Bounded Rationality Can Increase Parking Search Efficiency

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ABSTRACT

The search for parking space in busy urban districts is one of those routine human activities that can benefit from the widespread adoption of pervasive sensing and radio communication technologies. Proposed parking assistance solutions combine sensors, either as fixed infrastructure or onboard vehicles, wireless networking technologies and mobile social applications running over smartphones to collect, share and present to drivers real-time information about parking availability and demand. One question that arises is how does (and should) the driver actually use such information to take parking decisions, *e.g.*, whether to search for on-street parking space or drive to a parking lot and, in the latter case, which one. The paper is, hence, a performance analysis study that seeks to capture the highly behavioral and heuristic dimension of drivers' decisions and its impact on the efficiency of the parking search process. To this end we model drivers as agents of *bounded rationality* and consider *lexicographic heuristics*, an instance of the fast and frugal heuristics developed in behavioral sciences such as psychology and biology, as the mechanisms for their decisions.

We analyze the performance of the search process under these heuristics and compare it against the predictions of normative game-theoretic models assuming fully rational strategically acting agents. We derive conditions under which the simpler heuristic decision-making rules outperform the complex norms and show their satisfaction under a broad range of scenarios concerning the fees charged for the parking resources and their distances from drivers' destinations. The practical implications of these results for parking assistance solutions are identified and thoroughly discussed.

MobiHoc'14, August 11–14, 2014, Philadelphia, PA, USA.

ACM 978-1-4503-2620-9/14/08 ...$15.00.
http://dx.doi.org/10.1145/2632951.2632955.

1. INTRODUCTION

The increasing integration of advanced sensing and wireless technologies with urban infrastructures transforms dramatically the way citizens access and interact with them. At the same time, smartphones and other smart mobile devices turn their owners to potential mobile sensing platforms and engage them actively in the generation and distribution of various kinds of information. The two trends combined fuel the concept of smart city, whereby fundamental daily activities and operations are carried out more efficiently in favor of individual citizens and the society as a whole.

The parking space forms an instance of urban resource that is daily accessed and shared by multiple drivers. Often scarce in places such as shopping areas and business districts, it has to be properly managed to avoid congestion effects and the unfavorable consequences of cruising for parking, *e.g.*, waste of time and fuel and environmental burden [1]. Parking assistance systems seek to address the parking problem by expanding the reach of pervasive computing within the city roads and turning them to smart spaces. Combining sensors at the parking spots or onboard vehicles and radio communication technologies, often including the vehicular network [2], they collect and distribute information about parking demand and supply to the vehicular nodes. More recent approaches to the assisted parking search, such as [3] or [4], add a social media layer over the vehicular network. The drivers can use a mobile social application running on their smartphones to share their knowledge about parking space with other application users and even handover parking spots to save the overheads of parking search.

Despite the amount of work on the proposal and design of parking assistance systems, surprisingly little attention has attracted their actual performance. Directly relevant to the performance issue is the amount of information that *can* or *should be* used when selecting parking resource, *e.g.*, on-street or parking lot(s), or searching for an on-street parking spot. The major assumption is that these decisions involve, in one way or another, humans. The drivers are those who actually make decisions when the parking assistance system only provides information about parking resources without making recommendations. But even when they are assisted by on-board mounted software agents, the human factor is present either through some offline profile description or, again, through direct participation in online decision-making.

The paper, thus, focuses on the decision-making task and the management of supplied information by the drivers. It seeks to answer how *efficient* is the parking search process when drivers employ *plausible* heuristics to choose between different parking resources. The efficiency of parking search is assessed by how much drivers end up paying to get a parking spot, including the overhead costs due to needless cruising. For the decision-making process, we consider two major approaches. The first one views drivers as fully rational entities who process all information at hand and act strategically so as to maximize their return. The second approach is inspired by the significant body of work in behavioral sciences such as psychology and biology on a family of heuristics called *fast* and *frugal* heuristics. These heuristics are generally simple, seek to *describe* the actual cognitive tasks involved in the decision and approach humans as boundedly rational *satisficers* rather than optimizers: namely, when faced with a choice problem, they seek for a good enough alternative rather than the best one and they do so without necessarily processing all information available to them.

It has been repeatedly shown that fast and frugal heuristics describe well human choices in various settings [5][6]. Less intuitively, they have been shown to perform comparably and sometimes better than more complex choice selection models such as linear regression or classification and regression trees. In our work, we essentially propose and analyze some of the most popular heuristics in the context of two representative instances of the parking resource selection problem. In the first one, drivers have to chose between the scarce but cheap on-street parking spots and the more expensive yet abundant parking lot(s) space. The second instance features two distinct parking lots, located at different distances from the drivers' common destination, adding distance as a second decision criterion beyond cost.

Methodologically, we follow the same steps in both problem instances: first, we formulate and analyze the games that emerge under strategic fully rational decision-making, then we analyze the performance of the parking search application under the heuristics and finally we compare them. For the single-attribute problem instance we derive analytically the conditions that let heuristics result in higher efficiency. Our results suggest that in most realistic scenarios the use of decision heuristics results in more efficient parking search process than when the drivers (or the parking assistance software agents) act strategically in line with the game-theoretic prescriptions.

To the best of our knowledge, this is the first study importing tools and modeling approaches from the field of cognitive psychology to a problem that has been treated for years by transportation engineers and, more recently, also by computer scientists. Section 2 summarizes the problem setting, results that are known about it and background on the cognitive heuristics. Sections 3-4 then contain the paper's main contributions. Section 3 devises and analyzes the priority heuristic for choosing between the cheap but scarce on-street parking and the spacious yet more expensive parking lot(s). It also compares its outcomes with those of the game-theoretic model in [7]. Section 4 considers the problem version with two types of parking lots. It formulates and analyzes the game emerging under the assumption of strategic decision-makers, devises the decision heuristic drawing on data from surveys and compares the two models under different parameter sets. We discuss the implications of our work for actual parking assistance systems in Section 5.

