Utility-based power control for interference mitigation in a mixed femtocell-macrocell environment

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Abstract— **Femtocell networks are about to be deployed for the improvements in the cellular coverage and capacity. However, this deployment is non trivial (mainly) because of the extra interference that they cause to the macrocell users that share the same portion of the spectrum. This paper proposes a non cooperative power control approach for interference mitigation in a mixed femtocell-macrocell environment. We define objective functions that are different for each type of user and we compute the transmission powers that maximize the objective function of each user. Finally, we propose a distributed iterative power control scheme that, starting from any initial power vector, converges to these transmission powers.**

***Index Terms*— Signal-to-Interference-plus-Noise-Ratio, distributed algorithm**

I. INTRODUCTION AND MOTIVATION

RADIO spectrum, though a non-exhaustible resource, is limited and can be neither stored nor transferred from place to place. Spectrum can become easily congested even in a small wireless communication system, where few entities coexist and interfere with each other [1]. This is the reason why entities that share a portion of the spectrum, e.g., Access Points (APs), Base Stations (BSs), Mobile Nodes (MNs) etc., are competitors. The Quality-of-Service (QoS) realized typically depends on the Signal-to-Interference-plus-Noise-Ratio (SINR), and an entity's target may not be always achieved if another entity is trying to achieve its own and interferes. Radio Resource Management is a key required functionality in order to utilize the radio spectrum in a fair and efficient way. This can be achieved by adjusting at least one of the following parameters: transmission power, modulation scheme, wireless channel used, etc.

New wireless standards such as 3GPP's High Speed Packet Access (HSPA) and Long Term Evolution (LTE) achieve considerable improvements in system capacity and throughput. However, the deployment of macrocells results in high operational expenses and capital expenditures. A way to solve this problem is to deploy a large number of smaller and cheaper cells which are called femtocells. Femtocells are small base stations that connect to a mobile operator network using residential DSL or cable broadband connections [2]. Femtocells contribute to a better coverage, as the indoor users that they support experience superior indoor reception and achieve better data rates than the macrocell users. This permits them to lower their transmission powers, which is beneficial for the prolongation of their battery life.

One of the biggest challenges for the successful deployment of femtocells is the interference mitigation that they provoke to the macrocell users when they share the same frequency bands (which is the typical case). If the level of interference is not controlled, the deployment of femtocells is problematic. Consequently, the adoption of radio resource management techniques is of crucial importance to alleviate the problems of this femtocell-macrocell interference.

We focus on transmission power control, i.e., controlling the transmission power to achieve a specific target. It is a widely applied dynamic radio resource management technique which is successfully adopted for achieving interference mitigation. This may have significant benefits, such as increasing the system capacity, decreasing the energy consumption and meeting QoS demands.

In this work, we formulate a non cooperative power control game with a view to alleviating the consequences of the interference in this mixed femtocell-macrocell environment. We assume that each (either femtocell or macrocell) user aims at maximizing its own objective function and propose a utility-based distributed power control algorithm that converges to the transmission powers that each user should use so as to maximize its own objective function.

II. TRANSMITTER POWER CONTROL: A REVIEW

In this section, we shall provide a central taxonomy of power control algorithms. Further details can be found in [3] and references therein. The key feature is whether a power control algorithm was designed for a voice network or a data network.

Power control algorithms were firstly applied in voice networks. In case that the noise of the channel could be neglected, the idea was to maximize the minimum (and minimize the maximum) Signal-to-Interference-Ratio (SIR). On the other hand, in case that the noise of the channel could not be neglected, the idea was to find a power distribution so that the SINR targets of all the links could be satisfied. For both cases, distributed iterative schemes were presented that can always find out a solution, in case that there is a feasible one.

