Plane Cover Multiple Access: A New Approach to Maximizing Cellular System Capacity

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Abstract—**We develop a new Media Access Control strategy, called Plane Cover Multiple Access (PCMA), that provides a means of allocating wireless bandwidth in a packet based cellular system. PCMA seeks to maximize the number of parallel transmissions among cells by defining virtual cells in which users transmit using a given reuse factor. By keeping the reuse factors low, system throughput can be maximized. We show that the throughput of a simple system designed using PCMA is up to 82% more efficient than CDPA, the best known alternative for the cellular, mobile environment.**

Keywords— **wireless, packet switching, frequency reuse, multiple access, cellular, Reuse Partitioning**

I. Introduction

THE growth of demand for wireless information services as we enter the new millennium is astounding. Not only are increasing numbers of individuals signing up for wireless voice service, but they are also demanding data services including e-mail, instant messaging, and Web browsing. In fact, it is predicted that within several years the number of wireless Internet subscribers will exceed the number of wired subscribers. Such a forecast is easy to believe, since the wired world has seen an enormous shift to data applications compared to voice, for both business and personal use. Given such an undeniable trend in wireless, current approaches of sending data over circuit-switched cell phone channels cannot be economically viable for long. It is well known that the bursty nature of data traffic demands packet-based transmission for efficiency, while voice can also be supported over a packet network [1]. If the network supports quality of service guarantees, voice performance can match that of conventional circuit switched networks. However, even if wireless providers remain with circuit-switching for voice, they will inevitably need to convert much of their allotted spectrum to packet-based transport in order to stay competitive.

The literature abounds with proposals to increase the efficiency of conventional circuit-switched, cellular voice systems [2]. Also, many proposals for packet access in a single-receiver radio environment have been advanced. However, there has been relatively little research into protocols to support data services over a multicell packet-based transmission system. The challenge here is to devise a TDMA-based access scheme which avoids the well-known inefficiencies of fixed allocation of resources across cells and implements dynamic resource sharing depending on traffic demands while being flexible enough to accommodate retransmissions of failed transmissions due to intercell collisions.

Even if users generated traffic intensities that resulted in packets being sent every time slot on all channels, standard one-seventh reuse would have poor system utilization, since each cell could only make use of $1/7$ of the time slotted channels, for maximum system utilization of 0.142. One-seventh reuse will be used for comparison purposes since it is used in real systems, and it is a baseline against which all other resource allocation strategies can be judged. However, when designing actual protocols, schemes based on the Reuse Partitioning [3] idea should be kept in mind. Such schemes have been shown to permit more single channel users by permitting more aggressive reuse than 1/7 in many cases, subject to channel interference conditions being met.

Consequently, to obtain better utilization of channels, a model could be investigated which includes two key assumptions. First of all, multiple users in the same cell share the same channel. Decades of experience with data networks have revealed that the resulting ”statistical multiplexing” leads to better efficiency. Consequently, the investigation of a scheme in which all users can potentially access time slots in all channels is a logical next step. Such a scheme is true TDMA over a single logical channel. Secondly, the reuse of time slots should be assumed to be variable, not fixed at a factor such as 7. In general, to achieve greater system utilization, more intense reuse must be invoked. One possibility is to permit more intense reuse to all users in the system, while also providing a means of tolerating the increased interference level, by either using an interference-robust modulation (e.g., spread-spectrum as in CDMA [4]) or providing error recovery mechanisms (e.g., by retransmission as in CDPA [5]). A second approach is to permit more intense reuse only to those users for which transmission quality will not be compromised. An example of such an approach is Reuse Partitioning, proposed in [3] for circuit-switching, in which users closer to base stations are granted more intense reuse since their signals are still received strongly enough to overcome inter-cell interference. In [6] a scheme is proposed to permit more aggressive use of time slots used to transport packets. The reuse is accomplished by applying a form of reuse partitioning under the assumption of sectorized antennas at the base stations and of narrowbeam antennas at the subscribers. As the authors focus on fixed-point wireless Internet applications, no mobility is considered and no Rayleigh fading is included in the analysis. Our work in this paper, on the other hand, while based on similar principles, addresses cellular systems with mobile users and applies to a more general scenario. We present an approach to seeking optimal reuse for mobile users in wireless *packet* environments called Plane Cover Multiple Access (PCMA).

PCMA avoids the high price of splitting resources ahead of time by allocating them to users based upon a per-user reuse factor. We define a function $f(x)$, whose domain is the set of all points in the plane, such that a user located at position x transmits using reuse $f(x)$. We define a *virtual cell* to be the set of all points within a system cell (the area served by a single base station) having the same reuse factor. Users in better position for signal reception (perhaps they are closer to base stations) are granted a lower reuse factor. Then, a greater number of such users can transmit on a system-wide basis. For example, the system can allow one user, with reuse factor equal to 1, per cell to simultaneously transmit. This is possible, since we can guarantee with acceptably high probability that each transmission will be received. In order to receive a given transmission, even in the presence of interfering transmissions in nearby cells, packet capture is exploited. Reuse factors greater than one are also considered. In this case a group of virtual cells assigned reuse factor $N > 1$ share their resources such that at most $1/N$ of them are simultaneously transmitting. For this reason, use of such a reuse factor is also often referred to as $1/N$ reuse in the literature.

A basic difference between our proposed scheme and those based on Reuse Partitioning is essentially in its being packet-oriented, which provides a greater degree of flexibility in the management of traffic. Also, the use of transmission resources in a dynamic TDMA fashion allows statistical multiplexing, soft handoff and macrodiversity to be supported. Finally, the circuit-switched schemes based on Reuse Partitioning [2] attempt to assign users to channels for which there will be no interference, or in the case of packets, no collisions. In PCMA there does not have to be any such global scheduling. The dynamic nature of the wireless channel and the randomness of transmitter locations within virtual cells permit many packets to be captured and correctly received. For packets not captured retransmission is used. Since collisions are expected, more aggressive reuse can be pursued, possibly leading to greater performance.

In this paper, we show how a statistical model of packet capture can be invoked which allows us to accurately predict the probability of successful packet transmission in a variety of scenarios. By tuning the system operation based on these success probabilities, we can achieve greater reuse of resources.

The remainder of this paper is structured as follows. Section 2 indicates the basic system assumptions. Section 3 contains a thorough analysis of several versions of PCMA. Section 4 explores the delay characteristics of PCMA. Section 5 contains a discussion including implementation issues. Section 6 is the conclusion.