2. BACKGROUND

A typical dilemma faced by drivers when approaching their destinations in busy urban areas is: Should they invest time and effort in searching for cheaper on-street parking space or should they drive straight ahead towards one of the more expensive parking lots? The first option involves the risk of failing to find a vacant spot and eventually paying an excess cost due to cruising in terms of fuel consumption and time wastage.

2.1 The parking spot selection game

In [7], this dilemma is formulated as a resource selection problem. On-street parking space and parking lot(s) are two discrete types of parking resources with per time unit costs c_1 and c_2, respectively, with $c_1 < c_2$. An additional excess cost c_{exc} becomes relevant when vehicles end up in a parking lot after failing to seize an on-street parking spot. The vehicular nodes are viewed as fully rational agents that determine their strategies taking full advantage of the available information to them. The outcome of their actions and the respective payoffs depend on the decisions of all nodes so that their interaction is formulated as a game, the *parking spot selection game*:

DEFINITION 1. *The* Parking Spot Selection Game *is a tuple* $\Gamma_1(N) = (\mathcal{N}, \mathcal{R}, (A_i)_{i\in\mathcal{N}}, (w_j), j \in \{1, 2\})$*, where:*

- $\mathcal{N} = \{1, ..., N\}$, $N > 1$ *is the set of drivers searching for parking space,*
- $\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$ *is the set of parking spots;* $\mathcal{R}_1$ *is the set of on-street spots, with* $R = |\mathcal{R}_1| \geq 1$*;* $\mathcal{R}_2$ *is the set of spots in the parking lot(s), with* $|\mathcal{R}_2| \geq N$*,*
- $A_i = \{1, 2\}$ *is the action set for each driver* $i \in \mathcal{N}$*, action "1" denoting search for on-street parking space and "2" driving directly to a parking lot,*
- $w_1(\cdot)$ *and* $w_2(\cdot)$ *are the cost functions of the two actions, respectively.*

where

$$w_1(n) = min(1, R/n)c_1 + max(0, 1 - R/n)(c_2 + c_{exc}) \quad (1)$$

and $w_2(n) = c_2$, $n \in [1, N]$.

It is shown in [7] that at the Nash Equilibrium (NE) states of the game, whether pure or mixed, the vehicular nodes tend to over-compete for the scarce on-street parking space. Namely, the vehicles, N^{NE}_{cmp}, that choose to compete for on-street parking outnumber its supply, R, so that some of them end up in a parking lot only after incurring the excess cruising cost, c_{exc}. More formally, the game $\Gamma_1(N)$ has:

- a single pure strategy NE, $N^{NE}_{cmp} = N$, or equivalently a single symmetric mixed NE with $p^{NE}_{cmp} = 1$, if $N \leq N_0$
- $\binom{N}{\lfloor N_0 \rfloor}$ pure NE with $N^{NE}_{cmp} = \lfloor N_0 \rfloor$, if $N > N_0$ and N_0 is not an integer,

- $\binom{N}{N_0}$ pure NE with $N_{cmp}^{NE} = N_0$ and $\binom{N}{N_0-1}$ pure NE with $N_{cmp}^{NE} = N_0 - 1$, if $N > N_0$ and N_0 is an integer,
- one symmetric mixed NE with $p_{cmp}^{NE} = N_0/N$, if $N > N_0$

where

$$N_0 = \frac{c_2 + c_{exc} - c_1}{c_{exc}} R \quad (2)$$

In either case, and given that $N_{cmp}^{NE} = N \cdot p_{cmp}^{NE}$, the aggregate cost (fees+excess cost) paid by the drivers equals

$$C_{agg}^{NE} = min(R, N)c_1 + max(0, N_{cmp}^{NE} - R)(c_2 + c_{exc}) + (N - N_{cmp}^{NE})c_2$$

2.2 Fast and frugal decision heuristics

The NE is a solution concept with strong implications for the properties of decision-makers: they are agents with perfect processing capacity to analyze the available information and assess the alternatives presented to them. Most importantly, they can also analyze the possible strategies of other agents and identify the equilibrium choice, from which they have no reason to unilaterally deviate.

In this paper we are interested instead in a breed of heuristics developed in behavioral sciences such as psychology and biology, called fast and frugal heuristics [8]. Fast and frugal heuristics are models for making decisions, that: (i) rely heavily on core human capacities (such as memory recognition and recall); (ii) do not necessarily use all available information, and process the information they use by simple computations (such as using only one piece of information); (iii) are easy to understand, apply, and explain.

An important family of fast and frugal heuristics are the *lexicographic* heuristics, whereby the user inspects the attributes in a specified order and makes a decision based on the first attribute that allows for a decision to be made, without consulting other attributes. For example, consider the *priority heuristic* for choices among risky gambles [9]. Let us say we want to choose one of X = $(x_{min}, p; x_{max}, 1-p)$ and Y = $(y_{min}, q; y_{max}, 1-q)$, where for X the numerical outcome x_{min} is obtained with probability p and x_{max} is obtained with 1 - p and $0 < x_{min} < x_{max}$ (and analogously for Y).

If $|x_{min} - y_{min}| > thr_1 \cdot max\{x_{max}, y_{max}\}$, no other attributes are inspected and the gamble with the higher value on its minimum outcome is chosen. Otherwise, if $|p - q| > thr_2$[1], a decision is made in favor of the gamble with the lower probability of minimum outcome without considering attributes beyond the second. In the opposite case, the gamble with the higher maximum outcome is chosen (and if the maximum outcomes are equal, a choice is made randomly).

Such lexicographic heuristics describe well people's choices of consumer goods such as microwaves and apartments [5]. And, Katsikopoulos and Gigerenzer [6] have analytically shown that the priority heuristic predicts a host of major empirical phenomena in risky choice such as violations of the common consequence and common ratio axioms as well as the fourfold pattern of risk attitudes [10]. Another model of bounded rationality for a risky choice between two gambles, similar to the priority heuristic, was developed by Rubinstein in [11]. Both models attempt to model the underlying psychological processes and do not transform values or probabilities. But, unlike the priority heuristic, Rubinstein's model does not employ limited search and is not lexicographic.

