The strategy vector $\mathbf{s} = (s_1, s_2, s_3, s_4, s_5, s_6, s_7) = (T_2, W, T_7, W, W, W, W)$ is a maximal strategy vector since none of the nodes 2, 4, 5, 6, and 7 can have a successful transmission without interfering with at least one active transmission. However, it is not a NE under payoff model 1 since node 2, being selfish, will transmit to either node 4 or node 5.

Under payoff model 2, it is easy to check that the above strategy vector is a NE. In general, the following properties hold: *(i)* All maximal strategy vectors are Nash Equilibria

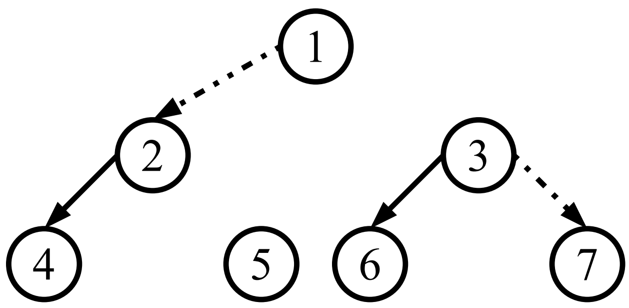


Fig. 5.2: Indicative Nash Equilibria for the network of Fig. 5.1. The full arrows indicate the active transmissions at a NE under payoff model 1. The dashed arrows indicate the active transmissions at a NE under payoff model 2.

under payoff model 2. *(ii)* All Nash Equilibria under payoff model 1 are Nash Equilibria under payoff model 2.

We now argue that, in this game, there is at least one NE, regardless of the payoff model used. Indeed, it is straightforward to construct a NE for each tree D2D network [78]. For example, in Fig. 5.2 we have sketched a NE for the perfect tree network of 7 nodes under both payoff models. The corresponding strategy vector under payoff model 1 is $\mathbf{s} = (s_1, s_2, s_3, s_4, s_5, s_6, s_7) = (W, T_4, T_6, W, W, W, W)$ and the corresponding payoff vector is $\mathbf{u} = (0, 1-c, 1-c, 0, 0, 0, 0)$. We can check that, after reaching this strategy vector, no node can improve its utility on its own. The strategy vector under payoff model 2 is $\mathbf{s} = (s_1, s_2, s_3, s_4, s_5, s_6, s_7) = (T_2, W, T_7, W, W, W, W)$ and the corresponding payoff vector is $\mathbf{u} = (1-c, 1-e, 1-c, 0, 0, 0, 1-e)$. We can check that, after reaching this strategy vector, no node can improve its utility by simply changing its strategy on its own.

Moreover, a desired property of any NE of this game is that it is Pareto optimal, meaning that no node can improve its payoff without deteriorating the payoff of at least one node at the same time [30]. Note that, in general, NE is not Pareto optimal [30]. However, in our game, the Pareto optimality property of any NE holds under both payoff models.

5.3.4 Finding a Nash Equilibrium

As a NE always exists, the question is how we can find it using a distributed scheme. A standard approach for finding a NE is by applying the best response scheme [84]. In this scheme, each node chooses the strategy that, given the strategies of all other nodes, maximizes its payoff. Unfortunately, as discussed in Chapter 4, the best response scheme does not necessarily converge to a NE for this particular game as it may lead to oscillations. As a counter-example for our case, let us consider a simple network consisting of 2 nodes: {1-2}. It is straightforward to check that, at a NE, either node 1 will transmit to node 2 or vice versa. If both nodes choose as their initial strategy to wait, the best response strategy for each node is to transmit, which will lead to a collision. Then, the best response strategy for both of them will be to wait, in the next round the best response strategy for both of them will be to transmit, and so on. Therefore, the algorithm will never converge to one of the two Nash Equilibria of this game.

Next, we discuss a distributed iterative algorithm, called Scheme 1, that aims at finding a NE. Firstly, we discuss Scheme 1 under the payoff model 1. Initially, each node has $|\mathbf{D}_i| + 1$ strategies, where $\mathbf{D}_i$ is the set of its 1-hop neighbouring nodes. Each strategy is selected with a probability equal to $\frac{1}{|\mathbf{D}_i|+1}$. Each strategy has the same probability since, under our model, a node is not interested in transmitting at a particular node, but rather wants to transmit a packet to any of its 1-hop neighbouring nodes. The algorithm is executed in rounds. Initially, nodes select their strategies simultaneously. Then, each node i that transmits learns from its destination node d_i whether its transmission was successful or not and computes its payoff on this round.

At the next round, each node i that had a successful transmission transmits to the same node d_i. This imposes some limitations on the strategies of the 1-hop neighbours of i and d_i for the next round. More specifically: *(i)* None of the 1-hop neighbours of i should transmit to i in the following round as no successful transmission can arise. *(ii)* None of the 1-hop neighbours of d_i (except, of course, i), that are also 2-hops neighbours of i, can transmit to d_i in the following round as no successful transmission can arise. The above piece of information is passed through the exchange of local 1-hop multicast messages that are sent by i and d_i correspondingly.

On the other hand, each node that did not have a successful transmission takes into account these limitations to decide its strategy in the next round. Let $\mathbf{V}_i$ be the set of unavailable neighbouring nodes for node i, meaning that i cannot transmit successfully to any of them in the next round and $\mathbf{D}_i - \mathbf{V}_i$ be the set of available nodes. If $\mathbf{D}_i - \mathbf{V}_i = \oslash$, then node i should wait in the next round. Else, it chooses to wait with a probability equal to

$$\frac{|\mathbf{V}_i| + 1}{|\mathbf{D}_i| + 1}.$$

Similarly, the probability to transmit is equal to

$$1 - \frac{|\mathbf{V}_i| + 1}{|\mathbf{D}_i| + 1} = \frac{|\mathbf{D}_i| - |\mathbf{V}_i|}{|\mathbf{D}_i| + 1}.$$

The motivation under this choice is that, as a node cannot transmit to nodes in $|\mathbf{V}_i|$, the probability of transmitting to nodes in $\mathbf{V}_i$ is transferred to the probability of waiting. As in the initialization phase, the probability of transmitting to a particular node remains $\frac{1}{|\mathbf{D}_i|+1}$. The algorithm ends when each node either has a successful transmission or waits and cannot have a successful transmission.

Under the payoff model 2, there are only two differences in Scheme 1: *(i)* Not only each node i that transmits successfully but also each node d_i that receives successfully will not change its strategy in the next round. This is because a successful reception leads to the biggest payoff under this payoff model as shown in Table 5.1. *(ii)* The scheme ends when each node either has a successful transmission/reception or waits with no packet addressed to it and cannot have a successful transmission.

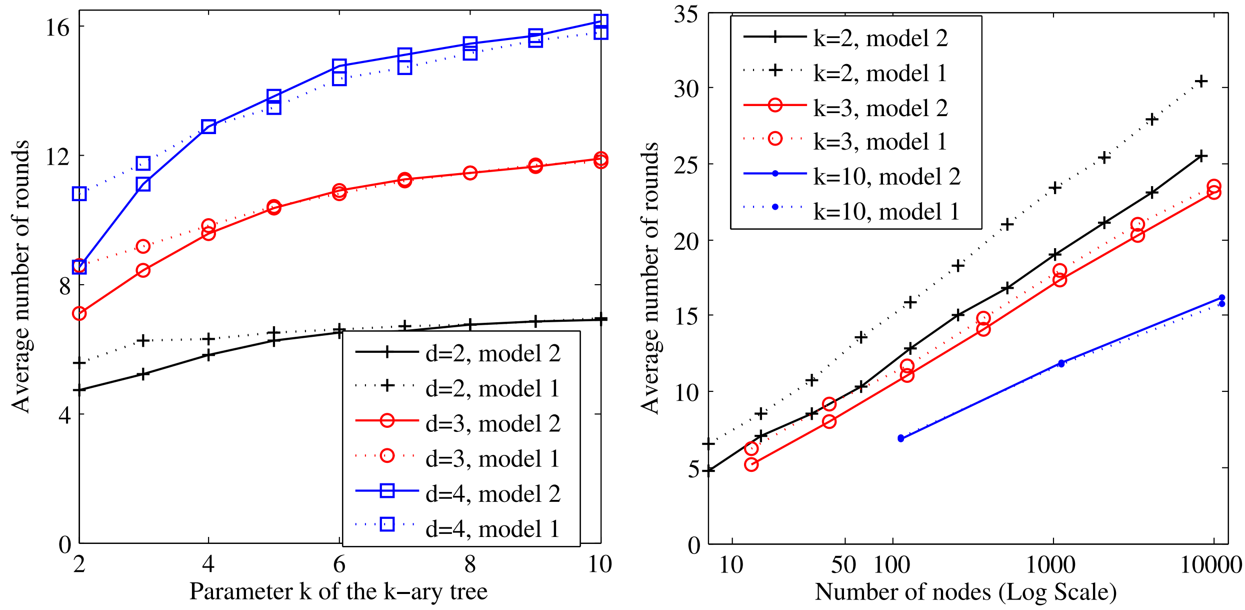
Due to the fact that the stopping condition of Scheme 1 corresponds to a strategy vector that is a NE, it is certain that, if Scheme 1 ends at a particular round, a NE will be reached, under both payoff models. The other possibility is that the maximum number of iterations will be reached without a NE. However, as we show in the next section, Scheme 1 converges to a NE after a very modest number of iterations for all examined cases.

5.3.5 Performance Evaluation

We have simulated Scheme 1 to evaluate its performance when the D2D network forms a perfect k-ary tree, *i.e.*, each non-leaf node has exactly k 1-hop next level neighbours and all leaf nodes are on the same depth d. For example, the tree in Fig. 5.1 is a perfect 2-ary tree. It is easy to show that a perfect k-ary tree of depth d has $\frac{k^{d+1}-1}{k-1}$ nodes. We simulated k-ary trees having a few nodes up to more than 10,000 nodes and, for each k-ary tree, we performed 1,000 simulations. The maximum number of rounds per simulation was set to 50 and Scheme 1 found a NE in all simulations for every k-ary tree.

The first set of simulations is used to evaluate the average number of rounds so that Scheme 1 converges to a NE versus the number of nodes, parameter k, and depth d. Fig. 5.3a presents these results for trees of depth 2, 3, and 4. We can see that Scheme 1 converges fast to a NE performing at most 16 iterations. The results are similar for both payoff models. We notice that, for a given parameter k, the number of rounds to converge to a NE is increasing with the depth d. This is natural since more nodes compete for spectrum access. However, the increase is quite slow. Moreover, for a given depth d, the number of rounds slightly increases with parameter k. This is natural, since, again, more nodes compete for spectrum access. However, the effect of parameter k is smaller than the effect of parameter d, implying that the depth of the tree influences more the convergence speed of Scheme 1 than the density of the nodes in a particular level.

We then present the average number of rounds to converge to a NE as a function of the number of nodes of trees with k=2, 3, and 10. In Fig. 5.3b the curves do not start from or end to the same number of nodes, as we study trees of depth at least 2 (trees of depth 1 are trivial to be resolved) that contain at most ≈10,000 nodes. The average number of



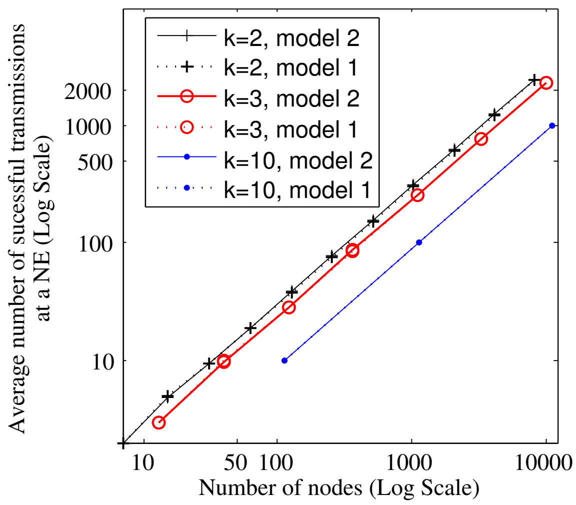
(a) Average number of rounds for convergence to a NE vs. parameter k and depth d.

