A Simple End-to-End Throughput Model for 802.11 Multi-Radio Multi-Rate Wireless Mesh Networks

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***Abstract*—We address the problem of estimating the end-to-end throughput of UDP flows in a multi-radio, multi-channel, and multi-rate 802.11 wireless mesh network (WMN). Estimates of the end-to-end throughput of flows in a WMN can facilitate the design of traffic-aware routing and channel assignment algorithms. We introduce a simple and intuitive model that captures key features of 802.11 WMNs such as contention, interference, and performance degradation due to links with low transmission rates. We use simulation to validate the accuracy of the proposed model in various scenarios for a network topology containing multiple transmitters and wired gateways.**

***Index Terms*—Wireless channel contention, multi-rate, wireless multi-hop networks.**

I. INTRODUCTION

Wireless mesh networks (WMNs) based on IEEE 802.11 offer quickly deployable and inexpensive wide area coverage, by utilizing multi-hop wireless paths to gateway nodes with a wired connection to an external network. Deployment scenarios include city-wide, campus, and indoor networks offering ubiquitous broadband access, which can carry large amounts of traffic, often with QoS requirements, that stress the network backbone. The ability to estimate, in a simple way, the end-to-end throughput of flows in WMNs can facilitate the design of efficient routing, channel assignment, admission and load control procedures to improve network performance.

The work of [1] develops a rather complex probabilistic model that can estimate the throughput of end-to-end flows in an ad hoc network. However, the model neglects physical layer issues that would have further complicated the analysis. The model was extended in [2] to the case of nodes with multiple receivers. Although the above model captures starvation phenomena inherent in CSMA-based random access protocols, it does not account for issues such as multiple transmission rates and multiple channels. Similar to the above, [3] proposes a probabilistic model that focuses on hidden node phenomena in a single radio wireless mesh network. The work of [4] investigates the maximum throughput of a path in an ad hoc network, when the rates of existing flows are known and assumed constant. The work of [5] explicitly considers multiple rates when estimating the maximum path capacity in an ad hoc network. Both [4], [5] assume that each link is traversed by a single flow. Other works, such as [6], consider multiple rates when estimating the available capacity per node, in order to perform end-to-end admission control.

We propose a simple model for estimating the end-to-end throughput of flows, that captures important aspects of multi-radio, multi-channel, and multi-rate WMNs, and non-saturated traffic conditions. A key feature of the model is its simplicity, which allows efficient implementation and quick estimation of the end-to-end throughput.

II. END-TO-END THROUGHPUT MODEL

Consider a network with static routers, each equipped with one or more 802.11 radio interfaces. Routers with a wired connection are gateways that provide connectivity to an external wired network. The network can be represented as a graph $G = (V, E)$, where V is the set of wireless interfaces and E is the set of directional links between them. We assume that traffic is generated by UDP senders, which limit their rate to the end-to-end throughput with which the mesh network forwards their traffic. Let F be the set of flows in the network. We assume that the channels assigned to the interfaces and a flow's path (set of wireless interfaces) are known. Moreover, we consider single-path flows; however, a multi-path flow can be modeled as multiple single-path flows. The end-to-end throughput of a flow $f \in F$ is $x_f = \frac{b_f}{T}$, where b_f is the number of bits from flow f served in time T.

Let $N(v)$ be the set of neighboring wireless interfaces whose transmissions can conflict with interface v's transmissions, due to the CSMA-based access protocol, and includes v. Hence, $N(v)$ contains all interfaces that are inside v's carrier sense range, and use the same channel as v. Transmissions from the interfaces in $N(v)$ determine the aggregate channel occupation time seen by v. The total time that interface v sees the channel occupied with packet transmissions, including transmissions of its own packets, is

$$T_v = \sum_{u \in N(v)} \sum_{f \in F_u} b_f T_{u,f} \, ,$$

where F_u is the set of flows whose packets are forwarded by interface u, and $T_{u,f}$ is the time needed for interface u to forward a single bit from flow f. Each pair (u, f) uniquely determines the link e over which interface u transmits flow f's packets, and the time needed to forward a single bit can be estimated from

$$T_{u,f} = T_e = \frac{\overline{T}_{oh} + \frac{p}{r_e}}{p} \, , \tag{1}$$

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where $\overline{T}_{oh}$ is the average MAC and physical layer overhead of the 802.11 protocol, p is the packet size in bits, and r_e is link e's transmission rate in bps [7].

Each wireless interface contends with other neighboring interfaces operating in the same channel, and seeks to transmit as many packets as possible. Hence, we could express the objective of a wireless interface v as follows:

$$\text{Maximize} \sum_{f \in F_v} b_f \,, \tag{2}$$

subject to

$$\sum_{u \in N(v)} \sum_{f \in F_u} b_f T_{u,f} \leq T \,, \tag{3}$$

$$0 \leq b_f \leq B_f \quad \forall f \in F_v \,, \tag{4}$$

where B_f is the maximum number of bits from flow f that can be transmitted in time T, which is determined by the bottleneck link in the path of flow f. Equation (3) represents the constraint that an interface cannot transmit when the channel is fully utilized. In the proposed model defined by (2)-(4), the left-hand side of (3) captures contention between interfaces operating in the same channel, and $T_{u,f}$ is given by (1), which captures multi-rate operation. The optimization in (2) is over b_f; in the next section we describe an algorithm for calculating the values of b_f that achieve the optimum.