[1]The value 0.1 is used for both thresholds thr_1 and thr_2 in [9] as a global constant across all humans.

Another example of a lexicographic heuristic describing people's *multi-attribute* choices under certainty is deterministic elimination by aspects (DEBA) [12]. In DEBA, the decision maker first orders the attributes according to the magnitude of their importance weights $(x_1, x_2, ...)$. She then inspects the value of all alternatives on the first attribute x_1. If all alternatives score equally on x_1, this attribute is eliminated. If exactly one alternative outperforms the rest on x_1, this alternative is chosen. Otherwise, all alternatives with a value on x_1 less than the maximum are eliminated and the process is repeated for the second attribute x_2. The process continues this way until an alternative is chosen. If more than one alternatives are left after all attributes are inspected, then one of them is chosen randomly.

The performance of lexicographic heuristics, in terms of accuracy or utility maximization, has been studied, via computer simulations and mathematical analysis in datasets from business, medicine and psychology. Overall, three major stylized facts have emerged [13]. First, there are small differences in performance between heuristics and more complex benchmarks such as linear regression, neural networks, classification and regression trees or naive Bayes. Second, simple heuristics often have higher performance in out-of-sample prediction. Third, each one of heuristics or benchmarks outperforms the other under certain conditions.

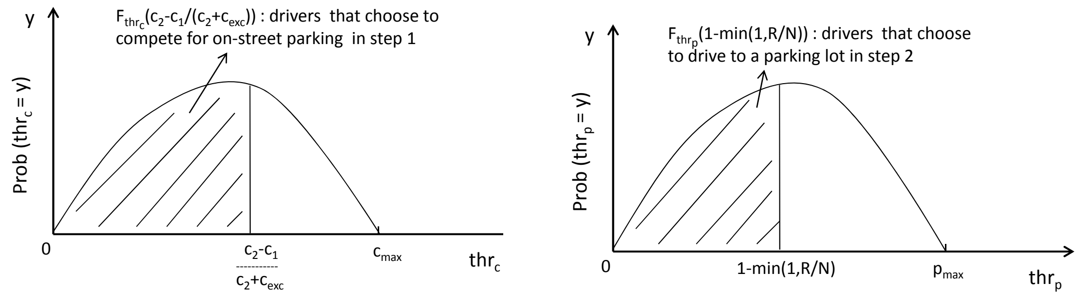
In what follows, an implementation of the priority heuristic guides the drivers' decisions between the two parking resources, when those consider only the fees charged for them. In section 4, we let decisions take account of both fees and distances of the resources from the drivers' destinations.

3. THE PRIORITY HEURISTIC FOR SELECTING PARKING RESOURCE

3.1 Description of the heuristic

We devise an implementation of the priority heuristic for the parking choice problem drawing on its generic description in Section 2. Since the problem setting involves losses (costs) rather than gains, the heuristic prescribes that drivers consider first the minimum cost related to each parking alternative; then the probabilities that these costs emerge out of their choices; and, finally, if no decision is made by that time, the maximum costs they may incur when choosing the one or the other alternative. More specifically, invoking the notation in Section 2.2, the on-street parking space can be viewed as the risky gamble $X = (-c_1, min(1, R/N); -(c_2 + c_{exc}), 1 - min(1, R/N))$, whereas the option of parking lot as the fixed-outcome gamble $Y{=}{-}c_2$ with probability 1. Then,

- (step 1) drivers decide to search for on-street parking space if the minimum possible costs of the two alternatives, c_1 and c_2 respectively, differ by more than a percentage thr_c of the worst-case cost they may incur as a result of their selection ($i.e.$, $c_2 + c_{exc}$).
- (step 2) In the opposite case, $i.e.$, $u = \frac{c_2 - c_1}{c_2 + c_{exc}} \leq thr_c$, they postpone their decision until after considering the probabilities of minimum costs: if their difference exceeds a threshold thr_p, they head for a parking lot.
- (step 3) Otherwise, when $1 - min(1, R/N) < thr_p$ so that neither the second criterion helps them reach a decision, they compare the maximum possible costs, $c_2 + c_{exc}$ and c_2, and decide in favor of the parking lot.



a. Parking fees/cost

b. Parking demand

Figure 1: Probability distributions for the sensitivity of drivers to the minimum parking cost (left) and the respective probabilities (right).

Table 1: Conditions for more efficient parking resource selection under the priority heuristic.

A *vs.* B	Corresponding r_{dc} range	
$A > B$	$r_{dc} < B$	(C1)
	$r_{dc} \geq A$	(C2)
	$max(B, \frac{N \cdot F_{thr_c}(\frac{c_2-c_1}{c_2+c_{exc}})-R}{R}) \leq r_{dc} < A$	(C3)
$A < B$	$r_{dc} \geq B$	(C4)
	$r_{dc} < A$	(C5)
	$A \leq r_{dc} < min(B, \frac{N-R}{N \cdot F_{thr_c}(\frac{c_2-c_1}{c_2+c_{exc}})-R})$	(C6)
$A = \frac{N-R}{R}, \qquad B = (1+\frac{c_2}{c_{exc}}) \cdot F^{-1}_{thr_c}(R/N)$		

As mentioned in Section 2.2, the original generic description of the priority heuristic in [9] recommends a single driver-agnostic constant (*i.e.*, 0.1) for both threshold parameters, thr_c and thr_p. We argue instead that these values vary across drivers reflecting differences in their financial status, the reason of their trip (business or leisure) and their individual preferences. This heterogeneity of drivers is modeled statistically through two probability distribution functions, f_{thr_c} and f_{thr_p} respectively. If F_{thr_c} and F_{thr_p} are the respective cumulative distribution functions, the number of drivers competing for on-street parking spots equals (ref. Fig. 1)

$$N^{PH}_{cmp} = N \cdot F_{thr_c}(\frac{c_2-c_1}{c_2+c_{exc}}) \tag{3}$$

whereas the drivers selecting the parking lot alternative are

$$N^{PH}_{PL} = N \cdot \Big(1 - F_{thr_c}(\frac{c_2-c_1}{c_2+c_{exc}})\Big) \tag{4}$$

Note that the partitioning of drivers into the two groups is independent of the distribution f_{thr_p}. Drivers who do not decide to compete for on-street parking space in step 1 will end up in a parking lot either in step 2 or in the last step. What f_{thr_p} determines is the portion of drivers deciding in favor of a parking lot in step 2 rather than in the last step.