(b) Average number of rounds for convergence to a NE vs. number of nodes.

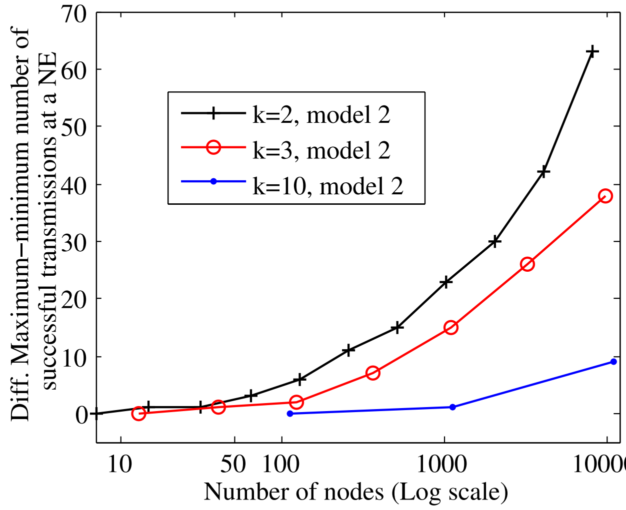
Fig. 5.3: Average number of rounds for convergence to a NE as a function of *(i)* parameter k and depth d of the k-ary tree and *(ii)* the number of nodes.

rounds to converge to a NE increases almost linearly with the logarithm of the number of nodes and this is the reason that the convergence is very fast. This is true for both payoff models. Finding a NE under payoff model 1 demands marginally more rounds to converge, which is rather expected since more strategy vectors correspond to a NE under payoff model 2. It is interesting that trees with $k = 2$ need more rounds than trees with $k = 10$. This is justified as follows: Consider a tree of around 100 nodes. It can be constructed either as a 10-ary tree of depth 2 or as a 2-ary tree of depth 7. As we see from Fig. 5.3a, the effect of the depth is bigger than the effect of the number of nodes in a particular level and this means that, for similar number of nodes, a longer tree demands more rounds to converge to a NE.

We then examine the average number of successful transmissions at a NE as a function of the number of nodes, fixing parameter k. We present these results in log-log scale in Fig. 5.4a. As expected, the number of successful transmissions increases linearly with the number of nodes. For both payoff models, the results almost coincide. Again, for similar number of nodes, we notice that the number of successful transmissions is larger for longer trees (*i.e.*, nodes with smaller parameter k). In Fig. 5.4b, we plot the difference of the NE with the maximum number of successful transmissions minus the NE with the minimum number of successful transmissions. We plot the results only under payoff model 2, as the results from payoff model 1 are very similar. The motivation is to examine whether there are



(a) Average number of transmissions at a NE.



(b) Maximum/minimum number of successful transmissions.

Fig. 5.4: Analysis of the average/maximum/minimum number of successful transmissions at a NE as a function of the number of nodes in a k-ary tree.

Nash Equilibria that are (non-)preferable under this metric due to the fact that significant fewer/more transmissions take place. Indeed, as simulations show, any NE under a k-ary tree setup is almost equally preferable. For example, consider a 2-ary tree of depth 12 that has on average 2487 successful transmissions. The worst NE involves 2456 successful transmissions and the best NE involves 2519 successful transmissions. The (plotted) difference of 63 corresponds to 2.5% fewer successful transmissions than the best possible case, which is insignificant. This means that there is no need to drive a solution towards a class of desirable NE under this metric.

5.4 Linear D2D Networks

5.4.1 System Model

We now consider a linear network that consists of N nodes $\{1 - 2 - \cdots - N\}$, where each node i can communicate with either its left-neighbouring node $i-1$ (L transmission) or its right-neighbouring node $i+1$ (R transmission). Using the same collision model with the tree networks, when $i < N-1$ an R transmission is successful iff nodes $i+1$ and $i+2$ have chosen to wait, whereas for $i > 2$ a L transmission is successful iff nodes $i-1$ and $i-2$ have chosen to wait. The same conditions hold, mutatis mutandis, for $i = 1, 2, N-1, N$.

This setup can be easily modelled as a non-cooperative graphical game with the players being the nodes and the strategy s_i of a player i being one of the following: $\{R, L, W\}$. We apply the payoff model 1 of Table 5.1.

5.4.2 Nash Equilibria Properties

In this section, we state two theorems that specify useful properties of strategy (sub)vectors at a NE.

Theorem 6. *Let $s = (s_1, s_2, \ldots, s_N)$ be a strategy vector that corresponds to a NE with $s_i = R$. Then:*

1. *If $i = N-1$, then the subvector (s_{N-1}, s_N) is equal to (R, W).*

2. *If $i = N-3$, then the subvector $(s_{N-3}, s_{N-2}, s_{N-1}, s_N)$ is equal to (R, W, W, L).*

3. *If $i \leq N-4$, then the subvector $(s_i, s_{i+1}, s_{i+2}, s_{i+3})$ is equal to either (R, W, W, L) or (R, W, W, R), where, in the second case, the R transmission of node $i+3$ satisfies this theorem as well.*

Proof. For each case, it is enough to show that the following two conditions hold:

- Condition A: Nodes whose strategy appears in one of the above subvectors do not have motivation to change unilaterally their strategies.

- Condition B: There is no other strategy subvector that fulfills condition A with $s_i = R$.

1. It is straightforward to verify that condition A holds. As for condition B, the only other possible subvector is (R, L) which does not fulfil condition A since it leads to a collision.

2. Concerning condition A, indeed, no node can improve its payoff by changing its strategy on its own. Concerning condition B, if node $N-2$ or node $N-1$ chooses to transmit, condition A cannot be satisfied since there will be a collision with the R transmission of node $N-3$ and it would be motivated to refrain from transmitting. If node N chooses W, condition A is not satisfied as node $N-1$ has motivation to choose R.

3. Regarding condition A, we have already discussed the case *(R, W, W, L)*. As for the case (R, W, W, R), this subvector fulfils condition A only if the subvector that starts with node $i+3$ fulfils condition A as well. As for condition B, we have argued on why no other subvectors may arise in the previous paragraph.

As a final comment, it is worth mentioning that if $i = N-2$, there is no NE where node i makes an R transmission. This is due to the fact that the subvector $(s_{N-2}, s_{N-1}, s_N) = (R, W, W)$, *i.e.*, the only subvector that corresponds to a successful R transmission, is collision-free but cannot be part of a NE, since node $N-1$ is motivated to choose R. Therefore, node i cannot choose R at a NE. $\square$

Theorem 7. *Let $s = (s_1, s_2, \ldots, s_N)$ be a strategy vector that corresponds to a NE with $s_i = L$. Then:*

1. *If $i = 2$, then the subvector (s_1, s_2) is equal to (W, L).*

2. *If $i = 4$, then the subvector (s_1, s_2, s_3, s_4) is equal to (R, W, W, L).*

3. *If $i \geq 5$, then the subvector $(s_{i-3}, s_{i-2}, s_{i-1}, s_i)$ is equal to either (R, W, W, L) or (L, W, W, L), where, in the second case, the L transmission of node $i - 3$ satisfies this theorem as well.*

Proof. The proof is similar to the proof of Theorem 6, so we omit it. □

5.4.3 Finding a Nash Equilibrium

We have already described Scheme 1, than can be used for finding a NE in tree D2D networks. This scheme can still be used for linear D2D networks, as linear networks are special types of tree networks. In Scheme 1, a node is interested in learning only whether it has a successful transmission or not, exchanging messages with up to its two-hop neighbours. This information is not sufficient to guarantee that a node that has a successful transmission will not need to change its strategy at a following iteration of the algorithm. For example, consider 3 nodes that form a linear network $\{1-2-3\}$ and the strategy vector $(s_1, s_2, s_3) = (R, W, W)$. With Scheme 1, node 1 will choose R in the next iteration, even if no NE with an R transmission for node 1 can arise. This has two undesirable effects for the nodes: It is a waste of resources and delays the convergence to a NE.

We propose Scheme 2, a more sophisticated scheme where nodes have motivation to exchange messages with up to their three-hop neighbours to alleviate the shortcomings of Scheme 1. The core of Scheme 2 is based on Propositions 1 and 2 that are closely related to Theorems 6 and 7.

Proposition 1. *Let $s = (s_1, s_2, \ldots, s_N)$ be a strategy vector with $s_i = R$. Then:*

1. *If $i = N - 1$ and (s_{N-1}, s_N)=(R, W), the algorithm will end up at a NE where this equality holds.*

2. *If $i = N - 3$ and $(s_{N-3}, s_{N-2}, s_{N-1}, s_N)$ is equal to (R, W, W, L), the algorithm will end up at a NE where this equality holds.*

3. *If $i \leq N - 4$ and $(s_i, s_{i+1}, s_{i+2}, s_{i+3})$=(R, W, W,L), the algorithm will end up at a NE where this equality holds.*

4. If $i \leq N-4$ and $(s_i, s_{i+1}, s_{i+2}, s_{i+3})$=(R, W, W,R), the algorithm will end up at a NE where the strategy vector includes either this subvector or the subvector (R, W, W,L).

Proposition 2. *Let $s = (s_1, s_2, \ldots, s_N)$ be a strategy vector with $s_i = L$. Then:*

1. If $i = 2$ and (s_1, s_2)=(W, L), the algorithm will end up at a NE where this equality holds.

2. If $i = 4$ and (s_1, s_2, s_3, s_4)=(R, W, W,L), the algorithm will end up at a NE where this equality holds.

3. If $i \geq 5$ and $(s_{i-3}, s_{i-2}, s_{i-1}, s_i)$=(R, W, W, L), the algorithm will end up at a NE where this equality holds.

4. If $i \geq 5$ and $(s_{i-3}, s_{i-2}, s_{i-1}, s_i)$=(L, W, W, L), the algorithm will end up at a NE where the strategy vector includes either this subvector or the subvector (R, W, W,L).

Scheme 2 aims at identifying strategy subvectors that are guaranteed to be part of the eventual NE. Nodes that belong to these subvectors do not change any more their strategies, having completed their statuses. The rest of them go on updating their strategies by taking into account the limitations due to successful transmissions that we discussed in Scheme 1. Clearly, if a node has a unique strategy left as an option, then it completes its status as well. So, when a node has a successful transmission, it transmits at the same direction at the next transmission round only if its strategy is part of a strategy subvector mentioned in Theorems 6 and 7. Otherwise, it flips a coin to decide upon its strategy. When all nodes complete their statuses, a NE has arisen and the algorithm ends. We present the pseudocode of Scheme 2 in Section 5.6.