II. System Assumptions

For simplicity of exposition we assume that separate frequency bands are used for the uplink (mobile to base) and downlink (base to mobile) directions. Furthermore, we assume that there is no a priori splitting of the spectrum into "channels" in the frequency domain. Rather, we view all uplink spectrum as one wide channel, or bit pipe. Resource sharing is accomplished by using TDMA. Thus, we form a series of equally sized time slots such that each slot can transmit exactly one packet. For simplicity of analysis, the overhead will be ignored. Since there is no splitting into channels, each cell could permit a transmission in each time slot. However, it will be seen that throughput, measured in successful packets/slot/cell, can be maximized by regulating transmissions, either by imposing a reuse factor on time slots or by limiting the rate at which transmissions can occur.

Users generate packets at varying rates and transmit them in the available time slots. For real-time traffic there are deadlines by when successful transmission must take place, while for best-effort service there are no such deadlines. In the latter case, however, we would like to transmit as soon as possible. We assume that a base station can successfully receive at most one packet per time slot.

We assume that each base station uses polling to assign slots to mobiles in its cell, and the overhead for the polling is minimal because the cells are small in size. The base stations do scheduling of mobile transmissions using whatever discipline they choose. We assume that at the start of each time slot the base sends a mobile ID number indicating which mobile in the cell is permitted to transmit in that slot. Since only one mobile per time slot is given permission there are no intra-cell collisions. The idea is for the base station to optimally schedule packet transmissions so that all packets meet their deadlines. Thus, the highest number of packets can be transmitted before their respective deadlines, maximizing system performance. We do not at present examine specific scheduling algorithms as in [7]. Rather, we focus on general techniques to schedule transmissions depending on mobile's positions within the system cells and the effects of such techniques on system throughput.

We assume a radio propagation model that accounts for Rayleigh fading resulting from multipath and an η-th power-loss law. The received power, W_R, from a transmitter at a distance r is given by

$$W_R = \alpha^2 A r^{-\eta} W_T \quad (1)$$

where α^2 is an exponentially distributed random variable with unit mean, which accounts for attenuation of the signal due to multiple transmission paths. The $Ar^{-\eta}$ term accounts for the power loss law, where A and $r^{-\eta}$ represent the constant and distance dependent attenuations, respectively. Finally, W_T is the transmitted power, which is multiplied by the terms accounting for multipath fading and path loss to give the received power. Note that in the literature η is typically assumed to be approximately 4. See for example [8].

The capture model we use assumes that a receiver can correctly receive a packet from intended transmitter x whose received power is W_x as long as we have $W_x / \sum_{i \neq x} W_i > b$, where W_i represents the received power of (interfering) transmitter i, and b is the threshold parameter. For example, $b \simeq 10$ dB is commonly used in GSM, where a Signal-to-Interference ratio of about 10 is deemed adequate for sufficient reception quality. In general, the threshold model is only used to make analytical computations possible. In [9] a simulation study was done using BPSK modulation and 511 bit packets that investigated how closely various capture thresholds model the actual probabilities of successful packet reception. Even in the absence of any error-correcting coding, it was found that using a capture threshold as low as 5 dB, or slightly greater than 3, still leads to pessimistic evaluations for the probability of successful packet reception. Consequently, we shall also use capture threshold $b = 4$ in the analysis, since it certainly is a lower bound on the performance achievable in practice.

III. Analysis

In order to derive system throughput, we need to compute the probability of a successful packet transmission. Using the model presented in the last section, this probability, P_s, can be expressed as

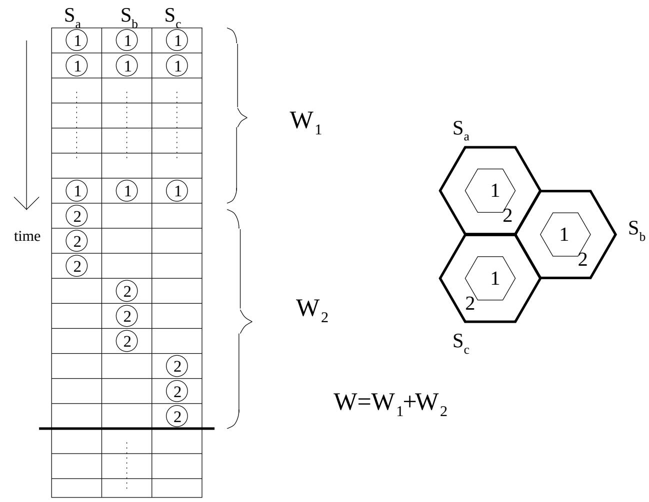

Fig. 1. How time slots are used by users in various locations in system cells.

$$P_s = P\left[\frac{\alpha_0^2 A r_0^{-\eta} W_T}{\sum_{i=1}^{M} \chi_i \alpha_i^2 A r_i^{-\eta} W_T} > b\right] \quad (2)$$

where M is the number of co-channel system cells considered, χ_i is a binary random variable which has value 1 if a transmitter is active in cell i, and 0 otherwise. The index "0" is used to represent the intended receiver while index "i" represents an interferer from cell i, $i \neq 0$.

Because of the dependence of P_s on the distribution of the r_i's we can affect the probability of a successful packet transmission in a given time slot by constraining the possible locations of interferers through scheduling. This is done by using virtual cells of different reuse factors, whose transmissions are scheduled independently. Furthermore, the dependence of P_s on the χ_i's allows us to further control the probability of success by changing the traffic intensity, G, which is the probability that $\chi_i = 1$ for all i. This is where the analysis of PCMA is unique. In the PCMA protocol we have the flexibility of setting up various virtual cells and of regulating the reuse factors and traffic intensities used by each one. Thus, our analysis will be able to show the relative performance of various virtual cell configurations.

Figure 1 is an illustration of one possible configuration of virtual cells. [1] Looking at a given system cell, such as S_a, all points contained in a concentric sub-cell are assigned reuse 1 by the reuse function, and thus constitute a virtual cell entirely contained in the system cell. All other points in the system cell are assigned reuse 3, and form

[1] We assume that W is the total bandwidth available per frame in the time domain, and that it therefore represents a number of time slots that must be shared by users in different virtual cells.

their own virtual cell. Under this assignment one user from each virtual cell of reuse 1 can transmit in parallel on a systemwide basis. Users in the other virtual cells, however, must use a reuse factor of 3 to divide time slots among themselves systemwide, as seen in Fig. 1, although different reuse factors and/or more levels of virtual cells are certainly possible.

As in [5] we can derive system throughput, S, as a function of G, the offered traffic load in terms of average packet transmissions per unit time per cell, and of $P_s = P_s(\varrho)$, the probability of successful packet reception from the intended mobile, which is in turn a function of ϱ, the position of the mobile relative to the base station. In computing S and G we will normalize all system cells to unit side length. For analytical convenience, as in [5], we approximate cells as circles of unit radius and, therefore, area equal to π.