3.2 Comparison with the game model $\Gamma_1(N)$

It is then convenient to analyze the efficiency of the parking search process under the game and the priority heuristic models as a function of the *decision certainty ratio* $r_{dc} = (c_2 - c_1)/c_{exc}$. This ratio grows as the parking fee difference increases or the cruising cost, *e.g.*, the risk related to competing for on-street parking, declines. It, thus, reflects the overall attractiveness of the on-street parking alternative. Theorem 1 summarizes the comparison of the two decision-making models.

Theorem 1. *The parking search process when drivers' choices are driven by the priority heuristic is more efficient than when they act as fully rational strategic agents under the conditions listed in Table 1.*

Proof. As already mentioned in Section 2.1, whenever $N > R$, the number of competing drivers induced by the strategic game exceeds the on-street parking capacity, *i.e.*, $N^{NE}_{cmp} > R$. On the contrary, under the priority heuristic, this number may generally be greater or smaller than R. We consider these two cases separately.

$N^{PH}_{cmp} < R$: This implies that

$$r_{dc} < (1 + \frac{c_2}{c_{exc}}) \cdot F^{-1}_{thr_c}(R/N) \tag{5}$$

and the difference in the aggregate cost induced by the two decision-making models is

$$\begin{aligned} \Delta C_{agg} &= C^{PH}_{agg} - C^{NE}_{agg} \\ &= (R - N^{PH}_{cmp})(c_2 - c_1) - (N^{NE}_{cmp} - R)c_{exc} \end{aligned} \tag{6}$$

If $N^{NE}_{cmp} = N_0 < N$, *i.e.*, $r_{dc} < N/R - 1$, then

$$\Delta C_{agg} = -N^{PH}_{cmp}(c_2 - c_1) < 0 \tag{7}$$

without additional conditions.

On the other hand, if $N^{NE}_{cmp} = N$, *i.e.*,

$$r_{dc} \geq N/R - 1 \tag{8}$$

$$\Delta C_{agg} = R(c_2 - c_1 + c_{exc}) - N(F_{thr_c}(\frac{c_2-c_1}{c_2+c_{exc}})(c_2-c_1) + c_{exc})$$

is negative when $r_{dc} < (N - R)/(N \cdot F_{thr_c}(\frac{c_2-c_1}{c_2+c_{exc}}) - R)$ so that, in combination with (8), r_{dc} must satisfy

$$\frac{N-R}{R} \leq r_{dc} < \frac{N-R}{N \cdot F_{thr_c}(\frac{c_2-c_1}{c_2+c_{exc}}) - R} \tag{9}$$

This interval of values is non-empty only when

$$r_{dc} < F^{-1}_{thr_c}(\frac{2R}{N})(1 + \frac{c_2}{c_{exc}}) \tag{10}$$

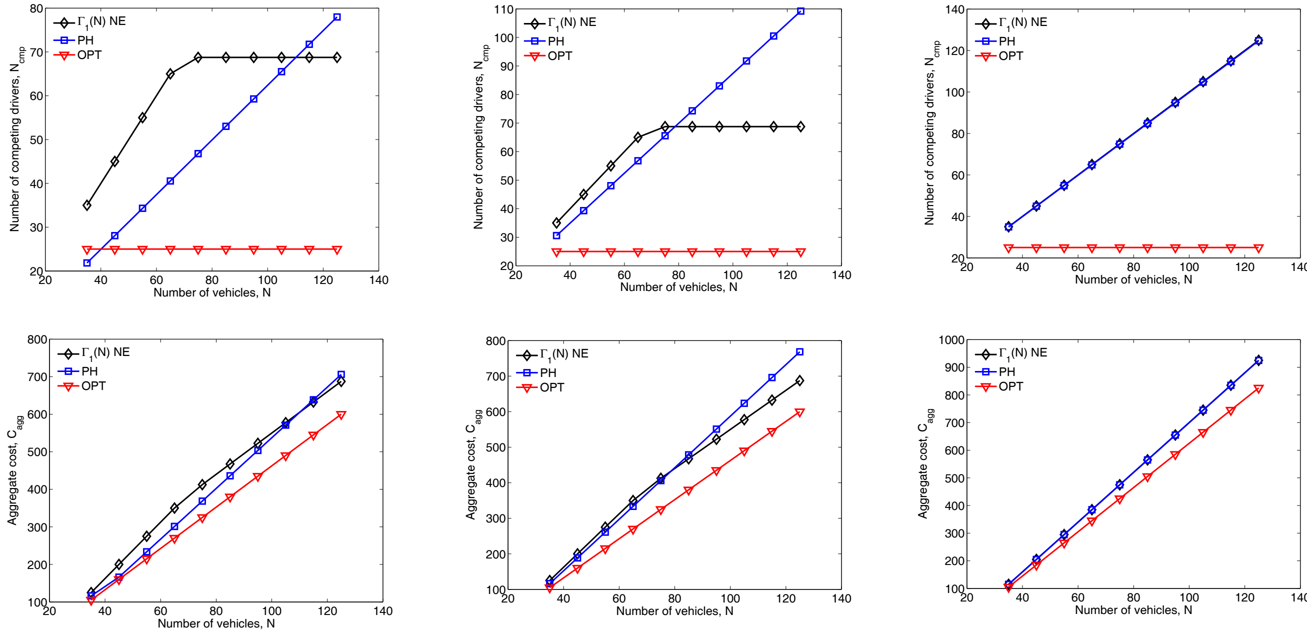
Constraint 10 is inactive since, for $N^{PH}_{cmp} < R$, it is dominated by constraint (5).