A. End-to-end throughput estimation

We will refer to the *first* wireless interface traversed by a flow as *source interface*. For downlink flows, source interfaces are located in the gateways, whereas for uplink flows, source interfaces are located in routers with the UDP senders. Note that two or more flows can have the same source interface.

Non-bottlenecked flows satisfy the inequality $b_f < B_f$. Initially, all flows are non-bottlenecked. For saturated flows, i.e. flows with UDP senders that can utilize all the throughput offered by the mesh network, initially $B_f = \infty$; in the case of non-saturated flows where the sender can transmit with maximum rate R, initially $B_f = R \cdot T$.

We use an iterative water-filling procedure for calculating the values of b_f, to achieve the target (2). In every round of the procedure, the number of bits sent by each *source interface* is increased by the same amount S; this reflects the fair channel access among contending interfaces, which can belong to the same multi-radio node, or to different nodes. If more than one non-bottlenecked flows have the same source interface, then the amount S is equally shared among these flows, which increase their b_f by the same amount, provided that (4) is satisfied. If the increase would violate (3) for some interface v, then all non-bottlenecked flows traversing the interface are marked as bottlenecked, and the values of b_f for these flows are set to satisfy $T_v = T$; the resulting value of b_f will be the new value of B_f, which is the maximum number of bits the bottlenecked flow f can transmit in time T, and $x_f = \frac{B_f}{T}$ is its end-to-end throughput. For the evaluation, we considered $S = 1$ Kbit and $T = 1$ sec; however, the throughput estimates are not sensitive to the exact values of these parameters.

The above procedure does not model asymmetric scenarios of 802.11 networks, such as the Flow In the Middle, and the Far and Near Hidden terminal scenarios discussed in [1]. However, it can be modified to capture such phenomena by utilizing the geometric relationships among interfaces that characterize the existence of the aforementioned scenarios; the extended procedure will be the focus of a follow-up paper.

B. Transmission rate estimation

The transmission rate of a wireless link is modelled as a step function of the signal-to-interference-and-noise ratio (SINR), estimated at the receiver. The minimum SINR required to sustain a rate r is $SINR_{min}(r) = R_s(r) - N$, where R_s is the receiver's sensitivity and N is the system noise at the receiver. Assuming only the presence of path loss, for a link between i, j, the SINR at receiver j can be expressed in dB as $SINR_j = P_i + G_{i,j} - 10 \cdot \log(N_j + I_j)$, where P_i is the transmission power at interface i, $G_{i,j}$ is the signal attenuation along the path from interface i to j, N_j is the noise power at the receiving interface j, and I_j is the interference power at j; I_j includes the interference from interfaces that are outside the carrier sense range of interface j, and operate in the same channel as j. We calculate the attenuation over a path of length d using $G(d) = G(1) + 10 \cdot \log(d^n)$, where $G(1)$ is the path loss at one meter and n is the path loss exponent.

When two interfaces j, k operate on non-orthogonal channels, the adjacent channel interference that k causes to j can be calculated using $I_{k,j} = P_k \cdot \xi_{k,j} \cdot G_{k,j}$, where factor $\xi_{k,j}$ depends on the inter-channel spectral distance, the channel width, and the spectral mask [8]; $I_{k,j}$ is added to I_j above.

We estimate the aggregate interference level I_j by means of an additive model: First, we group all wireless interfaces according to their assigned channel. Among the interfaces of each resulting group, excluding the group that includes interface j, we find the single interface that causes the maximum interference on j, which is the interface closest to j. By summing all the interference levels found in the previous step we obtain an estimate of the interference at j.

III. EVALUATION

Next we evaluate the accuracy of the proposed model using the Omnet++ 4.0 simulator. The simulations considered IEEE 802.11b, the receiver sensitivity for rates $(11, 5.5, 2, 1)$ Mbps was $(-83.01, -84.02, -88.41, -92.92)$ dBm [9], the noise $N = -90$ dBm, and the path loss model in Omnet++ was adjusted to match the loss model presented above with exponent $n = 2.9$. Furthermore, the packet size was 1500 bytes, and the sending rate of each UDP flows was gradually increased until it reached the value of the flow's bottleneck link.