5.4.4 Performance Evaluation for the Unicast Case

We have simulated Scheme 1 and Scheme 2 to evaluate their performances under linear D2D networks of various sizes (from 5 nodes up to 1000 nodes). For each size of the network, we have executed 10,000 simulations. We focus on the time taken by our schemes to converge to a NE. The first set of simulations is used to evaluate the average number of iterations needed so that the schemes converge to a NE versus the size of the network. We compare Scheme 2 with two variations of Scheme 1: *(i)* A scheme that uses an unbiased coin when a node needs to decide whether to transmit or not. *(ii)* A scheme that uses a biased coin giving higher probability to transmit. We experimented with different values of the probability to transmit and we present the results for 2/3, which is a representative value

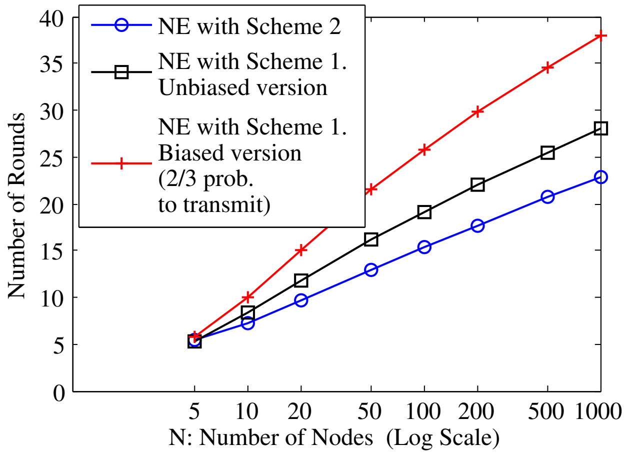


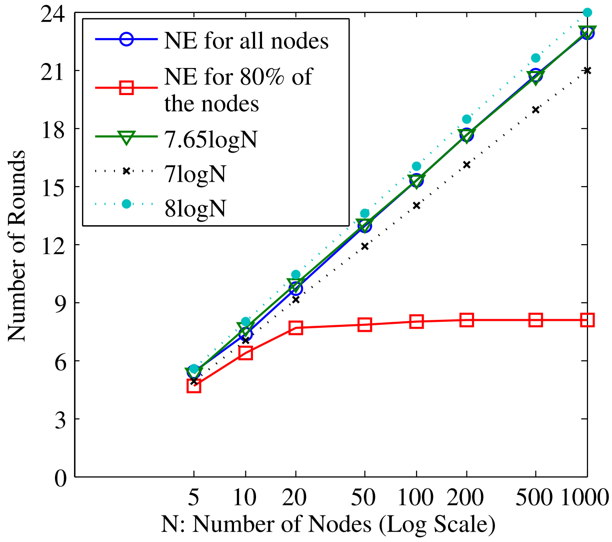
Fig. 5.5: Comparison of the proposed schemes. Average number of rounds for convergence to a NE under unicast.

for the trends that we notice. The motivation for this biased version is that, in principle, a node would prefer to transmit than to wait.

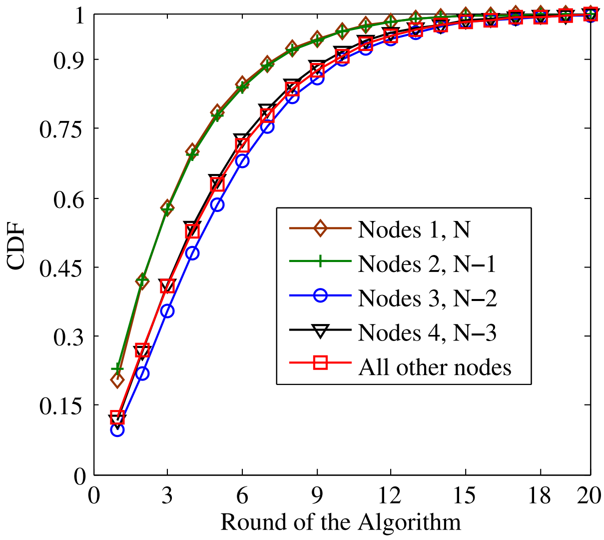
Fig. 5.5 presents the results. As expected, the number of rounds increases with the size of the network. Scheme 2 presents the best performance, ranging on average, from 5 rounds (for 5 nodes) to 23 rounds (for 1000 nodes). However, this increase is quite slow, e.g., augmenting the nodes from 200 to 500 demands only 3 more rounds on the average to find a NE. This means that even for networks that consist of many nodes, Scheme 2 converges fast. Actually, the increase is proportional to the logarithm of the number of nodes N of the network. Experimentally, we find that the average number of steps for the convergence to a NE is $\cong 7.65 \log_{10}(N)$ (see Fig. 5.6a). We also note that Scheme 2 converges to a NE without exceeding the maximum number of iterations (which was set to 50) with probability > 0.999 for all studied networks.

The unbiased version of Scheme 1 performs quite well, demanding a small number of extra rounds with respect to Scheme 2 to find a NE. The number of rounds is again proportional to the logarithm of the nodes of the network, however the constant multiplier is bigger than Scheme 2. On the other hand, the performance of the biased version of Scheme 1 is worse and deteriorates as the number of nodes increases. Moreover, about 5% of the simulations of big networks exceed the maximum number of iterations without converging to a NE. These undesirable features of the biased version are due to the fact that favouring nodes' probability to choose to transmit (even though, in principle, a node would prefer to transmit than to wait) increases also the probability for collisions and delays the convergence to a NE. This is the reason why the unbiased version performs better.

Next, we examine in which round, on average, 80% of the nodes have completed their statuses. We focus only on Scheme 2, since the convergence to a NE for a node that



(a) Average number of rounds for convergence to a NE.



(b) CDF of round at which a node completes its status.

Fig. 5.6: Performance evaluation for the unicast case for Scheme 2: Expected value and the Cumulative Distribution Function (CDF) of the convergence time.

uses Scheme 1 is not monotonic, meaning that it may change its status from complete to incomplete and vice versa. As Fig. 5.6a shows, for all studied networks that consist of at least 20 nodes, 80% of nodes will have converged to their final strategies in just 8 rounds. This means that, on average, 800 out of 1000 nodes will have converged to their final strategies by round #8 and only 200 of them will go on updating their strategies for up to round #23.

To further explore that issue for Scheme 2, we distinguish the nodes in five categories: *(i)* Node 1/node N, which have no left/right neighbour. *(ii)* Node 2/node $N-1$ which have one left/right neighbour. *(iii)* Node 3/node $N-2$ which have two left/right neighbours. *(iv)* Node 4/node $N-3$ which have three left/right neighbours. *(iv)* Every other node that has at least four left/right neighbours. We use this grouping motivated by the results of Theorems 6 and 7, as the nodes that belong to the same category are expected to have similar probabilities to participate in a strategy subvector that is guaranteed to be part of a NE. This is due to the fact that this probability depends on the number of left/right neighbours, so nodes that have the same number of left/right neighbours (0, 1, 2, 3, 4+ neighbours respectively) should be studied together.

Fig. 5.6b presents the results after 10,000 experiments on a network that consists of 10 nodes. The results are very similar for bigger networks as well. The horizontal axis corresponds to the round of Scheme 2 and the vertical axis to the probability that a node of each category will have completed its status up to that particular round. The fast convergence for the vast majority of the nodes is verified by these results. As we can see, at round #8, each node has a probability of more than 0.8 to have converged to its final strategy. Moreover, it is interesting to note that nodes have completed their status by round #12 with probability > 0.95 and also that nodes have completed their status by round #17 with probability > 0.99. Further analysis of this plot leads to the conclusion that all nodes have a significant probability to end up at a NE as transmitters, which is, in principle, a desirable property.

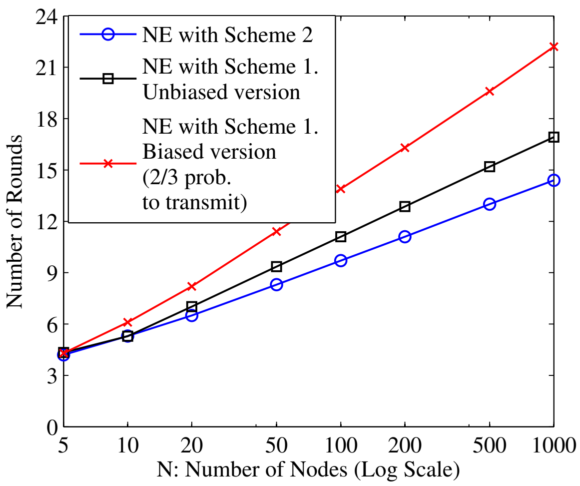
Finally, as the convergence to a NE for a node that uses Scheme 1 is not monotonic, the percentage of nodes that have completed their status in round $k+1$ can be smaller than in round k. The monotonic convergence to a NE is a great advantage of Scheme 2.

5.4.5 On the Nash Equilibria under Multicast Traffic

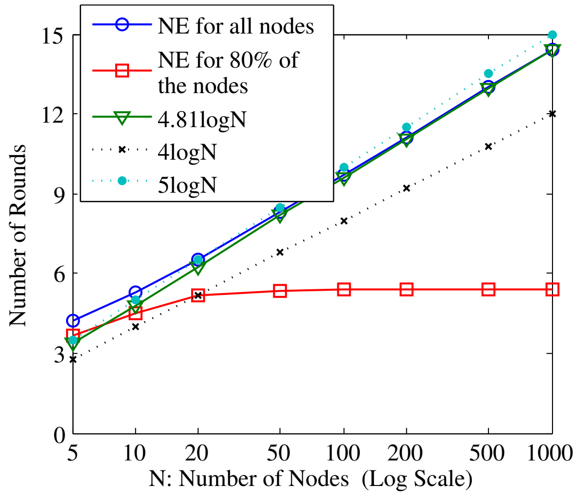
In this section, we study the multicast transmission scheme, where each node aims at sending, in a single broadcast transmission, its packet to all neighbours that are one hop away from it. Clearly, each node can choose between two strategies: to transmit (T) or to wait (W). Concerning the payoff, for each intermediate node i, there are some differences from the unicast case due to the fact that the transmission cost is equally divided to the number of nodes to whom the packet is sent. Therefore, if a node waits, its payoff is again 0; if it transmits and the transmission is successful for both neighbours (we call this state a fully-successful transmission), then its payoff is $2(1 - c/2) = 2 - c$; if one transmission is successful and the other fails (we call this state a semi-successful transmission), then its payoff is $1 - c/2 - c/2 = 1 - c$; if both transmissions fail, then its payoff is $-c/2 - c/2 = -c$.

Analysing the conditions for a successful transmission and using similar arguments with the unicast case, we find that a strategy subvector of the form $(s_{i-2}, s_{i-1}, s_i, s_{i+1}, s_{i+2}) = (W, W, T, W, W)$ should exist so that node i has a fully-successful transmission. If either $(s_i, s_{i+1}, s_{i+2}) = (T, W, W)$ or $(s_{i-2}, s_{i-1}, s_i) = (W, W, T)$ hold, then node i has a semi-successful transmission. Cases where a node has less than 2 left/right neighbours are treated similarly with the unicast case. At a NE, each node should either wait, or have a fully-successful transmission, or have a semi-successful transmission as, even in that case, it has no motivation to change its strategy to wait, as its payoff will be decreased from $1 - c$ to 0; note that $c \in (0, 1)$. Based on the above, we present the pseudocode of Scheme 2 for the multicast case in Section 5.7.

We then focus on how to find a NE under multicast traffic. As Scheme 1 can be applied directly without further changes, we highlight the changes that should be adopted for Scheme 2. A strategy subvector $(s_i, s_{i+1}, s_{i+2}, s_{i+3})$ or $(s_{i-3}, s_{i-2}, s_{i-1}, s_i)$ is guaranteed to be part of a NE of the network if it is of the form (T, W, W, T). This is true since these



(a) Average number of rounds for convergence to a NE under multicast for the proposed schemes.



(b) Convergence times of Scheme 2.

Fig. 5.7: Performance evaluation for the multicast case: Comparison of the proposed schemes and analysis of the average convergence time of Scheme 2.

transmissions will be (at least) semi-successful and the intermediate nodes that wait cannot have a (semi-)successful transmission.

We finally evaluate the performance of the schemes using the same metrics with the unicast case. Again, for Scheme 1, we present both an unbiased and a biased version. As Fig. 5.7a reveals, Scheme 2 converges very fast to a NE. The convergence is proportional to $k \log_{10}(N)$, where k is a coefficient and N is the number of the nodes of the network. Our simulations show that $k = 4.81$ approximates closely the results from the simulations for various sizes of the network (Fig. 5.7b). The unbiased version of Scheme 1 works quite well but this is not the case for the biased version. Our comments on Fig. 5.5 hold for Fig. 5.7a as well. Concerning the convergence of the 80% of the nodes of the network to a NE under Scheme 2, this is done in at most 6 transmission rounds (Fig. 5.7b).