$N^{PH}_{cmp} \geq R$**:** Now $r_{dc} \geq (1 + \frac{c_2}{c_{exc}}) \cdot F^{-1}_{thr_c}(R/N)$ and the aggregate cost under both decision-making models can be written as function of N_{cmp} as

$$C_{agg} = R \cdot (c_1 - c_2 - c_{exc}) + N \cdot c_2 + N_{cmp} \cdot c_{exc} \tag{11}$$

so that

$$\Delta C_{agg} = (N^{PH}_{cmp} - N^{NE}_{cmp})c_{exc} \tag{12}$$



a. $c_1 = 2, c_2 = 5.5, c_{exc} = 2, c_{max} = 0.8$ b. $c_1 = 2, c_2 = 5.5, c_{exc} = 2, c_{max} = 0.6$ c. $c_1 = 1, c_2 = 8, c_{exc} = 1, c_{max} = 0.8$

Figure 2: Number of drivers competing for on-street parking and aggregate cost as a function of the parking demand: R=25.

and it suffices to compare the numbers of drivers that end up competing for on-street parking space under the two models.

When $N_{cmp}^{NE} = N$, *i.e.*, $r_{dc} > N/R - 1$

$$\Delta C_{agg} = N \cdot (F_{thr_c}(\frac{r_{dc}}{1 + c_2/c_{exc}}) - 1) < 0 \tag{13}$$

without additional conditions. Otherwise, when $r_{dc} \leq N/R-1$, it must also hold

$$r_{dc} > \frac{N \cdot F_{thr_c}(\frac{r_{dc}}{1+c_2/c_{exc}})}{R} - 1 \tag{14}$$

Combining (5)-(14) we get Table 1. □

COROLLARY 1. *The parking search process when drivers' choices are driven by the priority heuristic is coherently more efficient than when they act as fully rational strategic agents as long as either:*

$$\frac{N \cdot F_{thr_c}(\frac{c_2 - c_1}{c_2 + c_{exc}}) - R}{R} < (1 + \frac{c_2}{c_{exc}}) \cdot F_{thr_c}^{-1}(R/N) < \frac{N - R}{R}$$

or

$$\frac{N - R}{R} < (1 + \frac{c_2}{c_{exc}}) \cdot F_{thr_c}^{-1}(R/N) < \frac{N - R}{N \cdot F_{thr_c}(\frac{c_2 - c_1}{c_2 + c_{exc}}) - R}$$

PROOF. By inspection of Table 1 (2nd column, lines 3 and 6), these conditions enforce negative ΔC_{agg} throughout the range of possible r_{dc} values. □

What are plausible choices for the distribution $f_{thr_c}(x)$? One such choice would be a parabolic function of the type $f_{thr_c}(x) = \alpha x^2 + \beta x + \gamma$ over $[0, c_{max}]$ implying that more drivers risk competing at medium values of the u ratio rather than at very low or high fee differences. HEnce, imposing $\int_0^{c_{max}} f_{thr_c}(x) = 1$ and $f_{thr_c}(0) = f_{thr_c}(c_{max}) = 0$, we get

$$f_{thr_c}(x) = \begin{cases} \frac{6x}{c_{max}^2}(1 - \frac{x}{c_{max}}) & \text{if } x \in [0, c_{max}] \\ 0 & \text{otherwise} \end{cases}$$

3.3 Numerical results

We validate the analysis of Section 3.2 and demonstrate the operational dynamics of the parking search process under the two decision-making models for different values for the fees charged c_1, c_2, the cruising cost c_{exc}, and the driver's sensitivity to their difference (c_{max}).

In Fig. 2a ($r_{dc} = 1.75$) the game model prescribes that $min(N_0 = 68.75, N)$ vehicles should be competing for on-street parking space. The priority heuristic, on the other hand, requires that a fixed 62% of the population should do so, resulting in under-utilization of on-street parking space for $N < 40$. The aggregate cost remains consistently lower under the heuristic even for demand/supply ratios N/R exceeding four and up to $N \leq 110$, where the condition (C3) of Table 1 is violated (for N=110, A=3.6, B=0.95, $N \cdot F_{thr_c}(\frac{c_2-c_1}{c_2+c_{exc}}) - R)/R = 1.7447$).

As drivers become more sensitive to the charged fee differences (Fig. 2b, c_{max}=0.6), the priority heuristic directs more of them to compete for on-street parking space. The aggregate paid cost exceeds that at the NE of the strategic game faster but not before the demand/supply ratio N/R exceeds 3 (for N=80, A=2.4, B=0.837, $N \cdot F_{thr_c}(\frac{c_2-c_1}{c_2+c_{exc}}) - R)/R$ = 1.796). The predictions of the two decision-making models are identical when the fee difference $c_2 - c_1$ a) makes the parking lot prohibitive for all drivers deciding heuristically; b) renders the expected cost of competition for on-street parking (1) smaller than the parking lot fee for any (realistic) value of parking demand. This is the case in Fig. 2c.