The offered channel load, G, consists of both new packet transmissions as well as packets that need to be retransmitted due to failed transmissions (which occur with probability $1-P_s(\varrho)$). Therefore, G varies with $P_s(\varrho)$ and so is itself dependent on ϱ. Specifically, the offered traffic density $g(\varrho)$ is a function of ϱ. The value of the total offered traffic in a cell can be computed by integration of the offered traffic density

$$G = \int_0^{2\pi} d\theta \int_0^1 r\, dr\, g(r,\theta) = \pi\, E[g(\varrho)] \qquad (3)$$

where ϱ has been expressed in polar coordinates r and θ to clarify the integration. In a similar fashion throughput can be computed from throughput density $s(\varrho)$ as

$$S = \int_0^{2\pi} d\theta \int_0^1 r\, dr\, s(r,\theta) = \pi\, E[s(\varrho)] \qquad (4)$$

where $s(\varrho)$ is again expressed in polar coordinates under the integral.

Throughput density is computed as the fraction of packets in the offered traffic load that are correctly received, or

$$s(\varrho) = P_s(G,\varrho) g(\varrho). \qquad (5)$$

A solution of (5) is possible if we mandate uniform throughput. Such is the practical case in which we do not want mobiles to receive preferred treatment based on location. To achieve fairness we require $s(\varrho) \equiv s$ and find that

$$S = \pi s \qquad (6)$$

by substituting into (4). Then, by solving (5) for $g(\varrho)$ (remembering that $s(\varrho) = s$) and substituting in the right side of (3) we find

$$G = \pi\, E\left[\frac{s}{P_s(G,\varrho)}\right] = \pi s\, E\left[\frac{1}{P_s(G,\varrho)}\right]$$

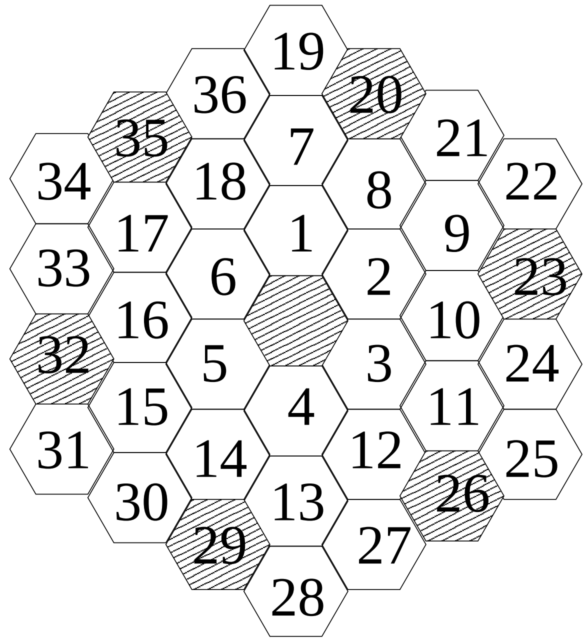

Fig. 2. Locations of the 36 interferers considered.

and then by substituting in equation (6) the throughput of a cell is found to be a function of G and is expressed as

$$S(G) = G\left(E\left[\frac{1}{P_s(G,\varrho)}\right]\right)^{-1}. \qquad (7)$$

As could be expected, throughput is a function of G. There is also a clear dependence on P_s, which in turn depends on G. Note that if P_s were identically 1, then we would have $S = G$, which is consistent with our intuition. In reality P_s will be less than 1, so that S will be strictly less than G.

A. Simple Case

We start with the simplest possibility, two sets of virtual cells, where one set uses reuse one and is made up of all users within a distance $\frac{1}{\sqrt{7}}$ of the base station. Because cells are normalized to unit radius, this size for virtual cells represents an extrapolation from one-seventh reuse and is thus akin to the approach used in basic Reuse Partitioning [3]. All users outside of the first set of virtual cells make up the second and final set of virtual cells which use a reuse factor N to be specified later.

Without loss of generality, let us consider any user, u in cell C_0 with base station B_0. During any given time slot when u has a packet to transmit, interference can come from nearby cells. Remember, under PCMA, a maximum of one user per cell is permitted to transmit in any given time slot. Thus, intracell interference is completely avoided and only intercell interference influences the probability of success.

We consider 36 potential interferering cells, i.e. we set $M = 36$ in (2) which corresponds to 3 rings of interfering cells around the intended cell as illustrated in Fig. 2. In Fig. 2 the patterned cells correspond to co-channel cells under the one-seventh reuse plan. The CDPA results consider interference from infinitely many cells in the uplink but only 18 cells in the downlink. We have chosen to consider interference from 36 cells in both directions for uniformity.

Considering 36 possible interferers and following the analysis presented in [5] gives

$$P_s(G, r_0) = \prod_{i=1}^{36} \left(1 - G \int_{R_{i1}}^{R_{i2}} \frac{2rdr}{(R_{i2}^2 - R_{i1}^2)(1 + b^{-1}(\frac{r_0}{r})^{-4})}\right) \quad (8)$$

where R_{i1} and R_{i2} are the nearest and farthest, respectively, possible distances of interferers from cell i to the intended base station, which depends on the virtual cell level being considered. Note that because of circular symmetry in the uplink direction, (8) does not depend on the position, ϱ, in terms of distance and angle, but only depends on the distance, r_0, of the intended mobile from its base station. This symmetry simplifies the calculation of throughput in (7).

By contrast, in the downlink direction there is no circular symmetry of interferers since the non-intended base stations which interfere have fixed positions. In this case the probability of successful packet reception is found as in [10] to be

$$P_s(G, \varrho) = \prod_{i=1}^{36} \left(1 - \frac{G}{1 + b^{-1}(\frac{r_0}{r_i})^{-4}}\right) \quad (9)$$

where the interference from the nearest 36 co-channel base stations is considered and r_i, $i = 1, \ldots, 36$, is the distance between the intended mobile, randomly located at position ϱ, and the i-th interfering base station. Even though the probability of successful packet reception depends on more than just the transmitter's distance from the intended base station, throughput can still be computed from (7).

Considering the virtual cells as defined above, we evaluated equation (7) using (8) and found that for capture threshold $b = 4$, maximum uplink throughput for first level virtual cells is 0.980. This throughput is achieved by letting $G = 1$ since throughput in this case is a strictly increasing function of G. This is because even though we are scheduling transmissions of users in first level virtual cells simultaneously, these cells are so small that there is little inter-cell interference, even when $G = 1$ and all first level virtual cells can transmit with probability 1. As a result, maximum G yields maximum throughput. We will see later that by increasing the size of the first level virtual cells, we create greater interference, forcing G to be reduced to decrease the interference and preserve maximum throughput. In this case, many more users can then enjoy reuse 1.