5.5 Conclusions

We focus on D2D networks where devices decide autonomously their strategy (either to transmit or to wait and receive data) using a graphical game model. In tree networks, and by using two alternative payoff models, we present Scheme 1, a distributed scheme that leads to an efficient NE and evaluate its performance by simulation. We show that the scheme converges fast to a NE and, each NE has about the same number of successful transmissions. In linear networks, we show that the analysis of the structure of a strategy vector at a NE is not only useful from a theoretical perspective, but can also be the key factor for developing a practical scheme with appealing properties. We propose Scheme 2 where devices communicate in a 3-hop neighbourhood. We show that Scheme 2 clearly outperforms Scheme 1 where devices exchange information in a 2-hop neighbourhood. Devices that apply Scheme 2 converge to a NE in a number of rounds that is proportional to the logarithm of the network size. Moreover, when devices in the neighbourhood end up in a strategy subvector that is a local NE, it is guaranteed that this will be part of the global NE of the network. This both reduces the waste of resources and contributes to the faster convergence to a NE.

Finally, although the emphasis of this work is on the theoretical foundations of the autonomous channel access problem, on the practical side, our approaches could be considered as an alternative to (recently patented) techniques for distributed scheduling of D2D transmissions using contention-based protocols [95], [96]. In situations where nodes always have packets to send to all of their neighbours, our schemes could be used as reservation protocols; in each slot, the nodes compete for medium access choosing a node to which they want to send a packet and, if they gain access, they transmit to this particular node. Further implementation details and discussion on signalling messages have been presented in the previous sections. Note also that our schemes are robust with respect to changes of the network (e.g., the positions of the nodes due to mobility), provided that these changes are much slower than the time needed to decide on the outcome of their choice. In any case, the nodes only need to know the number of their 1-hop neighbours (which may vary from slot to slot) to apply these schemes.

5.6 Appendix I: The Algorithm for the Unicast Case

Algorithm 3 presents the pseudocode of Scheme 2 that we discussed in Section 5.4.3. After the initial random choice of the strategies (lines 2-5), there are two big for-loops that are executed in each transmission round. In the first for-loop (lines 7-31), the algorithm examines cases 1-3 of Propositions 1 and 2. In the second for-loop (lines 32-43), it examines case 4 of Propositions 1 and 2. In the last two lines, it examines whether all nodes have completed their statuses or another transmission round should start.

Algorithm 3 Finding a NE through a distributed iterative scheme for the unicast case

1: Notation: C: Completed status, P: Pending status, N: Number of Nodes, S: Successful Transmission
2: **for** $i = 1 \rightarrow N$ **do**
3: i.status=P
4: each node i chooses randomly its strategy.
5: **end for**
6: **for** $k = 1 \rightarrow$ MAX_NUMBER_OF_ITERATIONS **do**
7: **for** $i = 1 \rightarrow N$ **do**
8: If node i has chosen R or L, node $i \pm 1$ informs it whether the transmission was successful or not. Each node i computes its payoff.
9: **if** i.transmission==$S \wedge i$.strategy==R **then**
10: **if** $i \leq N-3 \wedge (i+3)$.strategy!=W **then**
11: i.status=C, $(i+1)$.status=C, $(i+2)$.status=C
12: **if** $(i+3)$.strategy==L **then**
13: $(i+3)$.status=C
14: **end if**
15: **end if**
16: **if** $i == N-1$ **then**
17: i.status=C, $(i+1)$.status=C
18: **end if**
19: **end if**
20: **if** i.transmission==$S \wedge i$.strategy==L **then**
21: **if** $i \geq 4 \wedge (i-3)$.strategy!=W **then**
22: i.status=C, $(i-1)$.status=C, $(i-2)$.status=C
23: **if** $(i-3)$.strategy==R **then**
24: $(i-3)$.status=C

Algorithm 3 Finding a NE through a distributed iterative scheme for the unicast case (continued)

25: **end if**
26: **end if**
27: **if** $i == 2$ **then**
28: i.status=C, $(i-1)$.status=C
29: **end if**
30: **end if**
31: **end for**
32: **for** $i = 1 \to N$ **do**
33: **if** $i \geq 4 \land i$.status==$P \land i$.transmission==$S \land i$.strategy==$R \land (i-3)$.status==C **then**
34: i.status=C
35: **else**
36: i.next_strategy=L
37: **end if**
38: **if** $i \leq N-3 \land i$.status==$P \land i$.transmission==$S \land i$.strategy==$L \land (i+3)$.status==C **then**
39: i.status=C
40: **else**
41: i.next_strategy=R
42: **end if**
43: **end for**
44: Nodes that have completed their statuses send a local broadcast message to their neighbours along with their strategy.
45: **if** all nodes have completed their statuses, the algorithm ends at a NE. Else, the nodes that have incomplete status, update randomly their strategy by taking into account any limitations that are imposed by the strategy of the nodes that have completed their statuses (as discussed in the text).
46: **end for**

As a final comment, the algorithm uses the best response concept in the following cases: *(i)* When it identifies strategy subvectors that are guaranteed to be part of a NE (*i.e.*, cases where nodes complete their statuses). *(ii)* In lines 36 and 41 of the pseudocode. In both cases, no oscillations may arise and the adoption of the best response scheme will definitely lead to strategies that will be part of a NE.

5.7 Appendix II: The Algorithm for the Multicast Case

Algorithm 4 Finding a NE through a distributed iterative scheme for the multicast case

Notation: C: Completed status, P: Pending status, N: Number of Nodes, S: Successful Transmission

for $i = 1 \to N$ **do**
    i.status=P
    each node i chooses randomly its strategy.
end for
for $k = 1 \to$ MAX_NUMBER_OF_ITERATIONS **do**
    for $i = 1 \to N$ **do**
        If node i has chosen T, nodes $i \pm 1$ inform it whether the transmission was successful or not. Each node i computes its payoff.
        if i.transmission==S **then**
            if $i \leq N-3 \wedge (i+3)$.strategy!=W **then**
                i.status=C, $(i+1)$.status=C, $(i+2)$.status=C, $(i+3)$.status=C
            end if
            if $i == N-1$ **then**
                i.status=C, $(i+1)$.status=C
            end if
            if $i \geq 4 \wedge (i-3)$.strategy!=W **then**
                i.status=C, $(i-1)$.status=C, $(i-2)$.status=C, $(i-3)$.status=C
            end if
            if $i == 2$ **then**
                i.status=C, $(i-1)$.status=C
            end if
        end if
    end for
    Nodes that have completed their statuses send a local broadcast message to their neighbours along with their strategy.
    if all nodes have completed their statuses, the algorithm ends and this strategy vector is a NE for the network. Else, the nodes that have incomplete status, update randomly their strategy by taking into account any limitations that are imposed by the strategy of the nodes that have completed their statuses.
end for

Algorithm 4 presents the distributed iterative scheme that we discuss in Section 5.4.5. The only difference from the unicast case is that the part of the Algorithm 3 in lines 32-43 is not needed any more, as node $i \pm 3$ should not need to examine further its strategy and (probably) adopt a best response scheme in the next round. Corner cases with less than 3 left/right neighbours are treated similarly with the unicast case.

Chapter 6

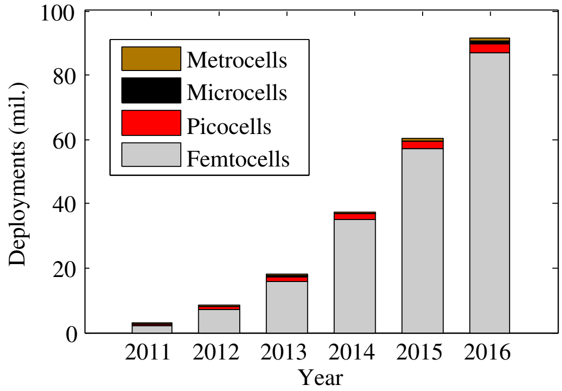
Power Control and Bargaining under Licensed Spectrum Sharing

6.1 Introduction and Motivation[1]

As we have discussed in Chapter 1, both the number of mobile devices and the volume of mobile data traffic are growing rapidly and, consequently, new communications paradigms have arisen to meet this demand. We analysed in Chapter 4 how small cells could contribute towards this direction. As Fig. 6.1 shows, in 2012, the number of small cell base stations was expected to reach almost 100 million by 2016 and the first LTE small cell had just been launched [98]; the deployment of small cells would be very beneficial for the widespread adoption of 4G (and then 5G) services, provided that adequate spectrum would be available for them. Noticing this trend, the operators started actively looking for opportunities to gain more licensed spectrum. However, licensing new spectrum to cellular operators through auctions [99] is no longer straightforward due to the scarcity of available spectrum and the time-consuming procedure of clearing such spectrum from its legacy usage [100]. A short term solution should be adopted to avoid delays in the global deployment of small cells.

In December 2012, the Federal Communications Commission (FCC), the responsible regulatory body in the USA, published a ground-breaking proposition [101]: It identified the 3.5 GHz band that was currently used by the U.S. Navy radar operations (but characterized by light usage) as a shared-access small cell band. In other words, the operators could jointly use this band, without having exclusive access. FCC also encouraged operators to identify how they could apply interference mitigation techniques including power control so that both small cell base stations that belong to the different operators and radars could coexist efficiently. Eighteen months after the publication of this proposition, in July 2014,

[1]This chapter is based on [97].



(a) Small cell deployment forecasts.

Small cell industry firsts		
First launch	Sprint Wireless (US)	September 2007
First enterprise launch	Verizon Wireless (US)	January 2009
First public safety launch	TOT (Thailand)	March 2011
First standardized launch	Mosaic (US)	February 2012
First LTE femtocell	SK Telecom (South Korea)	June 2012

(b) Small cell industry firsts.

Fig. 6.1: Small cell networks facts (based on [98]).

three major operators (Verizon, Ericsson, and Qualcomm) announced the first trials in the 3.5 GHz band[2], focusing on scenarios for complementary LTE-Advanced services. Similar efforts are planned in the 2.3 GHz band in Europe [102].

This idea, recently termed *licensed spectrum sharing* constitutes a complementary way to optimize spectrum usage other than the traditional approaches of either licensing spectrum or making it freely available. Licensed spectrum sharing is expected to be a key concept of 5G networks [13].

However, a great challenge to the widespread adoption of the licensed spectrum sharing paradigm is how the operators should interact with each other to satisfy their non-aligned interests [103]. In this chapter, we model this setup as a non-cooperative game among the wireless operators who aim at maximizing their revenues by using a simple charging scheme based on the Quality-of-Service (QoS) they offer [104].

Our contributions are the following: For the general case of N operators competing for downlink spectrum access, each one with one Base Station (BS) that transmits to one Mobile Node (MN), we propose a joint power control and bargaining scheme and discuss under which conditions it leads to operating points with higher payoffs for all operators than the traditional non-cooperating approach that leads to a Nash Equilibrium (NE). Furthermore, for the special case with 2 operators: *(i)* We show that this scheme will always lead to more preferable points than the NE for both operators. *(ii)* We prove that, through our scheme, the operating point that maximizes the social welfare (sum of payoffs) can always be reached. *(iii)* We compare its performance with a scheme based on linear pricing of the power, showing through simulations that we achieve better payoffs for most scenarios.

[2]Verizon, Qualcomm and Ericsson partner on field trials of 3.5 GHz spectrum sharing, last accessed: December 2014.

6.2 Related Work

Note that the problem of finding a more efficient point than the NE has already been studied in the broader context of wireless networks; in this section we briefly review some related approaches.

One direction is to consider a coalitional game [68]: Players that form the coalition act as a single entity, receive a common payoff, and then split it in a fair way using, e.g., the notion of the Shapley value. Then, the coalition is stable iff all players receive at least as much payoff as they would have received if they were on their own [68]. In our work, we do not assume coalitions among the operators, as this reflects reality more accurately.