4. TWO PARKING LOTS AND THE DISTANCE-COST TRADEOFF

The assumption underlying $\Gamma_1(N)$ is that the parking lots do not differentiate with respect to their location and fees. Equivalently, all parking lots are located within ap-

Table 2: Pure NE of $\Gamma_2(N)$ for different values of N

Range of N	NE states (N_1^{NE}, N_2^{NE})	Number of realizations	Conditions
$[1, N_{12}]$	$(N, 0)$	1	-
$(N_{12}, N_{12}+R_2]$	$(\lfloor N_{12}\rfloor, N-\lfloor N_{12}\rfloor)$	$\binom{N}{\lfloor N_{12}\rfloor}$	$N_{12} \notin \mathbb{N}^*$
	$(N_{12}, N-N_{12})$	$\binom{N}{N_{12}}$	$N_{12} \in \mathbb{N}^*$
	$(N_{12}-1, N-N_{12}+1)$	$\binom{N}{N_{12}-1}$	$N_{12} \in \mathbb{N}^*$
$(N_{12}+R_2, N_{13}+N_{23}]$	$(\lfloor \frac{N_{13}}{N_{13}+N_{23}}N\rfloor, \lceil \frac{N_{23}}{N_{13}+N_{23}}N\rceil)$	$\binom{N}{\lfloor \frac{N_{13}}{N_{13}+N_{23}}N\rfloor}$	$\frac{N_{13}}{N_{13}+N_{23}}N \notin \mathbb{N}^*$
	or $(\lceil \frac{N_{13}}{N_{13}+N_{23}}N\rceil, \lfloor \frac{N_{23}}{N_{13}+N_{23}}N\rfloor)$	$\binom{N}{\lceil \frac{N_{13}}{N_{13}+N_{23}}N\rceil}$	$\frac{N_{13}}{N_{13}+N_{23}}N \notin \mathbb{N}^*$
	$(\frac{N_{13}}{N_{13}+N_{23}}N, \frac{N_{23}}{N_{13}+N_{23}}N)$	$\binom{N}{\frac{N_{13}}{N_{13}+N_{23}}N}$	$\frac{N_{13}}{N_{13}+N_{23}}N \in \mathbb{N}^*$
$(N_{13}+N_{23}, \infty)$	$(\lfloor N_{13}\rfloor, \lfloor N_{23}\rfloor)$	$\frac{N!}{\lfloor N_{13}\rfloor!\lfloor N_{23}\rfloor!(N-\sum_{j=1}^{2}\lfloor N_{j3}\rfloor)!}$	$N_{13}, N_{23} \notin \mathbb{N}^*$
	(N_{13}, N_{23})	$\frac{N!}{N_{13}!N_{23}!(N-N_{13}-N_{23})!}$	$N_{13}, N_{23} \in \mathbb{N}^*$
	$(N_{13}-1, N_{23}-1)$	$\frac{N!}{(N_{13}-1)!(N_{23}-1)!(N-\sum_{j=1}^{2}N_{j3}-2)!}$	$N_{13}, N_{23} \in \mathbb{N}^*$
	$(N_{13}-1, N_{23})$	$\frac{N!}{(N_{13}-1)!N_{23}!(N-\sum_{j=1}^{2}N_{j3}-1)!}$	$N_{13}, N_{23} \in \mathbb{N}^*$
	$(N_{13}, N_{23}-1)$	$\frac{N!}{N_{13}!(N_{23}-1)!(N-\sum_{j=1}^{2}N_{j3}-1)!}$	$N_{13}, N_{23} \in \mathbb{N}^*$

proximately the same area (*e.g.*, business district) and they charge the same fee for their parking space so that drivers do not have strong preference for the one or the other.

In this section, we relax these two assumptions, introducing differentiation with respect to both charged fees and distances from travel destination. We consider two parking lots as alternatives to on-street parking. The first one is co-located with the drivers' common travel destination (distance $r_1 = 0$) and charges a fee c_3 per time unit. The second parking lot is further away at distance $r_2 > 0$ and charges a fee c_2, with $c_1 < c_2 < c_3$. On a more technical note, these assumptions turn the original single-attribute choice problem with essentially two alternatives into a two-attribute (cost, distance) choice problem with three alternatives. As before, we first formulate a game model for it, which becomes relevant under the highly normative assumption that that the vehicular nodes decide and act strategically and fully rationally. We then tailor the simpler DEBA heuristic (ref. Section 2) to the problem and compare the solutions it induces against the NE of the game model.

4.1 Strategic fully rational decision-making

The game model that captures the strategic interactions of drivers under full rationality stands in direct analogy with $\Gamma_1(N)$, as defined in Section 2.1.

DEFINITION 2. *The* Extended Parking Spot Selection Game *is a tuple* $\Gamma_2(N) = (\mathcal{N}, \mathcal{R}, (A_i)_{i\in\mathcal{N}}, (w_j), j \in \{1,2,3\})$, *where:*

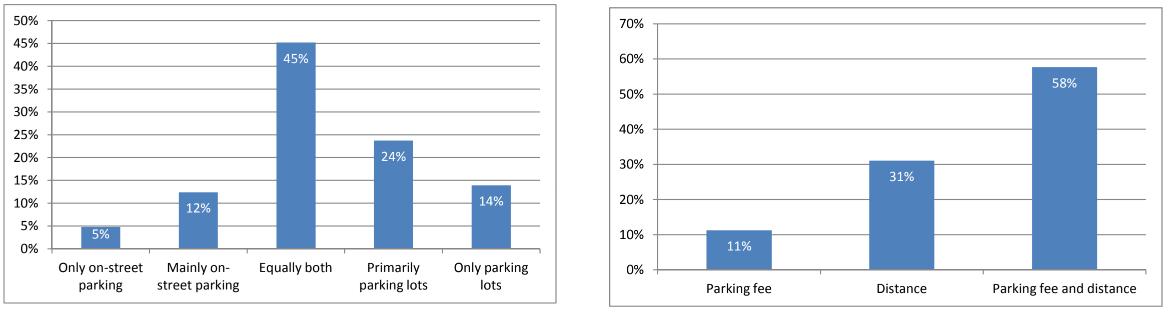
- $\mathcal{N} = \{1, ..., N\}$, $N > 1$ *is the set of drivers searching for parking space,*
- $\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2 \cup \mathcal{R}_3$ *is the set of parking spots;* $\mathcal{R}_1$ *is the set of on-street spots, with* $R = |\mathcal{R}_1| \geq 1$*;* $\mathcal{R}_2$ *is the set of spots in the more distant yet cheaper parking lot, with* $|\mathcal{R}_2| = R_2 > 1$ *and* $R_1 + R_2 < N$*; and* $\mathcal{R}_3$ *is the set of parking spots in the spacious but expensive lot that lies next to the travel destination, with* $|\mathcal{R}_3| = R_3 > N$.
- $A_i = \{1, 2, 3\}$ *is the action set for each driver* $i \in \mathcal{N}$*, action "1" denoting search for on-street parking space, "2" driving directly to the cheaper and "3" to the more expensive parking lot.*
- $w_j(\cdot,\cdot)$, $j = 1,2,3$ *are the cost functions of the three actions, respectively.*