In parallel, various power control algorithms were developed that are mostly suitable for data networks. The idea is that each user aims at maximizing its own utility function *U(.)* that expresses the (dis)satisfaction of a user that utilizes system resources. In the case of power control games, the general form of a utility function is $U_i(P_i, P_{-i}) = V_i(P_i, P_{-i}) - C_i(P_i)$, where V(.) is a value function that expresses the value that the link perceives and C(.) is a cost function that expresses the resources that it has to spend to achieve this value. P_i is the transmission user of user *i*, whereas $\boldsymbol{P_{-i}}$ is the vector of the transmission powers of all users except user *i*. Adopting the standard notation in the literature of power control games [4], in case of a non zero cost function, $U_i(P_i, P_{-i}) \equiv NU_i(P_i, P_{-i})$ and we call this quantity a *net utility* function.

By comparing these approaches, we could mention the following. Power control in voice networks is simple(r). It is SI(N)R-based and incorporates only this metric. Moreover, these SINR targets are “hard” in the sense that if the user cannot satisfy its target, then its value is zero. On the other hand, power control in data networks is (more) complex. It is (net) utility-based and may incorporate various metrics. Moreover, SINR targets are now “soft”, as a user may obtain a non zero utility value, even if the SINR that it perceives is lower than its desire.

However, it is well known that modern/ future wireless networks should provide both voice and data services. This means that it is time to (try to) unify these approaches with a view to providing algorithms that are suitable for networks with nodes that have heterogeneous targets and needs (a mixed femtocell-macrocell networks definitely belongs to that case). A good power control algorithm should be simple, fast and efficient and this implies that it should be flexible too. Adopting a hybrid approach that combines the simplicity of the SINR-based approaches with the powerful utility-based approaches is a non-trivial task that may lead to significant contributions. In the following section, we shall present an approach towards that direction.

III. Utility-Based Power Control For Mixed Femtocell-Macrocell Networks

A. System Setup and Problem Statement

Our topology consists of N_1 macrocell transmitters[1] and N_2 femtocell transmitters and all of them are using the same channel for their transmissions.

We choose the following objective functions for each femtocell and macrocell user:

Macrocell User Objective Function:

$$\max_i \left\{ U_i\left(P_i, P_{-i}\right) = B_i \ln(1 + SINR_i) \right\} \text{ subject to } 0 \le P_i \le P_{\max} \text{ and } 0 \le SINR_i \le SINR_{i\max} \quad (1)$$

Femtocell User Objective Function:

$$\max_i \left\{ NU_i\left(P_i, P_{-i}\right) = B_i \ln(1 + SINR_i) - \lambda_i P_i \right\} \text{ subject to } 0 \le P_i \le P_{\max} \text{ and } 0 \le SINR_i \quad (2)$$

Each macrocell user *i* uses a utility function which is a logarithmic function of the user’s SINR. This logarithmic function is weighted by a positive user-specific parameter B_i, that corresponds to the user’s “desire” for SINR. This utility function can be interpreted as being proportional to the Shannon capacity. Moreover, there are 2 constraints: The transmission power should not exceed P_{max} and the SINR of each user should belong to the interval $\left[0, SINR_{i\max}\right]$.

On the other hand, each femtocell transmitter *i* uses a *net utility function*. Apart from the utility part (which is the same with the macrocell users), there is a linear pricing function of P_i that defines the “price” that user *i* has to pay for using a specific amount of power. As previously, the transmission power should not exceed P_{max}. This net utility function is inspired by the one proposed in [4].

The reasons that we choose different objective functions for each category of users are the followings: Macrocell users have a higher priority to be served by the mobile operators. They can use any transmission power up to P_{max} (without paying for their choice) to overcome the extra interference that is caused by the femtocell users. On the other hand, femtocells are deployed by indoor users for their self interest. Consequently, a pricing function is used to discourage them from creating high interference to the macrocell users. However, as femtocells have generally higher demands for QoS, there is no maximum SINR constraint for them. This means that depending on the conditions (e.g., when the outdoor users are very distant), they can increase their SINR (and consequently, their throughput and data rate) as much as possible. It seems rational that the above heterogeneous characteristics could not be expressed successfully by a sole utility function (even if we use different values of the parameters for femtocells and macrocells).

As a final comment, we point out that the idea of using different objective functions for femtocell and macrocell users has already proposed in [5]. However, their approach is highly related to SINR. They demand that the SINR of each user *i* belongs to an interval $\left[SINR_{i\min}, SINR_{i\max}\right]$. In case that the SINR targets of macrocell users cannot be achieved, femtocell users are obliged to adjust their targets to the interval $\left[c * SINR_{i\min}, c * SINR_{i\max}\right], 0 \prec c \prec 1$.