Another direction is the application of the Nash Bargaining Solution (NBS) with a disagreement point, which is typically the NE [68]. In [105], Leshem and Zehavi compute the NBS in the context of the interference channel when there are two players and show through simulations that it significantly outpeforms the NE. In [106], Alyfantis, Hadjiefthymiades, and Merakos apply power control in the uplink using the utility function that has been proposed in [52]. They find the NBS where all players achieve equal Signal-to-Interference plus Noise Ratio (SINR) and discuss how the the powers of the MNs can be driven to this operating point, which is the socially optimal solution. In our work, we assume that the operators are not willing to reveal their utility functions (*i.e.*, their powers and all their associated gains). This is necessary for the computation of the NBS. Moreover, in the general case where there are N operators, the complexity of computing the NBS is significant. We believe that it is more realistic to consider an approach such as ours, where the level of information that is needed for finding a more efficient operating point than the NE is smaller.

Finally, pricing has been used as a way to find a more efficient NE. In [33], Alpcan et al. use as a utility function the throughput minus a linear function of the power. They show that, when the number of players N is lower than $L-1$, where L is the spread factor of the system, then the game admits a unique NE and their scheme converges to it. We will compare our approach with this scheme, showing that we can derive better results in terms of both payoff per operator and sum of payoffs. Moreover, a qualitative advantage of our approach is that it can be used for any spread factor $L \geq 1$. Note also that, with this scheme, it is impossible for all operators to achieve higher payoff than the NE payoff; even for the case of 2 operators, one of them will always be lower than the NE payoff. This is not the case for our scheme, since, by definition, all players will get at least as much as the NE payoff.

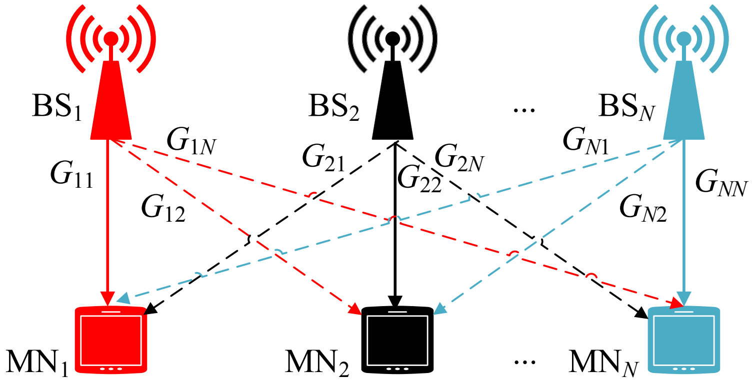


Fig. 6.2: Each operator i owns one Base Station, BS_i, and serves one Mobile Node, MN_i. We denote the path gain between BS_i and MN_j as G_{ij}.

In [107], pricing is used as a way to maximize the sum of payoffs. The authors show that the utility function we have used belongs to a family of utility functions named Type II utilities. They then prove, by using properties of supermodular games, that their approach maximizes the social welfare when the number of players N=2. Our scheme achieves the maximum sum of payoffs as well, provided that the maximum possible power reduction is asked for in the bargaining phase. The advantage of our approach is that the required level of cooperation is lower. Indeed, with our scheme, a node i should only know the exact level of the interference that it receives from node j to decide upon the level of its offer. This information (which, for the case of 2 operators, can be easily computed by the uplink) is also needed in [107]. Moreover, in [107], each node should also know the pricing profile of the other node (*i.e.*, how much that node charges for the interference it receives) in order to update its transmission power. In the general case with N operators, with our scheme, node i still only needs to know the same information as with the case of 2 operators. On the other hand, in [107], the level of the information increases significantly: node i should know the exact level of interference experienced by all other N-1 nodes, as well as their pricing profiles.

6.3 System Model

We consider N operators sharing a channel of bandwidth B at a common physical area. We focus on the downlink, as the traffic in this direction is typically heavier; however, our approach can be applied to the uplink as well. As Fig. 6.2 shows, operator i owns one Base Station (BS), BS_i, and serves one Mobile Node (MN), MN_i. We consider only one MN per operator, assuming that each operator still has its own exclusive band, where it serves the rest of its MNs. Note that our approach is also directly applicable to the case of multiple

Table 6.1: Game formulation.

Set of players	Set of nodes $\mathbf{N} = \{1, 2, \ldots, N\}$
Strategy of player i	$P_i \in [0, P_{\max}]$
Utility function for player i	U_i=$c_i B \log(1 + \text{SIR}_i)$

BS/MN pairs per operator provided that there is network planning so that BSs of the same operator do not interfere with each other. Dealing with co-interference (*i.e.*, interference from BSs of the same operator) in the shared spectrum band is left as future work.

Each operator i controls the discrete power $P_i \in \{P_{\min}, \ldots, P_{\max}\}$ of BS_i and charges MN_i proportionally to the throughput that it receives. The throughput of MN_i is defined as $T_i = B \log(1 + \text{SIR}_i)$, where SIR_i is the Signal-to-Interference Ratio and $G_{ji} \in (0, 1)$ is the path gain between BS_j and MN_i:

$$\text{SIR}_i = L \frac{G_{ii} P_i}{\sum_{j \neq i}^{N} G_{ji} P_j}.$$

Since we assume an interference-dominated environment, we ignore the thermal noise power.

In Table 6.1, we model this setup as a non-cooperative game with the players being the N operators. The strategy of each player i is the selection of the transmission power P_i; the payoff that it receives is $U_i = c_i T_i$, where c_i is a positive constant. We assume that MN_i is interested in downloading files, meaning that it is willing to pay more for a better download rate. For simplicity and ease of exposition, we assume that each MN has neither a minimum nor a maximum data rate requirement.

Each player aims at maximizing its payoff. It is easy to check that this game has a unique Nash Equilibrium (NE), at which all BSs transmit at $P_{\max}$ [108]. Let U_i^* be the NE payoff for player i and U_i' be its payoff at another operating point. The questions that we address are the following:

1. Is there another operating point where for each player i its payoff $U_i' \geq U_i^*$ and, for at least one of them, the strict inequality holds?

2. If so, how can we find it?

6.4 Analysis

We assume that the operators, though still selfish, decide to cooperate by applying a joint power control and bargaining scheme, in particular by using in part of the revenue accumulated from the services they have offered to their associated MN in the past. In this case, one operator, say OP_1, makes a "take it or leave it" offer to another one, say OP_2, of the form: "I offer you $e_{1,2}$ units if you reduce your power by a factor of M".

Defining how OP_2 is chosen is not critical, and goes beyond the scope of this work: A simple idea is that OP_1 chooses randomly OP_2. Alternatively, if OP_1 has some information on the exact level of interference that MN_1 receives from each other BS, it can make a more targeted offer, e.g., to the OP_2 that causes the greatest amount of interference.

Clearly, for the bargain to be beneficial for both operators, the following two conditions must hold:

$$U_1' - e_{1,2} \geq U_1^* \Leftrightarrow \tag{6.1}$$

$$c_1 B \log\left(1 + L\frac{G_{11}P_{\max}}{\sum\limits_{j\neq 1,2}^{N} G_{j1}P_{\max} + G_{21}\frac{P_{\max}}{M}}\right) - e_{1,2} \geq c_1 B \log\left(1 + L\frac{G_{11}P_{\max}}{\sum\limits_{j\neq 1}^{N} G_{j1}P_{\max}}\right). \tag{6.2}$$

$$U_2' + e_{1,2} \geq U_2^* \Leftrightarrow \tag{6.3}$$

$$c_2 B \log\left(1 + L\frac{G_{22}\frac{P_{\max}}{M}}{\sum\limits_{j\neq 2}^{N} G_{j2}P_{\max}}\right) + e_{1,2} \geq c_2 B \log\left(1 + L\frac{G_{22}P_{\max}}{\sum\limits_{j\neq 2}^{N} G_{j2}P_{\max}}\right). \tag{6.4}$$

From (6.2) and (6.4), when the corresponding equalities hold, we can compute the maximum offer, $e_{1,\max}$, that OP_1 is willing to make, as well as the minimum offer, $e_{2,\min}$, that OP_2 is willing to accept.

If $e_{1,\max} \geq e_{2,\min}$, then OP_1 can find an offer that OP_2 will accept. If a successful negotiation takes place, then BS_1 transmits at $P_{\max}$ and BS_2 transmits at $\frac{P_{\max}}{M}$. In this case, each operator that does not take part in the negotiation increases its payoff as well. This is due to the fact that the throughput of their associated MN is increasing, as they receive less interference from BS_2. Otherwise, no successful bargaining can take place, and all nodes continue to transmit at $P_{\max}$, as this is the NE operating point.

This joint power control and bargaining scheme could be used instead of the default power control scheme that the devices normally follow. A description of the modifications that are needed (at a high level) follows: *(i)* BS_1 sends a signal to BS_2, *i.e.*, the BS to which BS_1 wants to make an offer. BS_1 specifies its offer and the power reduction it requests. *(ii)* BS_2 evaluates the offer as described above and sends a signal to BS_1 with its decision. *(iii)* If BS_2 accepts the offer, it reduces its power to the requested level. Otherwise, it automatically applies the standard power control scheme that it adopts. Note that BS_1 does not need to modify its power control scheme in response to the decision of BS_2.

Steps *(i)*-*(iii)* are repeated in each transmission round. A complementary protocol should define whether a BS (and its associated operator) will make or receive an offer in each round. We will present a simple scheme where BS_1 makes successive offers to BS_2. When BS_1 is not interested in making offers any more to BS_2, a different pair should be selected (and BS_1 may then receive an offer). Other models can be considered as well. For example, in Chapter 3, we have adopted a different model where, in each round, nodes independently decide whether they are interested in making or receiving an offer and then broadcast their status to the rest of the nodes. Finally, at a different time-scale, bank transactions among the operators should take place to exchange the agreed amount of money for each successful negotiation.

An operator is interested in knowing: *(i)* Given a power reduction M, can it make a successful offer? *(ii)* If so, which is the minimum offer that it should make (clearly, this one will maximize its payoff)?

Note that if the operator knew all the path gains and other parameters, then it could easily compute whether it could make an offer or not and, if so, which would be the optimal offer (*i.e.*, the one that will maximize its payoff). However, in the general case, the operator cannot "guess" whether its offer for a requested power reduction will be accepted. A distributed strategy is to start by making its maximum offer to the other operator. All quantities for the computation of $e_{1,\max}$ from (6.2) can be computed by OP_1. Note that this means that the interference that BS_2 creates to MN_1 should be estimated. Since each operator knows that all other operators transmit at $P_{\max}$, the only element that should be estimated is the path gain G_{21}. This is already known for the case of two operators. In the general case, it can be estimated using pilot transmissions. Note also that in case that the path gains are varying with time (due to mobility, fading, etc.), our approach continues to hold provided that the changes in the topology are much slower than the time needed for the operators to make a decision using our scheme.

If the offer is rejected, then the operator has no motivation to make another offer for this requested power reduction, and it should choose a different operator to negotiate with. Otherwise, in subsequent rounds of negotiations, it can reduce its offer by small amounts, to see if it can further improve its payoff. If this is the case, the operator that accepted the offer will have its payoff reduced; however, its payoff will be still higher than the NE payoff without bargaining (otherwise, it would have rejected the offer). In any case, the player that receives an offer is in a more privileged position than the player that makes the offer in the sense that, as its payoff is reduced, it could decide to take the risk of rejecting the offer, so that the operator that makes the offer starts to increase its offer again. This is due to the fact that an operator cannot estimate the minimum offer that it should make.

A different strategy for the node making the offer would be to start with a (random) offer, e.g., with a fraction of the maximum offer that it could make, and then increase it by small amounts. Then, the operator should stop offering the first time that its offer gets accepted. However, the disadvantage of this strategy is that some rounds may be wasted with the BSs transmitting at the NE without bargaining, since some offers (in the worst case, all offers) may be lower than the minimum offer that the other operator could accept.