The analysis of throughput for the second level virtual cells is almost identical, except that now we use a reuse factor greater than one. From the possibilities $N = 3$, 4, and 7, we chose $N = 3$ and found (again with $b = 4$) that maximum throughput is 0.903 when $G = 1$. It is clear that in both cases we want to keep the traffic intensity, G, at 1. The reason this gives maximum throughput is that we are taking additional steps to reduce interference. For first level virtual cells we are keeping the size so small (basing it on the minimal interference one-seventh reuse plan) that there is little inter-cell interference. For second level virtual cells we employ reuse 3 to minimize interference. In CDPA, by contrast, only retransmission is used to recover from interference, and there are no a priori strategies (like virtual cells) to reduce inter-cell interference. However, the load created by using retransmission in CDPA is so great, that keeping G below 1 is actually required to achieve optimal system throughput.

In order to achieve fairness for users in both sets of virtual cells, we need to consider how time slots are allocated between the virtual cells on a system-wide basis. In appendix A we derive an expression for system throughput in terms of the throughput achieved in the two sets of virtual cells, taking into account the fact that all users in the system must enjoy the same throughput, regardless of location inside or outside virtual cells. Using this expression with $N = 3, A_{in} = A/7, A_{out} = A - A_{in}$ and the two values for the optimal throughput in the two levels (0.980 and 0.903) yields an overall throughput efficiency of 0.334 which is about as good as CDPA, although this is the most simplistic version of PCMA. Next we adjust the size of the first level virtual cells to optimize system throughput.

B. Optimizing Virtual Cell Radius

Because we have a capture model at our disposal, we are not limited to having the radius of the first level of virtual cells be $\sqrt{(1/7)}$. In our previous results on PCMA, this limitation was present [11]. The only inner cell radii that could be used were those that corresponded to reuse $3, 4, 7,$ etc. in a conventional cellular system. The partitions were set up according to the formula

$$r_i = \sqrt{\frac{1}{N_i}}$$

where $N_i \in \{1, 3, 4, 7, \ldots\}$. Thus, the choices for radii were extremely limited. We avoid this limitation by exploiting capture and optimizing system throughput as a function of the radius of the first level virtual cells.

We assume, as in the previous section, that there are two sets of virtual cells. Users in first level virtual cells are permitted reuse 1. Any users outside these virtual cells make up the second level of virtual cells and must use reuse N, where $N \in \{3, 4, 7\}$. We refer to such a system as (1,N). By repeating the computations detailed in the previous section, by performing the integrals over the appropriate domains, we can compute the throughput for users in first and second level virtual cells, for any first level virtual cell radius, r. It should be noted that if $r = 0$ we have the case of $1/N$ reuse, which has system throughput efficiency of no more than $1/7 = 0.142$ for the case of $N = 7$. In the other extreme, $r = 1$, we have no users with $1/N$ reuse, but everyone uses reuse 1. This is the same situation as CDPA with full reuse [5], which has maximum throughput efficiency of about 0.367 with our capture model[2] and capture ratio $b = 4$.

If we plot total system throughput vs. radius of first level virtual cells and find that it is a strictly increasing function, then the maximum throughput would be obtained with $r = 1$, i.e. with CDPA. However, using $1/N$ reuse in the outer regions of the cells where CDPA performs poorly, we should, intuitively, achieve better system throughput. Therefore, it should be expected that the function has a maximum at some $r \in (0, 1)$. We now proceed to verify this intuition.

From Appendix A we know that system throughput, S, can be expressed as

$$S = A \left[\frac{N\, A_2}{f_2{}^{\star}} + \frac{A_1}{f_1{}^{\star}} \right]^{-1} \qquad (10)$$

where A is the area of a system cell, while A_i and $f_i^{\star}$ are area and throughput, respectively, of virtual cell i. Also, N is the reuse factor used among second level virtual cells. We computed expression (10) for various values of first level virtual cell radius for both uplink and downlink directions under a (1,7) system. The results are shown in the lower two curves of Fig. 3.

Notice that the result for $r = 0$ corresponds to straight one-seventh reuse, so throughput in this case is 0.141 for uplink, or about $1/7$. In this case the virtual cells using

[2]We assume that users in the intended cell inside a hexagonal virtual cell of radius r' are distributed uniformly about a circle with the same area as the hexagon. This is an analytical simplification that has been verified in [5] by simulation. We could also have used a circle inscribed in the virtual hexagonal cell as an upper bound on performance or a circle circumscribed about the virtual hexagonal cell as a lower bound on performance. Our choice represents a compromise between these two extremes which allows accurate results while providing for analytical tractability.

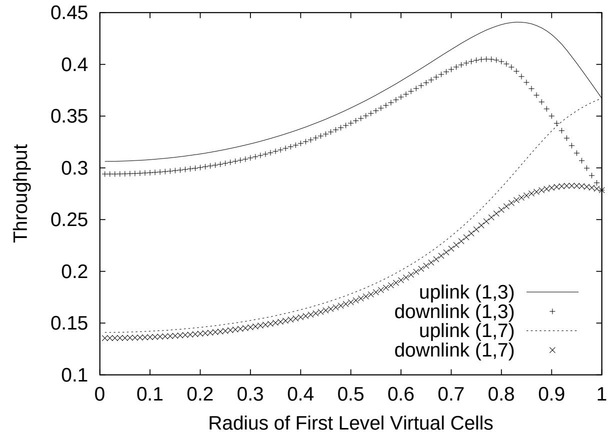

Fig. 3. Throughput results as a function of the radius of first level virtual cells for $b = 4$. Upper two curves use reuse $N = 3$ for second level virtual cells, while lower curves use reuse $N = 7$.

reuse 1 have size 0 and only second level virtual cells, using reuse factor 7 are used. Notice also that the result for $r = 1$ for the uplink was 0.367. This extreme represents using only reuse 1 throughout the system and corresponds to CDPA [5]. Our approach, in this uplink case, coincides exactly with CDPA. The reason is that reuse factor 7 is too conservative when b is as low as 4, and therefore should not be used. The greater reuse efficiency of reuse 1 more than compensates for its increased interference. Therefore, it is more efficient to use only reuse 1 throughout the system in the uplink. In the downlink there is an improvement possible if the first level virtual cells have radius of 0.94 so there are some users in second level virtual cells using reuse factor 7. However, the resulting improvement over CDPA in the downlink is only about 1.5%.