B. On the Computation of the Transmission Powers for Femtocells and Macrocells

So as to solve the optimization problem (2), we take the partial derivative w.r.t. P_i by replacing the SINR with its definition (G_{ij} expresses the link gain from transmitter *i* to receiver *j* and *n* is the noise of the channel):

$$\frac{\partial NU_i(P_i, P_{-i})}{\partial P_i} = \frac{B_i}{\left(1 + \dfrac{G_{ii} P_i}{\sum\limits_{j \ne i} G_{ij} P_j + n}\right)} \frac{G_{ii}}{\left(\sum\limits_{j \ne i} G_{ij} P_j + n\right)} - \lambda_i \cdot \quad (3)$$

$$= \frac{B_i G_{ii}}{\left(\sum\limits_{j \ne i} G_{ij} P_j + n\right) + G_{ii} P_i} - \lambda_i = \frac{B_i G_{ii}}{INTERF_i + G_{ii} P_i} - \lambda_i$$

By setting the partial derivative equal to zero, we find that there is a unique solution *x*:

$$x = \frac{B_i}{\lambda_i} - \frac{INTERF_i}{G_{ii}}. \quad (4)$$

Then, by applying the Fermat’s Theorem we study the monotonicity of this function. We distinguish the following cases:

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[1] The words “transmitter”, “user”, “node” and “link” are used interchangeably corresponding to the node that applies the power control algorithm.

a) The unique solution x belongs to the interval $[0, P_{\max}]$. In that case, Table I shows the monotonicity of the function.

TABLE I: STUDY OF THE MONOTONICITY OF THE NET UTILITY FUNCTION

P_i	$0 \quad x \quad P_{\max}$
$\frac{\partial NU_i(P_i, P_{-i})}{\partial P_i}$	$+ \quad 0 \quad -$
$NU_i(P_i, P_{-i})$	↗ ↘

There is a unique local maximum (which is also a global maximum), which corresponds to the transmission power x. The utility that each user will receive is:

$$NU_i(x, P_{-i}) = B_i \ln\left(\frac{G_{ii} B_i}{INTERF_i \cdot \lambda_i}\right) - B_i + \frac{INTERF_i \cdot \lambda_i}{G_{ii}} \cdot \tag{5}$$

b) In case that x<0, each node should transmit with $P_i = 0$ (and its utility will be 0).

c) In case that $x \succ P_{\max}$, each node should transmit with $P_{\max}$ (and its utility will be $B_i \ln(1 + SINR_i) - \lambda_i P_{\max}$).

Following the same process for the optimization problem (1), the partial derivative of (1) w.r.t. P_i is:

$$\frac{\partial U_i(P_i, P_{-i})}{\partial P_i} = \frac{B_i G_{ii}}{INTERF_i + G_{ii} P_i} \cdot \tag{6}$$

As this partial derivative is always positive, the utility function is a monotonic increasing function at the interval $[0, P_{\max}]$. So, the (global) maximum point is $P_{\max}$ and corresponds to a utility value

$$U(P_{\max}, P_{-i}) = B_i \ln\left(1 + \frac{G_{ii} P_{\max}}{\sum_{j \neq i} G_{ij} P_j + n}\right) \cdot \tag{7}$$

However, as there is also the constraint $SINR_i \le SINR_{i\max}$, the following condition must be also fulfilled:

$$\frac{G_{ii} P_i}{INTERF_i} \le SINR_{i\max} \Leftrightarrow P_i \le \frac{INTERF_i \cdot SINR_{i\max}}{G_{ii}} \cdot \tag{8}$$

By also taking into account that $0 \le P_i \le P_{\max}$, we find out that each macrocell node should transmit using the formula:

$$P_i = \min\left\{P_{\max}, \frac{INTERF_i \cdot SINR_{i\max}}{G_{ii}}\right\} \cdot \tag{9}$$

This is an equivalent formula of the well-known *Simplified Foschini-Miljanic formula with P_{max} constraint* [6]. However, the key difference is that, contrary to [6], where each node's value is either 0 (when the target is not achieved) or 1 (when the target is achieved), each user gets a non zero value even if it has not achieved its $SINR_{i\max}$.