6.5 Analysis for $N = 2$ Operators

We now investigate under which circumstances a successful bargain may arise, for the special case where there are $N = 2$ operators, denoted by OP_1 and OP_2, with a common charging parameter $c_1{=}c_2{=}c$. This case provides intuition about what happens in the general case. Furthermore, since in many markets there are indeed only 2 operators, it also is of practical interest.

Theorem 8. *Let* $q \triangleq \frac{G_{11}}{G_{21}}$ *and* $r \triangleq \frac{G_{22}}{G_{12}}$ *be the ratios of the path gain coefficient of the associated BS to the path gain coefficient of the interfering BS.*

1. *If* $M \geq \max\{1, \frac{r}{q}\}$*, then* $e_{1,\max} \geq e_{2,\min}$.
2. *If* $M \geq \max\{1, \frac{q}{r}\}$*, then* $e_{2,\max} \geq e_{1,\min}$.

Proof. We sketch the proof focusing on case 1 (case 2 is treated similarly). Starting from (6.2) and (6.4), the inequality $e_{1,\max} \geq e_{2,\min}$ becomes:

$$M^2 - \left(1 + \frac{r}{q}\right) M + \frac{r}{q} \geq 0 \Leftrightarrow (M-1)\left(M - \frac{r}{q}\right) \geq 0.$$

This holds for $M \geq \max\{1, \frac{r}{q}\}$. □

Note that as M expresses how many times the power will be reduced, it is by definition greater than 1. Therefore, if $r \leq q$, then, for any requested reduction of the power from OP_1, there will be an interval $[e_{2,\min},\ e_{1,\max}]$ where an offer will be accepted. If $r > q$, then this interval exists for $M > \frac{r}{q}$, therefore for some power reductions an offer will never be accepted.

A direct conclusion from Theorem 8 is presented in Proposition 3.

Proposition 3. *For any requested power reduction* M*:*

- *If* $r < q$ *then* OP_1 *can make a successful offer.*
- *If* $r > q$*, then* OP_2 *can make a successful offer.*
- *if* $r = q$*, then both operators can make a successful offer.*

In other words, through this joint power control and bargaining scheme, operators can always end up at a point that is more preferable for both of them than the NE where both transmit with maximum power $P_{\max}$.

A different approach would be the case where both operators make concurrent offers. We have not adopted it for two reasons. Firstly, in this case bargaining would be an (at least) a two-step procedure, which adds complexity and requires more coordination. In the first step, operators should announce the power reduction that they request. In the second step, they should announce their offers based on the requested power reductions. Note that with this two-step procedure, either both offers should be accepted, or none. Merging these steps to one may lead to payoffs that are lower than the NE without bargaining. This is due to the fact that the operators should know the requested power reduction before deciding upon the level of their offers.

Secondly, we can state and prove Theorem 9, that shows that our 1-direction bargaining is equivalent to a 2-direction bargaining scheme as described above. Therefore, due to its simplicity, it is preferable to adopt it.

Theorem 9. *Consider 2 operators,* OP_1 *and* OP_2*, that negotiate with each other using a 2-direction bargaining.* OP_1 *asks* OP_2 *to reduce its power* l *times and offers* e_1*.* OP_2 *asks* OP_1 *to reduce its power* m *times and offers* e_2*. Clearly,* $l \neq m$*, otherwise there is no point in bargaining since the final state will be equivalent to the original one. The following holds:*

1. *If* $l < m$*, then there is another 1-direction bargaining where only* OP_2 *makes an offer. It asks* OP_1 *to reduce its power* $\frac{m}{l}$ *times and offers* $e_2^{'}$=e_2-e_1*. This 1-direction bargaining is equivalent with the 2-direction bargaining, i.e., the operating point that* OP_1 *and* OP_2 *will end after the application of any of them will be the same.*

2. *If* $l > m$*, then there is another 1-direction bargaining where only* OP_1 *makes an offer. It asks* OP_2 *to reduce its power* $\frac{l}{m}$ *times and offers* $e_1^{'}$=e_1-e_2*. This 1-direction bargaining is equivalent with the 2-direction bargaining.*

Proof. At the NE, OP_1 and OP_2 receive correspondingly:

$$U_1 = c_1 B \log\left(1 + \frac{G_{11}}{G_{21}}\right), \tag{6.5}$$

$$U_2 = c_2 B \log\left(1 + \frac{G_{22}}{G_{12}}\right). \tag{6.6}$$

After the announcement of the offers, OP_1 should transmit at $P_1^{'}$=$\frac{P_{\max}}{m}$ and OP_2 at $P_2^{'} = \frac{P_{\max}}{l}$. Let $M = \frac{P_1^{'}}{P_2^{'}}$=$\frac{l}{m}$. Then, we can compute the revenue that they will receive if they accept the offers:

$$U_1^{'} = c_1 B \log\left(1 + \frac{MG_{11}}{G_{21}}\right) - e_1 + e_2, \tag{6.7}$$

$$U_2^{'} = c_2 B \log\left(1 + \frac{G_{22}}{G_{12}M}\right) + e_1 - e_2. \tag{6.8}$$

For a successful negotiation, (6.7) $\geq$ (6.5) and (6.8) $\geq$ (6.6).

Consider case 1, where $l < m \Leftrightarrow M < 1$. Clearly, e_2 should be bigger than e_1, otherwise OP_1 will never accept the offer. If OP_2 would have offered $e_2^{'}$=$e_2 - e_1$ asking for a power reduction of $\frac{m}{l} = \frac{1}{M}$ times, then we end up with (6.7) and (6.8). Case 2 is treated similarly and we omit the proof. $\square$

Having settled that 1-way negotiations are optimal, we return to the point of the efficiency of the resulting point after a successful negotiation. We state Theorem 10 that specifies the socially optimal operating point, *i.e.*, the one that maximizes the revenue sum.

Theorem 10. *The maximum sum of revenues of the operators corresponds to one of the following operating points:* A_1=(P_1, P_2)=$(P_{\max}, P_{\min})$ *or* A_2=(P_1, P_2)=$(P_{\min}, P_{\max})$.

Proof. Let $V = \frac{P_1}{P_2}$. We look for the global maximum of the function

$$f(V) = cB \log(1 + qLV) + cB \log\left(1 + L\frac{r}{V}\right), \tag{6.9}$$

where $V \in \left[V_{\min} \triangleq \frac{P_{\min}}{P_{\max}}, V_{\max} \triangleq \frac{P_{\max}}{P_{\min}}\right]$ and q, r, are defined in Theorem 8. Taking the first derivative of f and setting it equal to zero, we show that:

- When $V_{\min} < \sqrt{\frac{r}{q}} \triangleq t$, f is strictly decreasing in $[V_{\min}, t]$ and strictly increasing in $[t, V_{\max}]$. Therefore, its global maximum is either at $V_{\min}$, *i.e.*, at A_2, or at $V_{\max}$, *i.e.*, at A_1.

- When $V_{\min} \geq t$, f is strictly increasing in $[V_{\min}, V_{\max}]$, having its global maximum at $V_{\max}$.

- When $V_{\max} \leq t$, f is strictly decreasing in $[V_{\min}, V_{\max}]$, having its global maximum at $V_{\min}$.

$\square$

We now state Theorem 11 that clarifies when our bargaining scheme can lead to the socially optimal operating point.

Theorem 11. *Let* A_1 *(resp.* A_2*) be the point that maximizes the social welfare of the system. Then, if* OP_1 *(resp.* OP_2*) applies the bargaining scheme with* $M = \frac{P_{\max}}{P_{\min}}$, *it will reach* A_1 *(resp.* A_2*).*

Proof. Let A_1 be the global maximum of the function f, as defined in Theorem 10. By definition:

$$f(A_1) \geq f(A_2) \Leftrightarrow \tag{6.10}$$

$$\log(1 + qLV_{\max}) + \log\left(1 + \frac{Lr}{V_{\max}}\right) \geq \log\left(1 + \frac{Lq}{V_{\max}}\right) + \log(1 + LrV_{\max}). \tag{6.11}$$

After some algebra, (6.11) becomes $(q-r)V_{\max}^2 \geq q - r$, which holds when $q \geq r$, since $V_{\max} > 1$. From Proposition 3, when $q \geq r$, OP_1 can make a successful offer that leads to A_1. The proof for OP_2 is omitted. □

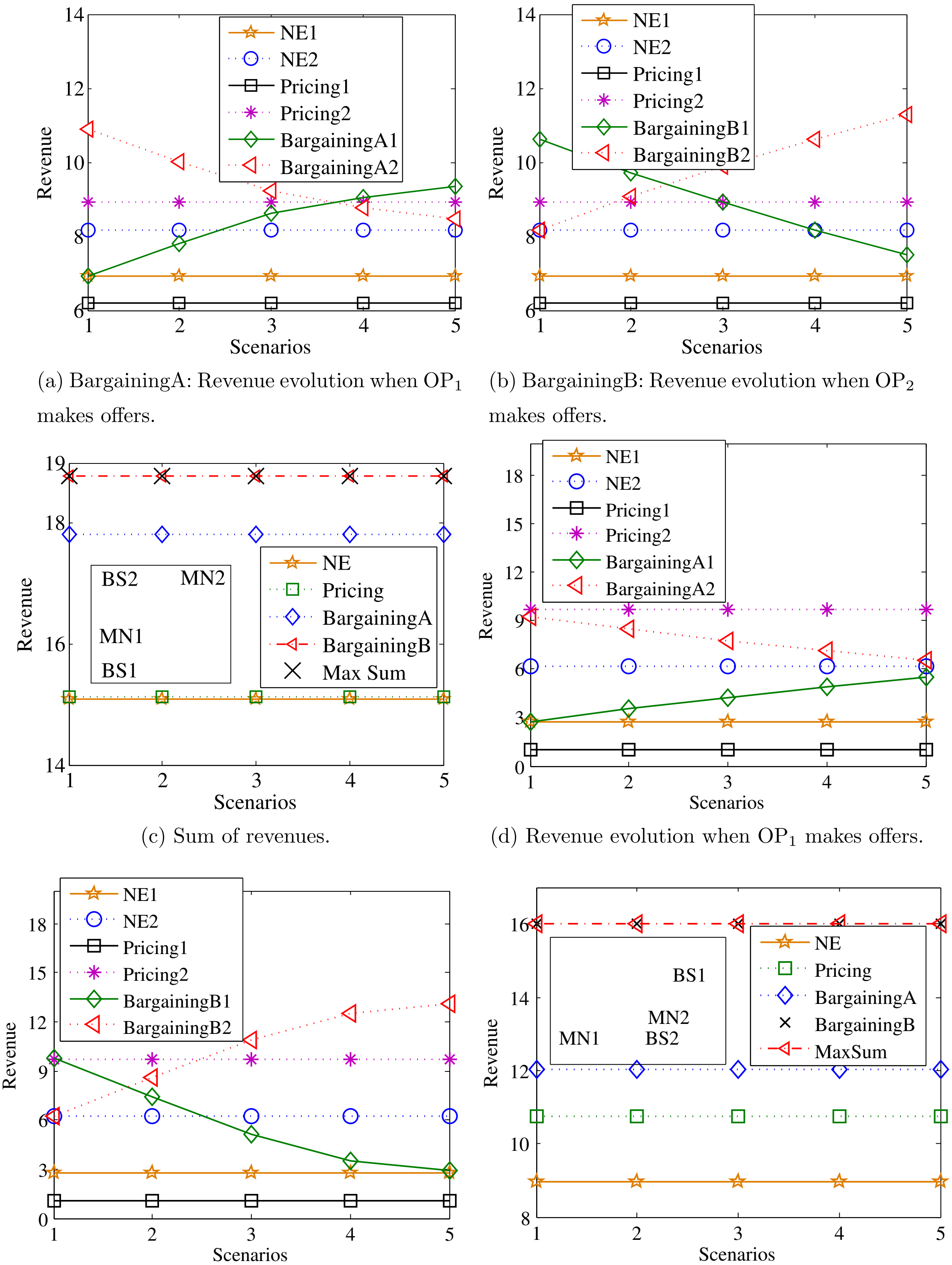
6.6 Performance Evaluation

We illustrate our bargaining scheme for N=2 operators, and when each operator asks for the maximum possible power reduction M=32 [109]. We present two variations: BargainingA, where OP_1 makes successive offers starting from a $e_{1,\max}$ offer and progressively reducing its offer each time by 15%, and BargainingB (similarly, but OP_2 makes offers). We compare them with the NE, the NE that arises after the application of pricing [33] with a linear pricing factor z (denoted as Pricing), and finally with a scheme that maximizes the sum of revenues (denoted as MaxSum) [107].