Because we can utilize any reuse factor in the second level of virtual cells, it seems plausible that using smaller reuse factors will lead to superior performance. In the upper curves of Fig. 3 we plot the results for reuse $N = 3$ for the second level virtual cells. This is a (1,3) system. The maximum occurs for $r = 0.83$ and provides system utilization of about 0.441. Again, the result for $r = 0$ corresponds to straight $1/N$ reuse, so throughput in this case is 0.306 or about $1/3$. Notice also that the result for $r = 1$ was 0.367. This extreme represents using only reuse 1 throughout the system and again corresponds to CDPA [5]. Our approach, therefore, represents a 20% improvement over CDPA in the uplink.

The downlink for the (1,3) system is also shown in Fig. 3. Here maximum throughput is 0.405 and occurs when the radius of the first level virtual cells is 0.77. This is a 45% improvement over CDPA in the downlink. Note

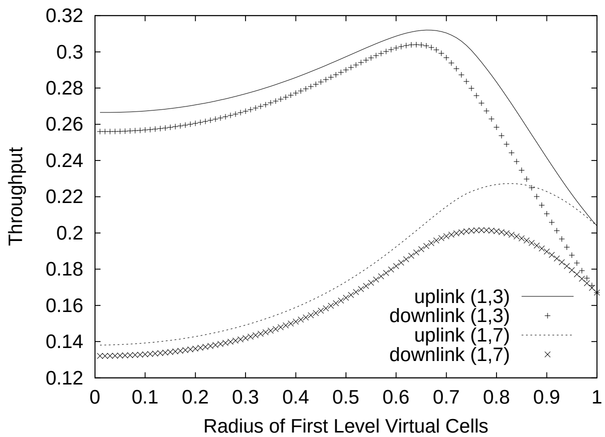

Fig. 4. Throughput results as a function of the radius of first level virtual cells for $b = 10$. Upper two curves use reuse $N = 3$ for second level virtual cells, while lower curves use reuse $N = 7$.

that in order to achieve maximum system throughput in the downlink direction, a smaller first level virtual cell radius must be used compared to the uplink.

We repeated the previous calculations using capture threshold $b = 10$ for uplink and downlink in both (1,7) and (1,3) systems. The results are shown in Fig. 4. Here, using either $N = 3$ or 7 as reuse for second level virtual cells leads to improved performance over CDPA, with the $N = 3$ case, i.e. a (1,3) system, giving a 49.5% improvement over CDPA in the uplink and an 82% improvement in the downlink. The optimal first level virtual cell radius is 0.66 in the uplink, while for the downlink it is slightly less at 0.64. In the (1,7) system the improvements were 11% and 21% in the uplink and downlink, respectively. For this system the optimal first level virtual cell radius is 0.83 in the uplink and 0.77 in the downlink. As was the case for the (1,3) system with $b = 4$, the optimal first level virtual cell radius is slightly less for the downlink direction.

Note that as was expected, for both $b = 4, 10$ performance improves as N decreases. The increased system throughput efficiency comes from the fact that a more efficient protocol is in use in the second level virtual cells. For example, using $N = 3$ provides better throughput than $N = 7$. This allows us to be less aggressive in pushing the size of the first level virtual cells, because a better protocol is in place in the second level virtual cells to “take up the slack.” This allows us to limit the size of the first level virtual cells to the area in which they perform best (for users closer to the center of the cell and the base station). We then use reuse 3 for users closer to the cell boundary. Reuse 3 is better suited for this job, since it is designed to combat interference by splitting resources. So using it in a situation where interference is expected to be high is quite logical. On the contrary, using one-third reuse for users near cell centers represents inefficiency, since interference is expected to be low for such users. We are thus receiving the corresponding benefits of two different protocols.

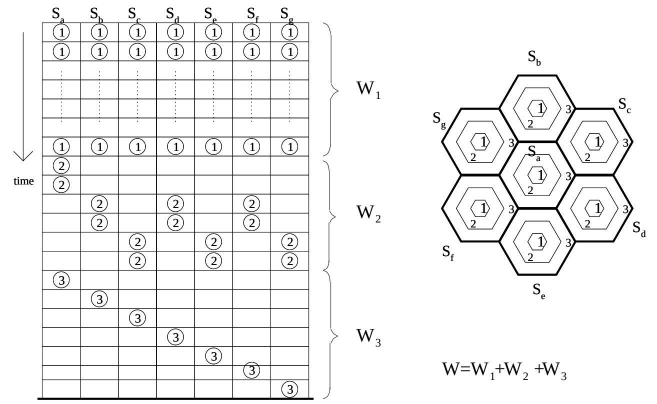

Fig. 5. How time slots are used by users in various locations in system cells in a (1,3,7) system.

C. *Multiple Levels of Virtual Cells*

This section investigates further improvement in performance by considering multiple levels of virtual cells. We start by considering a system with 3 levels of virtual cells using reuse factors 1, 3, and 7, a (1,3,7) system. As in the previous system users closest to base stations are in first level virtual cells and can transmit using reuse factor 1. This time, we split the remaining users into 2 sets of virtual cells, second and third level virtual cells. The users closer to the cell center are placed in the second level virtual cells and transmit using reuse factor 3 and all other users are placed in the third level virtual cells and utilize reuse factor 7 (see Fig. 5).

We computed system throughput for all possible sizes of the 3 virtual cells. For both capture thresholds, $b = 4$ and 10, performance was the greatest for both uplink and downlink when the virtual cells of reuse 7 had size zero. This case was the same as simply using 2 sets of virtual cells with reuse 1 and 3. One can see this result for $b = 10$ in Fig. 6, where the curves for (1,3,7) and (1,3) exactly coincide. The reason is that reuse 3 provides ample protection from co-channel interference so that reuse 7 is not necessary. The increase in interference suppression from reuse 7 is not enough to compensate for the inefficiency of the increase in resource splitting.

Therefore, we then considered another 3 virtual cell system, this time with reuse factors 1, 3, and 4. In this case improvement was achieved over the 2 cell system, but only in the case of capture threshold $b = 10$, in the uplink. This

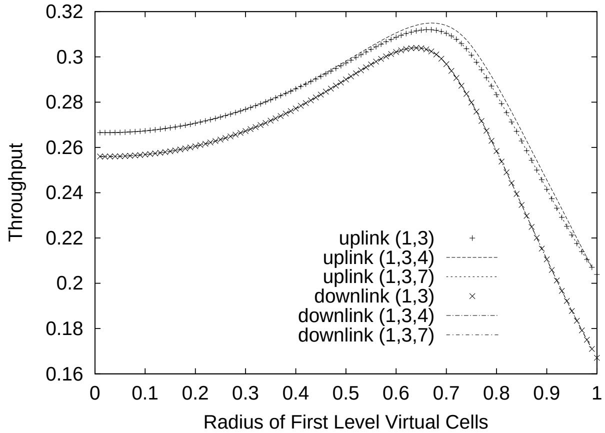

Fig. 6. Throughput results for three levels of virtual cells as a function of the radius of the first level virtual cells for $b = 10$.