Figure 1 presents the topology of the mesh network used for the evaluation. The network contains six wireless routers labelled $R_q, q \in [1, 6]$. All routers have one wireless interface, except R_5 which has two wireless interfaces. The wireless interfaces are labelled $v_m, m \in [1, 7]$. Figure 1 shows the four (out of eight) directional links, with e_{ij} denoting the link from interface i to interface j; these are the links along which traffic is forwarded from the four downlink UDP flows $f_k, k \in [1, 4]$, which originate at the two gateways R_1 and R_3. Links e_{12}, e_{34}, e_{35} operate on the same channel, which is orthogonal to the channel of link e_{67}. Furthermore,

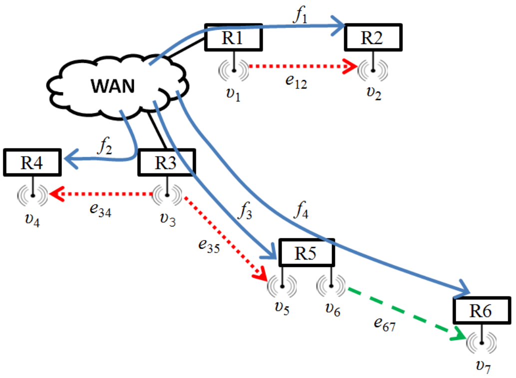

Fig. 1. Simulated network topology.

interfaces v_1 and v_3 belong to the same collision domain and therefore contend for channel access. Table I presents the three scenarios studied that differ in the transmission rate of links e_{12} and e_{67}, which is changed by adjusting the distance of the corresponding routers. All four flows $f_k, k \in [1, 4]$ are assumed saturated and active in all scenarios, except for flow f_3 which is not active in *Scenario c*. Figure 2 shows that there is excellent agreement between the end-to-end throughput of flows given by our model and the Omnet++ simulator, with the average relative difference being less than 5%.

Next we analyze each of the three simulated scenarios. In *Scenario a*, interface v_1 serves a single flow while interface v_3 serves three flows. Both interfaces have the same transmission rate, and contend for channel access since they are within each other's carrier sense range. The top graph in Figure 2 shows the results for this scenario. Since interfaces v_1 and v_3 share the channel fairly, they obtain an equal share of the channel, and flow f_1 served by v_1 achieves three times the throughput of the flows served by v_3. This shows that the proposed model can accurately estimate the end-to-end throughput in scenarios where contending interfaces serve a different number of flows.

In *Scenario b*, link e_{12} has rate 1 Mbps, which results in a prolonged channel occupation time for packet transmissions over link e_{12}. Since interfaces v_1 and v_3 have equal channel access probability, as shown in *Scenario a*, the throughput of all flows is reduced, due to e_{12}'s low rate. The middle graph in Fig. 2 shows that the end-to-end throughput estimated by the proposed model and simulation agree, hence the model can accurately capture the influence of multiple rates.

Finally, in *Scenario c* the transmission rate of link e_{67} is 1 Mbps, which again results in prolonged channel occupation times for transmissions over link e_{67}. Since link e_{67} operates on a channel that is orthogonal to the channel used by the other links, their performance is not affected by its low transmission rate. Also, in this scenario flow f_3 is inactive. The simulation results in the bottom graph of Fig. 2 indicate that flow f_4 achieves throughput less than 1 Mbps, because it is bottlenecked by the low rate link e_{67}. Furthermore, the aggregate throughput of flows f_2 and f_4 is approximately equal to the throughput of flow f_1, which shows that interfaces v_1 and v_3 again share the channel fairly. This holds because transmission opportunities of v_3 that cannot be utilized by the bottlenecked flow f_4, are used by f_2. This scenario shows that the proposed model can accurately capture the influence of bottleneck links on a flow's end-to-end throughput, and that resources that cannot be used by a bottlenecked flow, are used by other non-bottlenecked flows.

TABLE I
LINK TRANSMISSION RATES IN THE SIMULATED SCENARIOS

Link	Transmission Rate		
	Scenario a	*Scenario b*	*Scenario c*
e_{12}	11	1	11
e_{34}, e_{35}	11	11	11
e_{67}	11	11	1

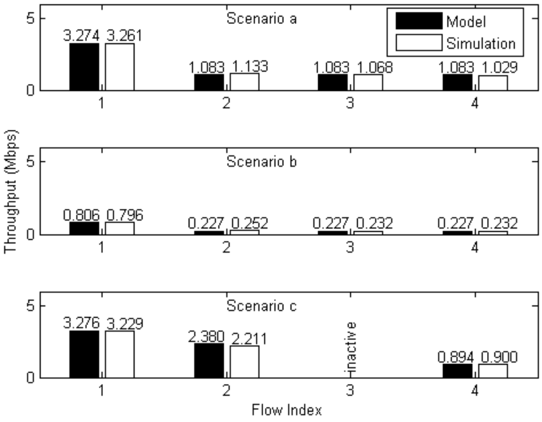

Fig. 2. Comparison of model and simulation results.

IV. CONCLUSION

We propose and evaluate a simple model for estimating the end-to-end throughput of flows, that captures important aspects of multi-radio, multi-channel, and multi-rate WMNs, and non-saturated traffic conditions. The model allows quick estimation of the end-to-end throughput, that can facilitate the design of traffic-aware resource management procedures such as routing, channel assignment, and load balancing.

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