All schemes are compared in terms of the revenue they achieve for the 2 operators. The notation Scheme$i$ refers to the payoff of OP_i with this scheme (e.g., BargainingA1 means that we compute the payoff of OP_1 with the scheme BargainingA); Scheme refers to the sum of payoffs (e.g., BargainingA means that we compute the sum of payoffs with this scheme).

In Fig. 6.3a, we present the operating points arrived at by BargainingA (the parameters for this particular topology are shown in the legend) for the topology of Fig. 6.3c. At each point, the revenues of both operators are larger than the NE revenues. At the first three points, they are larger than the Pricing scheme as well. Similar trends appear in Fig. 6.3b, with BargainingB. In Fig. 6.3c, we show that both schemes outperform strictly both NE and Pricing. Actually, BargainingB also maximizes the social welfare.

In Fig. 6.3d-6.3f, we present the same set of diagrams for the topology of Fig. 6.3f, where both MNs are closer to BS_2 than to BS_1. Note that, as shown in Fig. 6.3d, for BargainingA, we cannot find an operating point where both operators achieve higher payoff than Pricing. Still, all operating points are more preferable than the NE without pricing. For BargainingB, there are 3 such points that correspond to the last 3 scenarios as depicted in Fig. 6.3e. Again, both BargainingA and BargainingB achieve higher social welfare than Pricing.



(a) BargainingA: Revenue evolution when OP_1 makes offers.

(b) BargainingB: Revenue evolution when OP_2 makes offers.

(c) Sum of revenues.

(d) Revenue evolution when OP_1 makes offers.

(e) Revenue evolution when OP_2 makes offers.

(f) Sum of revenues.

Fig. 6.3: Revenue under NE, Pricing, BargainingA, BargainingB, and MaxSum. Common parameters: L=4, B=2, c=1, z=1.5. In Fig. 6.3a-6.3c, G_{11}=0.5, G_{21}=0.2, G_{12}=0.05, G_{22}=0.2. The topology is shown in Fig. 6.3c. In Fig. 6.3d-6.3f, G_{11}=0.2, G_{21}=0.5, G_{12}=0.5, G_{22}=0.95. The topology is shown in Fig. 6.3f.

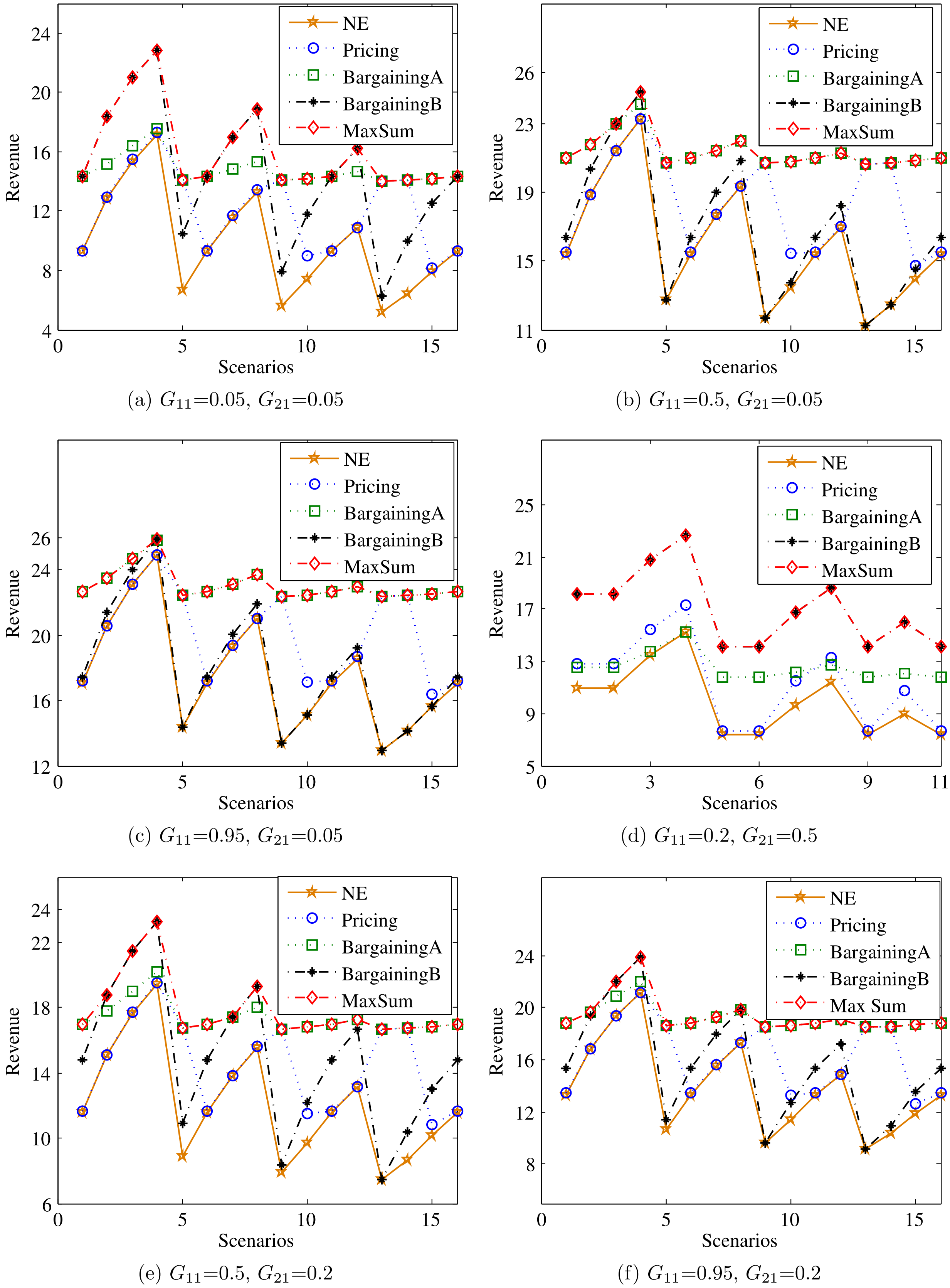
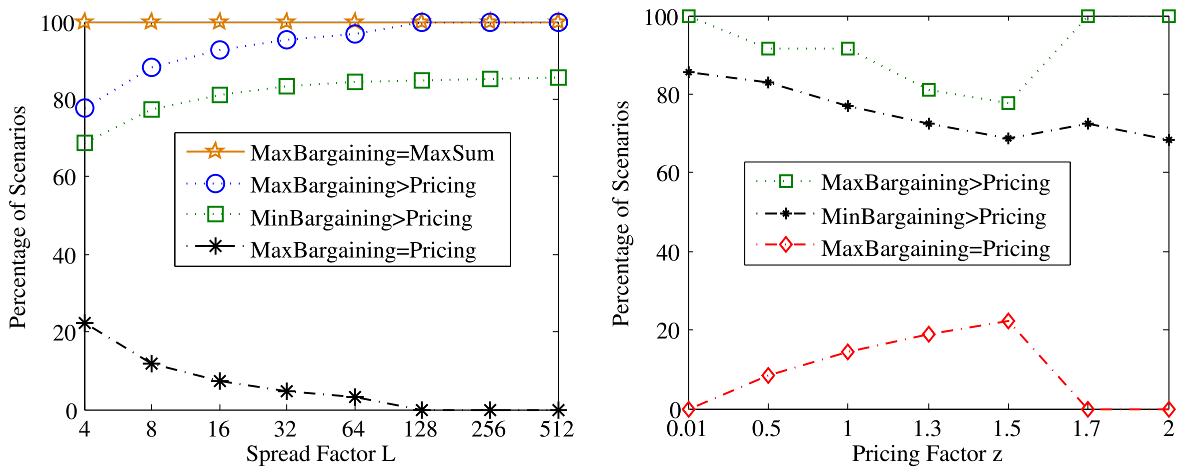


Fig. 6.4: Sum of revenues under NE, Pricing, BargainingA, BargainingB, and MaxSum. G_{12} & $G_{22} \in \{0.05, 0.2, 0.5, 0.95\}$, L=4, B=2, c=1, z=1.5.

Table 6.2: Scenarios of Fig. 6.4: Comparison of sum of payoffs for MaxBargaining, MinBargaining, and Pricing.

Figure	% MaxBargaining > Pricing	% MaxBargaining=Pricing	% MinBargaining > Pricing
6.4(a)	75	25	75
6.4(b)	75	25	62.5
6.4(c)	75	25	62.5
6.4(d)	100	0	60
6.4(e)	75	25	75
6.4(f)	75	25	68.75



(a) Sum of revenues as a function of L. Path gains $G_{ij} \in \{0.01, 0.06, 0.11, \ldots, 0.96\}$, z=1.5.

(b) Sum of revenues as a function of z. Path gains $G_{ij} \in \{0.01, 0.06, 0.11, \ldots, 0.96\}$, L=4.

Fig. 6.5: Sum of revenues under NE, Pricing, BargainingA, BargainingB, and MaxSum. B=2, c=1.

In Fig. 6.4, we present 6 diagrams that show the evolution of the sum of revenues. In each diagram, the ratio of the path gains for MN_1 is constant and we modify the ratio of the path gains for MN_2 as depicted in the corresponding legend. As specified by Theorem 11, in all scenarios, MaxBargaining=max{BargainingA, BargainingB} achieves the maximum sum of revenues.

In Table 6.2, we present, for each diagram, the percentage of scenarios that MaxBargaining and MinBargaining=min{BargainingA, BargainingB} outperforms Pricing, as well as the cases where MaxBargaining equals Pricing. In all diagrams, for the vast majority of scenarios at least MaxBargaining performs better than Pricing and, in many scenarios, this is the case for MinBargaining.

In Fig. 6.5a, out of the 160000 possible combinations of the path gains G_{ij} that belong to the set $\{0.01, 0.06, 0.11, \ldots, 0.96\}$, we have excluded the artificial scenarios where both MN_1 is closer to BS_2 than to BS_1 and MN_2 is closer to BS_1 than to BS_1. We have simulated the remaining 124000 scenarios. Simulations verify that in all cases MaxBargaining coincides with the MaxSum. Moreover, the sum of revenues with MaxBargaining strictly outperforms Pricing in 80% to 95% of the scenarios for small spread factors ($L \leq 64$) and 100% of scenarios for large spread factors. Furthermore, in the majority of scenarios (70% to 85%), even MinBargaining strictly outperforms Pricing.

In Fig. 6.5b, we present the sum of revenues as a function of the pricing factor z. Our experimental study reveals that the best pricing factor is 1.5. We have noticed the same trend for other spread factors as well. This is the reason that we have used this value of z for Pricing in Fig. 6.5a. For other values of z, the sum of revenues with Pricing is significantly lower.