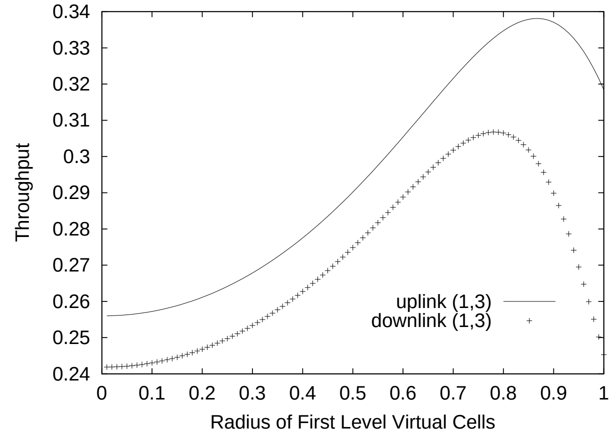

Fig. 7. Throughput results as a function of the radius of first level virtual cells for $b = 4$ and a propagation model including shadowing. Reuse 3 is used for second level virtual cells.

is due to the fact that for $b = 4$, capture is much more easily achieved so that reuse 3 is all that is needed for optimal system throughput. Thus, providing for the option of virtual cells with reuse 4 is unnecessary, and use of such high reuse cells is overly conservative and degrades throughput. Results for $b = 10$ are reported in Fig. 6. The maximum throughput achieved in the uplink was with 0.68 for the outer radius of the first level virtual cells, 0.94 for the outer radius of the second level virtual cells, and the third level virtual cells extending to fill the remainder of system cells (the third level virtual cells have outer radius 1, inner radius 0.94). The increase in system throughput over the 2 cell system is 0.93%. In the downlink direction there is no improvement over the 2 level (1,3) system. This can be seen in Fig. 6 where the downlink (1,3) and (1,3,4) curves coincide. Due to such minimal improvement in only one direction, the best choice for its level of complexity is most likely a system with two levels of virtual cells.

D. Two level System with Shadowing

The analysis thus far has featured a propagation model that does not account for shadowing due to terrain irregularities and obstructions. However, we can easily modify the model to include log-normal shadowing. Based on our findings without shadowing, we choose to investigate 2 level systems with reuse factors 1 and 3. We modify equation (1) by adding a term corresponding to shadowing so that received power becomes

$$W_R = \alpha^2 10^{0.1\xi} A r^{-\eta} W_T \tag{11}$$

where ξ is a Gaussian random variable with zero mean and variance σ^2.

In Fig. 7 we illustrate the results for uplink and downlink for capture threshold $b = 4$ and standard deviation $\sigma = 6$. It is immediately evident that the improvement relative to CDPA is less extreme. This could have been expected, however, since mobiles communicate only with the base station in their cell, and this is not necessarily the base station for which the least shadowing is experienced. The virtual cell structure is designed to increase throughput by taking advantage of the path loss law, which is directly related to cell position in terms of distance from the base station. However, distance from the base station does not directly affect shadowing. If our criteria for virtual cell membership were based upon shadowing as well as distance from the base station, performance would likely increase. Additionally, we could employ a best base station selection mechanism whereby the base station from which the strongest signal is received (taking shadowing into account) would be used, and it might not be the base station in the mobile's cell. We are currently investigating both strategies, following an approach as in [12].

IV. Delay Analysis

In order to complement the throughput analysis of the previous section we now present a simple delay analysis. The results will provide some notion of the delay experienced by users in a PCMA system and will also permit an investigation of the throughput vs. delay tradeoff. As presented in [13], we can use equation (7) to obtain an expression for expected number of transmissions for users in any given virtual cell. Rearranging equation (7) gives

$$E\left[\frac{1}{P_s(G,\varrho)}\right] = G/S \tag{12}$$

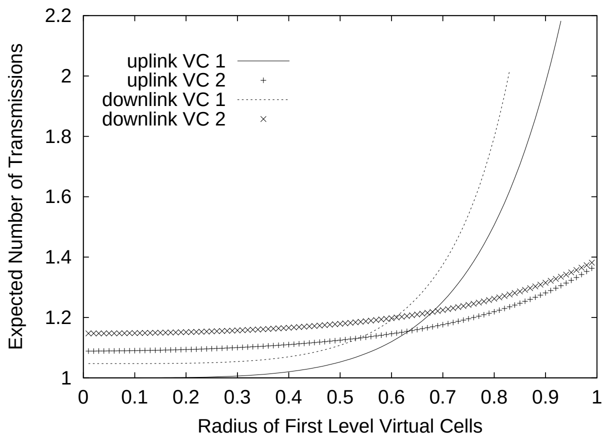

Fig. 8. Expected number of transmissions as a function of the radius of first level virtual cells for $b = 4$ in the uplink and downlink without shadowing.

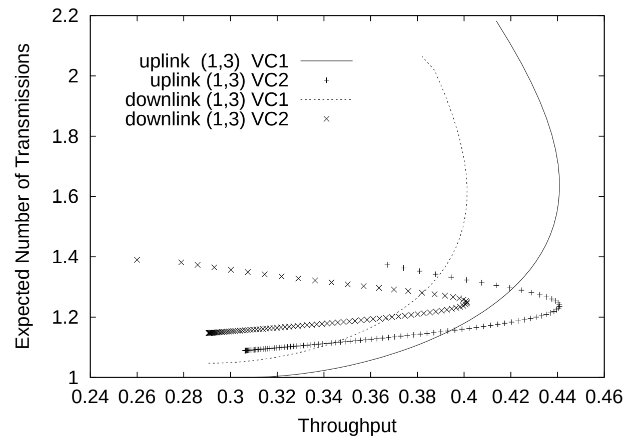

Fig. 10. Expected number of transmissions vs. throughput for $b = 4$ in the uplink and downlink without shadowing.