C. The Mixed Femtocell-Macrocell Utility-Based Distributed Power Control Algorithm

In this section, we provide the pseudocode of *the Mixed Femtocell-Macrocell Utility-Based Distributed Power Control Algorithm* that converges to the transmission powers that solve (1) and (2), starting from any initial power vector. It is worth mentioning that our algorithm is fully distributed in the sense that each node does not need to exchange info with other nodes so as to decide upon the level of its transmission power at the transmission round *k+1*. More specifically, each femtocell/ macrocell node needs to know the following info: a) its transmission power at the previous transmission round k, b) the values of the parameters G_{ii} and $SINR_{imax}$, c) The total interference that it has received at the previous transmission round and d) (only if it is a femtocell node) the values of the parameters B_i and λ_i. Elements a), b) and d) are already known to each transmitter, whereas element c) can be easily computed through the reverse link.

Algorithm 1: *The Mixed Femtocell-Macrocell Utility-Based Distributed Power Control Algorithm*

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Input:  Initial Transmission Power Vector $\mathbf{P_{init}}$, Noise $n$,
        SINR Target $\gamma_i^t$ of each link, $P_{\max}$
for k=1 to Max_Number_of_Iterations
    • each receiver $i$ computes the received power
      $P_{REC\,ji}(k) = G_{ji} P_j(k)$ from each transmitter $j$ and informs its
      transmitter $i$ for the total received power:
          $\sum_j G_{ji} P_j(k) + n \cdot$                                  (10)
    • each transmitter $i$ computes the quantity
          $SINR_i(k) = \frac{G_{ii} P_i(k)}{\sum_{j \neq i} G_{ji} P_i(k) + n} \cdot$     (11)
    • if $i$ is a macrocell transmitter, it updates its power using (9):
          $P_i(k+1) = \min\left\{P_{\max}, \frac{INTERF_i(k) \cdot SINR_{i\max}}{G_{ii}}\right\} \cdot$   (12)
    • if $i$ is a femtocell transmitter, it updates its power using (4):
          $P_i(k+1) = \frac{B_i}{\lambda_i} - \frac{INTERF_i(k)}{G_{ii}} \cdot$            (13)
        o if $P_i(k+1) < 0$, then $P_i(k+1) = 0$
        o if $P_i(k+1) \succ P_{\max}$, then $P_i(k+1) = P_{\max}$
    • if, for each $i$, $|P_i(k+1) - P_i(k)| \le e$, $e \to 0^+$,
        o break;
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Finally, we point out that this algorithm should lead to a Nash Equilibrium point, as the utilities of the users that correspond to the powers that solve (1) and (2) cannot be improved unilaterally.

IV. CONCLUSIONS AND PERFORMANCE EVALUATION PLANS

In this work, we presented an approach for the smooth coexistence of macrocell and femtocell users that share the same portion of the radio spectrum. We defined a power control game where each user aims at maximizing its own objective function, we determined the corresponding transmission powers and we provided a distributed algorithm that converges to them.

Our future plans include the detailed performance evaluation of our algorithm, by simulating its operation and obtaining its performance in terms of average utility per femtocell or macrocell user. System parameters from [5] will be used. Curves on the average throughput and the average SINR per user will also be sought. Moreover, we shall examine the convergence rate of our algorithm. Last but not least, we shall focus on the fairness of our scheme, using the

Jain's fairness index.[2] A key question is whether our scheme is biased against femtocell or macrocell users. We plan to examine the influence of the values of the user-specific parameters (B_i, λ_ι, $SINR_{imax}$) to the fairness of the system.

Another dimension for future work includes the combination of this work with [1], where we have presented a heuristic algorithm that applies power control by allowing negotiations among the nodes that have not achieved their (SINR) targets. As this scheme works on top of a power control algorithm, it would be interesting to apply it as a complementary approach in our case.

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