6.7 Conlusions

The goal of this chapter was to study the emerging concept of licensed spectrum sharing, where no exclusive rights are given to any single operator, under the prism of game theory. Assuming that the operators charge their customers based on the throughput that they offer to them, we define a non-cooperative game that has a unique Nash Equilibrium, where all operators transmit at P_{max}. Our work starts with the observation that the operators, though still selfish, have motivation to cooperate to end up at more efficient operating points that increase their revenues. We develop an incentive-based mechanism that enables this cooperation, by combining traditional power control with bargaining, using "take it or leave it" offers. Then, we show that, even in the general case, where N operators share the same portion of the licensed spectrum, there are conditions that guarantee that a more efficient operating point may arise. We then deepen our results for the special case of two operators. *(i)* We show that for any level of requested power reduction, at least one of the two operators can make an offer than can be accepted and leads to a more efficient operating point than the NE. *(ii)* We derive a set of bargaining strategies that lead to the operating point that maximizes the social welfare of the system, demanding less exchange of messages than the state-of-the art. *(iii)* We show that our scheme outperforms the standard idea of linear pricing of the transmission power as a way of finding more efficient operating points in terms of both revenues per operator and sum of revenues.

Chapter 7

Conclusions and Extensions

7.1 Conclusions

In this thesis, we study heterogeneous wireless networks that consist of autonomous nodes with possibly different QoS targets. These networks will be the norm in the forthcoming 5G era, and the efficient distributed management of the interference that arises due to the coexistence of these nodes is a prerequisite for their successful deployment. To combat this challenge, we combine powerful radio resource management techniques (power control and channel access) with game-theoretic concepts and tools. In this overall setting, we analyse such wireless networks aiming at two points: *(i)* We seek Nash Equilibria points. *(ii)* We use bargaining as a way to create incentives for nodes to find more efficient operating points than the Nash Equilibria; we propose schemes where nodes with various degrees of cooperation end up at these points and we study their properties and efficiency with mathematical analysis and simulations.

Besides our conclusions per chapter, we discuss here some general lessons learnt from this research:

- Our study in Chapters 3-6 confirms that nodes can coexist efficiently by simply deciding on their own whether to transmit or not and the level of their transmission power. In other words, even though modern wireless networks are complex and consist of nodes that may belong to different operators or have different targets, simple classical radio resource management methods such as power control and channel access that need minimal cooperation are enough for a significant reduction of the interference that arises in these challenging scenarios.

- In some cases, the performance of the nodes at a Nash Equilibrium (NE) point, which is the naturally resulting operating point, is acceptable and there is no need to seek a more efficient operating point. We observed this in Chapters 4 and 5. In Chapter 4, where we study a challenging environment with small cell nodes and macrocell nodes having different utility functions, extensive simulations that are based on realistic assumptions and topologies show that, in most scenarios, smooth coexistence of all nodes is feasible. In Chapter 5, where we study the channel access competition in linear and tree device-to-device networks, we show that any NE is Pareto optimal. Moreover, we find that each NE has about the same number of successful transmissions, meaning that, from a network operator's perspective, each NE is almost equally preferable.

- On the other hand, in some other cases, nodes achieve poor performance at a NE and a better operating point would be welcome. We notice this outcome in Chapters 3 and 6, where simple but well-adopted power control schemes lead to Nash Equilibria where players are unhappy with their performance; in Chapter 3 many players cannot achieve their SINR targets, whereas in Chapter 6 their revenues are small. To combat this problem, we introduce bargaining among unsatisfied players as a way to create incentives to further update their transmission power. Using "take it or leave it" offers, players negotiate pairwise in order to find more efficient operating points. As a result, in Chapter 3, more players achieve their targets, whereas in Chapter 6 the operators increase their revenues. We show that these joint power control and bargaining distributed schemes, besides being superior than the NE, perform significantly better than the well-adopted idea of applying pricing of the transmission power as a way to find better operating points. Finally, our bargaining scheme in Chapter 6 admits an appealing incentive-compatible feature: No player receives lower utility function than its utility at the NE. This is not the case for the pricing schemes.

- The level of cooperation among the nodes in modern wireless networks may influence their performance. In this thesis, we study non-cooperative game theory with nodes being selfish. This, of course, does not forbid a node to exchange information with other nodes. Indeed, increasing by even a small amount the number of nodes with which a node exchanges messages can have a significant impact to its performance. This is the case in Chapter 5, where we compare approaches with nodes exchanging messages in a 2-hop neighbourhood versus a 3-hop neighbourhood. For the latter case, we show that the nodes converge faster to a NE and their convergence is monotonic, meaning that the percentage of nodes that finalize their strategy is increasing per round.

- In such wireless networks, theoretical results can be directly transformed into distributed schemes of practical interest. We observed this feature in Chapters 5 and 6.

In Chapter 5, motivated by our study for the structure of the resulting Nash Equilibria, we propose the powerful scheme described in Section 5.4.3, where nodes exchange messages in a 3-hop neighbourhood. In Chapter 6, by deriving conditions for a successful bargaining, we propose schemes that are guaranteed to end up at more efficient points than the NE; we can even derive simple strategies, with lower communication overhead than pricing schemes, that maximize the social welfare of the network.

7.2 Directions for Future Work

In this section, we discuss some ideas for further research in the areas that we studied in this thesis.

One general direction is to study our approaches in the context of *repeated* non-cooperative games [68], where a given non-cooperative game is played multiple times by the same set of players. In this context, the game that is repeated is called the stage game. Note that the notion of repetition is different from the iterations that are needed so that a stage game converges to a NE. In a repeated game, there is an outer loop that corresponds to a different repetition of the stage game and, possibly, an inner loop that corresponds to the rounds of each repetition of the stage game, where nodes update their strategies aiming at arriving to a NE.

We present next a brief example of a repeated game formulation in the context of Chapter 5. Consider a linear D2D topology that consists of 4 nodes: $\{A - B - C - D\}$. (W, T, T, W) is a strategy vector that corresponds to a NE for the multicast case of this stage game under payoff model 1 (as defined in Table 5.1). However, if there were two repetitions of the game, nodes B and C could make an agreement to transmit in different rounds. For example, in round 1, the strategy vector would be (W, T, W, W), whereas in round 2 the strategy vector would be (W, W, T, W). Clearly, in both rounds, neither node A nor node D have motivation to change their strategy from W. Therefore, the payoff vector $\mathbf{u} = (u_A, u_B, u_C, u_D)$ with elements the sum of payoffs from these two rounds will be equal to $\{0, 2 - c, 2 - c, 0\}$. This is due to the fact that when a node waits (W), it receives a zero payoff. On the other hand, when a node transmits (T), it has a fully-successful transmission and its payoff is $2 - c$. On the other hand, if the nodes choose the strategy vector (W, T, T, W) in both rounds, they will receive $\{0, 2 - 2c, 2 - 2c, 0\}$, since, in each round, nodes B and C will have a semi-successful transmission that corresponds to a payoff of $1 - c$. Clearly, a more efficient point arises when nodes B and C exploit the fact that the game is repeated. Examining systematically these cases for both multicast transmissions and unicast transmissions is an interesting topic.

Another general direction is to consider coalitional game theory, a topic that we briefly discussed in Chapter 6. Instead of a non-cooperative formulation with bargaining, in the settings of Chapters 3 and 6, players could either seek the Nash Bargaining Solution or form coalitions. Towards this direction, the decision of which player is going to make or receive an offer to update its power can be based on a model of alternating offers [110], where players rotate their roles. Moreover, typical questions of coalitional game theory should arise and be exploited [68]: Is the grand coalition, *i.e.*, a coalition that includes all players, stable or players have motivation to form different coalitions? Do the players have incentives to compute the Shapley value that leads to a fair sharing of their revenues? If not, how should the players share their revenues?

From a practical perspective, it is interesting to compare the communication overhead of a scheme that leads to any of these solution concepts with the schemes that we propose in this thesis. It is worth mentioning that designing distributed schemes with acceptable convergence time is a prerequisite in modern heterogeneous networks irrespective of the level of cooperation among the nodes.

Besides the above general directions, we also point out some specific issues per chapter that could be explored further.

In Chapter 4, we made the typical assumption that the pricing coefficient c_i of the cost function in the game formulation (as defined in Table 4.1) is constant. It is interesting to examine the effect of c_i on the efficiency of the resulting NE. Towards that direction, an interesting application would be to model this two-tier small cell network as a Stackelberg game [111], where there is a hierarchy among the players and one or more players (the leaders) announce their strategies before the other players (the followers) choose their strategies. In the context of small cell networks, the leaders would be the operators that announce their pricing policy aiming at maximizing their revenues, whereas the followers would be the (small cell) mobile nodes aiming at maximizing their utility functions as defined in our setup.

In Chapter 5, a natural extension is to study the channel access competition problem in general D2D networks, where the underlying graph is neither a tree nor a line. It is an open issue whether the proposed schemes end up at a NE under these general setups or some modifications should be made by taking into consideration the existence of cycles in the graphs. Moreover, it would be interesting to derive bounds on the Price of Anarchy [112], *i.e.*, the ratio between the NE with the minimum sum of payoffs and a centralized solution that maximizes the sum of payoffs, for various topologies. We could use this metric of the efficiency of the NE to explore whether there are NE that are significantly more preferable than others.

In Chapter 3, it is interesting to explore different mechanisms that define the level of an offer and the conditions that should be fulfilled so that an offer gets accepted. Towards this direction, a straightforward alternative mechanism that could be adopted is the one proposed in Chapter 6. Moreover, when a Buyer i chooses a Seller j, i should know the receiving power from j to decide upon the level of its offer. This piece of information can be passed by a unicast message. If we had assumed that each Seller broadcasts this element to the set of Buyers, then i could choose j more efficiently (e.g., by making an offer to the one that creates more interference to it). Another approach for i would be to make parallel offers to multiple Sellers to reduce their powers so as to achieve its target.

In Chapter 6, it is an ongoing work to simulate scenarios with $N > 2$ operators that apply our bargaining mechanism as described in Section 6.4. Moreover, we plan to evaluate our mechanism in terms of social welfare, examining whether a theorem similar to Theorem 11 can be proved for N operators. Finally, a natural extension is to include the more realistic case where a customer has made an agreement with his operator that he will not be charged when his throughput is lower than some minimum value. In this case, each operator should firstly compute the minimum possible power (*i.e.*, the maximum possible power reduction M) that it can transmit at to guarantee the predefined minimum throughput $T_{\min}$. For example, for the case of $N = 2$ operators, the following condition should be satisfied for Operator 1:

$$B \log\left(1 + \frac{G_{11}}{MG_{21}}\right) \geq T_{\min}.$$

After some algebra, this is equivalent to:

$$M \leq \frac{G_{11}}{G_{21}} \cdot \frac{1}{2^{T_{\min}/B} - 1}.$$

Therefore, independently of the level of the received offer, there is an upper bound on the maximum power reduction M that an operator can negotiate. If the above condition holds, then the operator will take part in the bargaining using the formulas that we have presented in Section 6.5. A thorough performance evaluation of this scenario would be very interesting.

Appendix A

Abbreviations and Acronyms

2G	2^{nd} Generation
3G	3^{rd} Generation
4G	4^{th} Generation
5G	5^{th} Generation
BS	Base Station
CDF	Cumulative Distribution Function
CDMA	Code Division Multiple Access
D2D	Device-to-Device
DBFM	Distributed Bargaining Foschini-Miljanic
FCC	Federal Communications Commission
FDMA	Frequency Division Multiple Access
FM	Foschini-Miljanic
FPRP	Five-Phase Reservation Protocol
HD	High Definition
KKT	Karush-Kuhn-Taker
LTE	Long Term Evolution
MN	Mobile Node
NBS	Nash Bargaining Solution
NE	Nash Equilibrium
OP	Operator
QoS	Quality-of-Service
Rx	Receiver
SaS	Soft and Safe
SCBS	Small Cell Base Station
SCMN	Small Cell Mobile Node
SINR	Signal-to-Interference plus Noise Ratio
SIR	Signal-to-Interference Ratio
TDMA	Time Division Multiple Access
Tx	Transmitter
UBPC	Utility-Based Power Control
VANETs	Vehicular Ad hoc NETworks

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