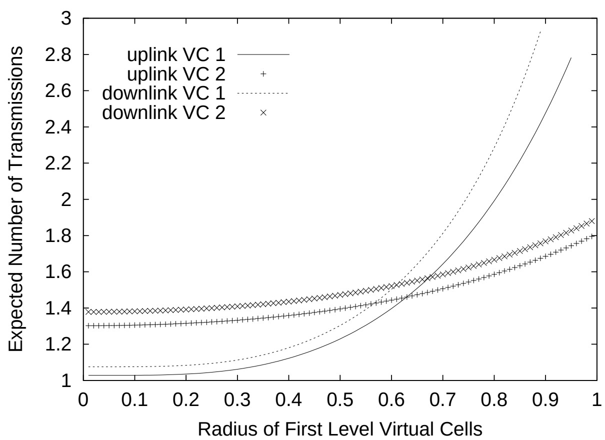

Fig. 9. Expected number of transmissions as a function of the radius of first level virtual cells for $b = 4$ in the uplink and downlink with shadowing.

where the expectation of $1/P_s$ is taken over all possible mobile positions and, therefore, is the expected number of transmissions, $E[N_t]$, to successfully transmit a packet. An exact measure of delay would require precise specification of a scheduling algorithm for base stations, which is outside the scope of this paper. Consequently, we focus on interpreting the current analytic result.

In Figs. 8 and 9 we report $E[N_t]$ for capture threshold $b = 4$ and different combinations of uplink and downlink with and without shadowing. In each case we consider a $(1, 3)$ system, since such a system has been shown to be the most efficient for its complexity.

As could be predicted, the expected number of transmissions increases monotonically for both virtual cells as the radius of the first level virtual cells increases. The reason is that P_s decreases since more users in first level virtual cells are potentially located closer to the system cell boundary. In the case of the second level virtual cells, fewer users are potentially located closer to the center of system cells. In either case, interference increases and results in a drop in P_s which thus increases the expectation of $1/P_s$. Conversely, $E[N_t]$ decreases monotonically as the radius of first level virtual cells decreases. Therefore, we can decrease delay by choosing smaller radii. Such an action does not come without a price, however, since throughput will decrease as a result.

In the case of the uplink without shadowing in Fig. 8, for example, optimal throughput is achieved when $r = 0.83$ and the corresponding $E[N_t]$ values for first and second level virtual cells are 1.62 and 1.24, respectively. If we decrease r to 0.75 the values fall to 1.36 and 1.196 i.e. 16% and 3.5% lower, respectively. At the same time throughput falls from 0.441 to 0.428, which is a reduction of less than 3%. Whether such policies are desirable is up to the system operator. Note that it is even possible to operate the system such that the expected number of transmissions is equal for both virtual cells. We simply use the radius where the two curves for the corresponding virtual cells intersect. From Figs. 8 and 9 it can be seen that the radius of the first level virtual cells must be set around 0.6 in all cases to achieve this goal. Such a policy in the case of uplink without shadowing would result, however, in a throughput reduction of 10.8%.

In Fig. 10 we plot the expected number of transmissions versus throughput for both uplink and downlink for capture threshold $b = 4$ with no shadowing. As in the previous figures we can see clearly how the first level virtual cells require a greater number of transmissions as a result of using more aggressive, universal reuse on a systemwide basis. What is readily apparent from this graph, additionally, is the great decrease in $E[N_t]$ for the first level virtual cells that comes at the price of only a slight drop in throughput. Given that we are assuming uniform throughput across all virtual cells, the expected number of transmissions for both virtual cells can be read from the graph by drawing a vertical line at a fixed value for throughput and seeing at which values the line intersects the appropriate curves in Fig. 10. In particular, by lowering the vertical line from the peak throughput value of 0.44 only slightly, we see a dramatic decline in expected number of uplink transmissions for the first level virtual cells. The decline for second level virtual cells is not as dramatic. However, it is quite convenient that the more dramatic decline is for the first level virtual cells, as these are the ones which experience much greater delay for the practical radii of first level virtual cells that maximize throughput.

V. Discussion

In the current presentation we have developed and analyzed a multi-access protocol that uses up to three levels of virtual cells. The performance improvement achieved is substantial compared to other protocols that have been proposed to support mobile users in a cellular, packet system. It is far superior to using a one-seventh reuse scheme as in many current systems. Actually, the final version of PCMA presented does not even use such a reuse in any of its virtual cells. In examples where reuse 7 was used, the performance was worse than when lower reuse factors (i.e. $1, 3, 4$) were adopted. Additionally, there is considerable improvement compared to CDPA, an alternative for the systems we are considering. The scheme considered in [6] is also shown to improve on CDPA. It involves invoking more intense reuse for certain users when possible, by exploiting a concept very similar to reuse partitioning. The focus of that paper is for fixed-point service as in wireless home Internet solutions. As such, the paper considers a different scenario from what is being considered here, i.e., no Rayleigh fading and the use of narrow terminal antenna beams, which are direct consequences of the absence of mobility. Furthermore, the base station antennas are assumed to be sectorized and the reuse strategy is directly based on this assumption. Additionally, the computation of reuse factors involves overhead that in the case of no mobility is easily justified, but could become impractical once fast fading or mobility were introduced so that optimal allocation is to be dynamically updated. The scheme in [6] can then only be applied to fixed point service, and would not work in a cellular system with mobile users. On the other hand, PCMA and the analytical methodology we have used are applicable to any wireless, cellular network supporting mobility. Our analysis accounts for dynamic environment conditions including mobility and Rayleigh fading. It also makes no assumptions regarding directional antennas, although our analysis could certainly be applied to systems with more specialized antennas.

The downlink results are particularly promising. The percentage of improvement over CDPA is even greater than in the uplink case. The reason is that interference is greater in the downlink, since a mobile closer to the cell boundary is effectively equally close to 2 different base stations, making reception difficult. Therefore, in this direction, using greater reuse factors for users nearer to cell boundaries is even more advantageous than for the uplink. The result is that PCMA normalizes the uplink and downlink by making their throughputs closer than is possible under other schemes. The greater downlink improvement is important since a greater percentage of traffic will likely be over the downlink. An example is Web browsing where most data is sent in the downlink.

It is clear from Figs. 8 and 9 that the curves for expected number of transmissions increase with much greater slope for first level virtual cells as their radius increases. This more rapid increase is due to the fact that first level virtual cells employ no reuse strategy. Therefore, as their size increases, more users are potentially located closer to cell boundaries, and could thus be extremely close to an interfering user in a neighboring system cell. For second level virtual cells, a greater percentage of users become closer to system cell boundaries, but they are separated from potential interferers by one layer of buffer cells due to their use of a reuse 3 strategy. Thus, the impact on increased interference is much less.

One implementation question is how base stations know in which virtual cell mobiles reside. After all, it is the base station that sends out permissions to transmit at different time slots for different virtual cells. In the current example of PCMA, since all virtual cells are centered symmetrically around the base stations, mobile's positions could be deduced by the polling mechanism based on received signal strength during the signaling portion of the protocol. All mobiles whose signal strength is above a given threshold are in the first level of virtual cells. Of the remaining mobiles, those whose signal strength is above a second threshold are in the second set of virtual cells, etc.

An advantage of this approach is that thresholds could be set based on permanent environmental characteristics of the area or even modified at run time based on the degree of success of mobile transmissions. For example, if the probability of successful transmission of mobiles in a given virtual cell is unacceptably low, an explanation could be that environmental conditions were causing users to be included in that virtual cell that should really be in a virtual cell of higher reuse. Then the threshold value could be increased making membership in the virtual cell more selective. Conversely, thresholds could be decreased if success probabilities are higher than expected, allowing more users to enjoy lower reuse.

Another possible solution to the mobile location problem would be to use external input such as a GPS system and have mobiles include their exact position as part of their polling response. An alternative would be for neighboring stations to all poll a single mobile and use all their signal strength measurements to triangulate and locate the exact mobile's position. Such more elaborate solutions might be needed in a PCMA system with non-symmetrical virtual cells.

Based on the numerical results of section III, optimizing system throughput in both uplink and downlink will necessitate the use of different virtual cell radii for each direction. The more hostile nature of the downlink does not permit the use of as large first level virtual cells as in the uplink case. We have assumed separate frequency bands for uplink and downlink, so this poses no conceptual problem: each frequency band can operate using its own virtual cell sizes. Even if this assumption is removed, there is still no problem in maintaining different virtual cell sizes. In such a case, distinct sets of time slots would be used for uplink and downlink. The base station would then schedule in each set of time slots differently, according to location of mobiles in virtual cells for that set's direction. Determining mobile membership in an additional set of virtual cells is done using the same technique as for the first set, so no additional complexity is introduced.

VI. Conclusion

We have presented and analyzed a wireless cellular multi-access strategy that provides superior system throughput to competing methods proposed for packet environments supporting mobility. As demand for wireless data services increases, a move to packet based transmission systems seems inevitable. Thus, our approach appears to represent the preferable means of designing multi-access protocols for future cellular wireless systems. By invoking a packet capture model we were able to present and analyze a protocol that uses various reuse factors for different users in the system. As such, our approach is similar to Reuse Partitioning in a conventional circuit-switched cellular system. However, our model allows "partitions" that we call virtual cells to have any size, not just the few sizes that correspond to reuse $3, 4, 7$, etc.

For capture threshold $b = 10$ the throughput performance of even a simple example of the PCMA technique shows a 49% improvement in the uplink and an 82% improvement in the downlink over CDPA, the best known alternative for the type of system under consideration. Also, PCMA brings the throughput performance of uplink and downlink directions closer than under CDPA, with the downlink being only slightly disadvantaged. Since highly used applications such as Web browsing make far greater use of the downlink, this balancing of downlink performance relative to the uplink is highly desirable. Further improvement can be seen by adding extra levels of virtual cells, but only in the uplink direction for capture threshold $b = 10$. Furthermore, since the improvement is small it is likely that implementations of the PCMA technique will use only two levels of virtual cells.

VII. Appendix A

In order to derive a general expression for system throughput, we introduce several definitions. First, assume two levels of virtual cells, inner and outer. Then, given a radius for the inner virtual cells, r, there are two functions for throughput, f_{in} and f_{out}. First, there is $S_{in} = f_{in}(G_{in})$ which gives throughput for the inner virtual cells as a function of the offered load, G_{in}, for the inner virtual cells. Secondly, there is $S_{out} = f_{out}(G_{out})$ which gives throughput for the outer virtual cells as a function of the offered load,G_{out}, for the outer virtual cells. Because disjoint sets of time slots are allocated to users from inner and outer virtual cells, we can choose distinct values for G_{out} and G_{in}. Therefore, we choose values that maximize S_{out} and S_{in}, respectively.

We set $f_{in}{}^{\star} = max_{G_{in} \in [0,1]} f_{in}(G_{in})$. Similarly for $f_{out}{}^{\star}$. In order to maintain consistent service quality throughout the system, we insist that the throughput per unit area, s, is equal for users in all virtual cells.

Therefore, we need

$$S_{out} = sA_{out} \tag{13}$$

and

$$S_{in} = sA_{in}. \tag{14}$$

The other requirement is that the total bandwidth available in the system, W, must be split between those in inner and outer virtual cells so that uniform s is maintained. In a given system cell, we will allocate W_{out} amount of bandwidth to users in the outer virtual cell and W_{in} to users in the inner virtual cell. Because users in the outer virtual cells are using reuse N, $N\ W_{out}$ must be reserved on a system-wide basis for such users. However, since users in inner virtual cells have universal reuse, only W_{in} amount of bandwidth must be reserved for them on a system-wide basis. So we have $N\ W_{out} + W_{in} = W$.

Thus,

$$S_{out} = W_{out} f_{out}{}^{\star} = sA_{out} \quad (15)$$

$$S_{in} = W_{in} f_{in}{}^{\star} = sA_{in}. \quad (16)$$

The left equalities hold since the throughput achieved in a given region depends on both the fraction of system bandwidth allocated to the region and the throughput value for that region, $f^{\star}$, in successes per slot. The right equality holds because of equations (13) and (14) above.

Now, solving for W_{out} and W_{in} gives

$$W_{out} = \frac{sA_{out}}{f_{out}{}^{\star}} \quad (17)$$

and

$$W_{in} = \frac{sA_{in}}{f_{in}{}^{\star}}. \quad (18)$$

Adding N times equation (17) plus equation (18) gives total system bandwidth on the left and an expression in terms of s on the right,

$$W = s[\frac{N\ A_{out}}{f_{out}{}^{\star}} + \frac{A_{in}}{f_{in}{}^{\star}}] \quad (19)$$

So,

$$s = S/A = W[\frac{N\ A_{out}}{f_{out}{}^{\star}} + \frac{A_{in}}{f_{in}{}^{\star}}]^{-1} \quad (20)$$

and

$$S/W = A[\frac{N\ A_{out}}{f_{out}{}^{\star}} + \frac{A_{in}}{f_{in}{}^{\star}}]^{-1}. \quad (21)$$

S/W gives the total system throughput per unit of bandwidth. Thus, this represents the overall system utilization.

We can also generalize to any number of virtual cells. In general, for k virtual cells we have that system utilization is

$$A\left[\Sigma_{i=1}^{k}\frac{N_i A_i}{f_i{}^{\star}}\right]^{-1} \quad (22)$$

where N_i, A_i, and $f_i{}^{\star}$ are the reuse factor, area, and maximum throughput, respectively, of virtual cell i.

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