With respect to the original game $\Gamma_1(N)$, cost functions have now to account for the non-monetary cost related to the distance of the three alternatives and the effort related to driving and walking from/to them. If $f_r(\cdot)$ is a function that monetizes this cost, it holds that $f_r(r_2) > f_r(r_1) > f_r(r_3) = 0$. Then $w_1(n_1, n_2)$ is given by (15)[2],

$$w_2(n_1, n_2) = min(1, \frac{R_2}{n_2})(c_2 + f_r(r_2)) + \lceil 1 - R_2/n_2\rceil^+(c_3 + c_{exc})$$

[2]The assumption in (15) is that drivers that fail to seize an on-street parking spot will first seek for parking space in the distant cheaper lot and only if this does not work out for them, they will resort to the expensive lot with guaranteed parking space.

$$w_1(n_1,n_2) = min(1,\frac{R_1}{n_1})\big(c_1+f_r(r_1)\big)+\lceil 1-\frac{R_1}{n_1}\rceil^+\Big(\frac{\lceil R_2-n_2\rceil^+\cdot\big(c_2+c_{exc}+f_r(r_2)\big)}{\lceil n_1-R_1\rceil^+ + \lceil n_2-R_2\rceil^+}+(1-\frac{\lceil R_2-n_2\rceil^+}{\lceil n_1-R_1\rceil^+ + \lceil n_2-R_2\rceil^+})(c_3+c_{exc})\Big) \quad (15)$$



a. Parking resource used

b. Criteria for selecting parking lot

Figure 3: Survey outcomes regarding the parking preferences of 1200 drivers in Nicosia, Cyprus.

and $w_3(n_1,n_2) = c_3$, where $\lceil x\rceil^+ = max(x,0)$ and (n_1,n_2) describes the state of the system, *i.e.*, the numbers of vehicular nodes that decide to compete for on-street parking capacity and drive towards the cheaper distant parking lot, respectively[3].

In principle, $f_r(\cdot)$ is driver-specific; extra driving and walking effort is not equally annoying to all drivers. Yet for computational tractability purposes, we restrict ourselves to symmetric users, assuming that $f_r(\cdot)$ has similar semantics for all users. Defining:

$$\begin{aligned} N_{12} &= \frac{c_2+f_r(r_2)+c_{exc}-\big(c_1+f_r(r_1)\big)}{c_{exc}}R_1 \\ N_{13} &= \frac{c_3+c_{exc}-\big(c_1+f_r(r_1)\big)}{c_{exc}}R_1 \quad \text{and} \\ N_{23} &= \frac{c_3+c_{exc}-\big(c_2+f_r(r_2)\big)}{c_{exc}}R_2 \end{aligned} \quad (16)$$

we can show that

THEOREM 2. *The game $\Gamma_2(N)$ has the pure NE listed in Table 2.*

PROOF. The proof is given in the Appendix A. □

PROPOSITION 1. *For any value of $N > R_1$, the NE of the game $\Gamma_2(N)$ induce over-competition for the on-street parking capacity.*

PROOF. The proof is given in the Appendix B. □

4.2 Heuristic decision-making

The game-theoretic model for the drivers' decisions among the three alternatives has normative rather than descriptive value. Besides acquiring and exhaustively processing information about the demand and supply of parking resources, vehicular nodes also need to "monetize" the cost related to the distance of parking resources from the drivers' destination. As a more realistic decision model, we introduce a heuristic that essentially modulates the generic DEBA heuristic in [12] with application-specific data obtained via a paper survey. The survey was carried out in Nicosia, Cyprus, during the summer months of 2012 with the participation of around 1200 drivers replying to more than twenty questions about their parking habits and preferences.

The replies of drivers about the parking resource type they make use of and their selection criteria are summarized in Figure 3. They reveal two distinct stages at their decision-making process as well as high heterogeneity regarding their decision criteria. First, the population of drivers is partitioned into five groups reflecting their preferences about the type of parking resource (on-street parking *vs.* parking lot) and the frequency of their usage. Then, drivers using parking lots either exclusively or occasionally, are further divided into three groups, depending on the criteria (cost and/or distance from destination) directing their decisions.

Technically speaking, let $i \in [1,5]$ and $j \in [1,3]$ index the groups of drivers emerging at the first and second stage, respectively, in left-to-right order as they appear in Fig. 3. For example, $i = 2$ denotes the group of drivers that primarily seek on-street parking and $j = 1$ the parking lot users who consider only the charged fees when selecting a lot. If p_i and q_j are the percentage volumes of the respective groups, the expected number of drivers who search for on-street parking space is:

$$N_1^H = N\cdot(p_1+\alpha p_2+0.5p_3+(1-\alpha)p_4) \quad (17)$$

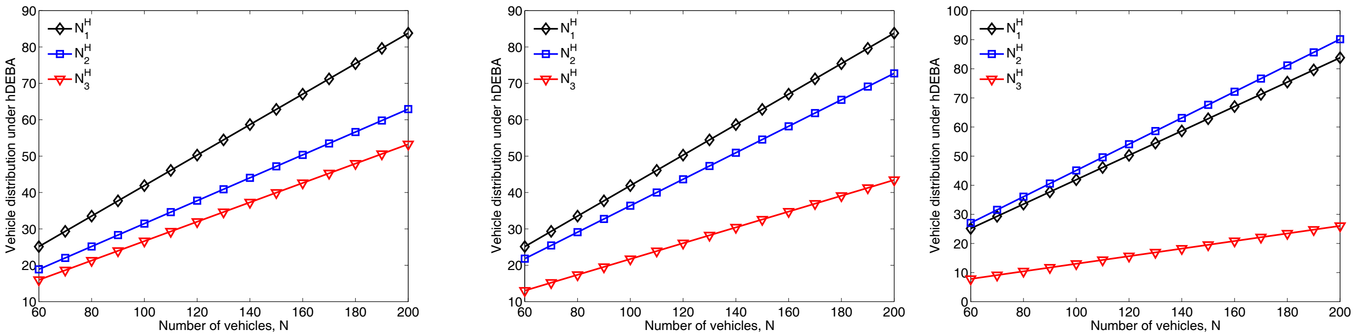
where $0.5 < \alpha \le 1$ is the quantitative interpretation of "primarily" in the responses of drivers. The plausible assumption in (17) is that all on-street parking spots are subject to the same charging policy throughout the area of interest and their distance from the drivers' common destination does not vary considerably.

Likewise, the expected number of users that will end up in a parking lot is

$$N_{PL}^H = N_2^H + N_3^H = N\cdot(p_5+\alpha p_4+0.5p_3+(1-\alpha)p_2) \quad (18)$$

The way these N_{PL}^H users further decide between the two parking lots is dictated by the right plot in Fig. 3. The cheaper but remote (expensive yet destination-adjacent) parking lot is chosen by the $N_{PL}^H\cdot q_1(q_2)$ drivers plus part of the $N_{PL}^H\cdot q_3$ drivers who account for both criteria in their decisions. We argue that these drivers essentially consider the two criteria sequentially: first, the distance, then the parking fees. If the distance of (walking time from) the remote lot is prohibitive (*e.g.*, beyond a driver-specific threshold thr_w), they drive to the destination-adjacent parking lot. Otherwise, they consider the difference in the fees charged

[3] Apparently a state $\{n_1,n_2\}$ implies that the remaining $N-n_1-n_2$ vehicular nodes select the more expensive parking lot.



a. $c_1 = 1.5, c_2 = 5, c_3 = 8, c_{exc} = 2, w_2 = 6$ b. $c_1 = 1.5, c_2 = 5, c_3 = 8, c_{exc} = 2, w_2 = 4$ c. $c_1 = 1.5, c_2 = 5, c_3 = 10, c_{exc} = 2, w_2 = 6$

Figure 4: Distribution of vehicles to the three parking resources when decisions use the hDEBA heuristic: R_1=25, R_2=40, $w_{min} = 3$, $w_{max} = 12$, $c_{max} = 5$.

by the two lots, $c_3 - c_2$. If this is "small" enough, (*e.g.*, below a driver-specific threshold thr_c), they pay the higher fee of the nearby lot to save the additional driving and walking overhead; otherwise, they drive to the remote parking lot.

As in section 3, we capture the heterogeneous sensitivity of drivers to the parking lot distance and fees statistically through negative parabolic distributions of the type $f(x) = \alpha x^2 + \beta x + \gamma$. We let thr_w vary in $[w_{min}, w_{max}]$ and enforcing $f(w_{min})=f(w_{max})=0$ we get $\alpha = -6/(w_{max} - w_{min})^3$, $\beta = -\alpha \cdot (w_{min} + w_{max})$ and $\gamma = \alpha \cdot w_{min} w_{max}$.

The fee difference threshold, on the other hand, is let vary in $[0, c_{max}]$ according to

$$f_{thr_c}(x) = \frac{6x}{c_{max}^2}(1 - \frac{x}{c_{max}}) \tag{19}$$

For given distance (*i.e.*, expected walking time) w_2 of the distant parking lot, the number of drivers that select it is

$$N_2^H = N_{PL}^H \cdot \Big(q_1 + q_3(1 - F_{thr_w}(w_2)) \cdot F_{thr_c}(c_3 - c_2)\Big)$$

whereas

$$N_3^H = N_{PL}^H \cdot \Big(q_2 + q_3 \cdot \big(F_{thr_w}(w_2) + (1 - F_{thr_w}(w_2)) \cdot (1 - F_{thr_c}(c_3 - c_2)\big)\Big)$$

will drive to the nearer and more expensive parking lot. The way the actual distance of the remote parking lot and the fee difference partition the drivers' population into the three parking resources is shown in Fig. 4. As expected, more parking lot users are directed to the cheaper remote one with parallel reduction of the expensive lot users as w_2 decreases and the fee c_3 of the expensive lot increases.

It is worth remarking that the proposed heuristic is essentially a driver heterogeneity-aware adaptation of the original DEBA heuristic; we, thus, call it hDEBA. The practice in lexicographic multi-attribute selection is to encode the three parking resource alternatives as vectors of three attributes (a_1, a_2, a_3) in order of decreasing importance weight (priority). The attribute with the highest priority corresponds to the parking resource type and is binary. An expected number of N_1^H drivers *implicitly* set $a_1 = 1$ if the type is "on-street parking" and $a_1 = 0$ if it is "parking lot"; the opposite holds for the remaining N_{PL}^H drivers. The second attribute corresponds to the distance from the destination and is turned to a binary one through the driver-specific threshold thr_w. Finally, the third attribute is the fee of the parking resource, which is conceptually set to zero or one by each driver depending on the relation of $c_3 - c_2$ to her threshold thr_c. Note that $thr_w \to 0$ for the $N_{PL}^H q_2$ drivers that decide exclusively on the basis of the distance attribute and $thr_w \to \infty$, $thr_c \to 0$ for the $N_{PL}^H \cdot q_1$ drivers who only consider the charged fees.

4.3 Strategic *vs.* heuristic predictions

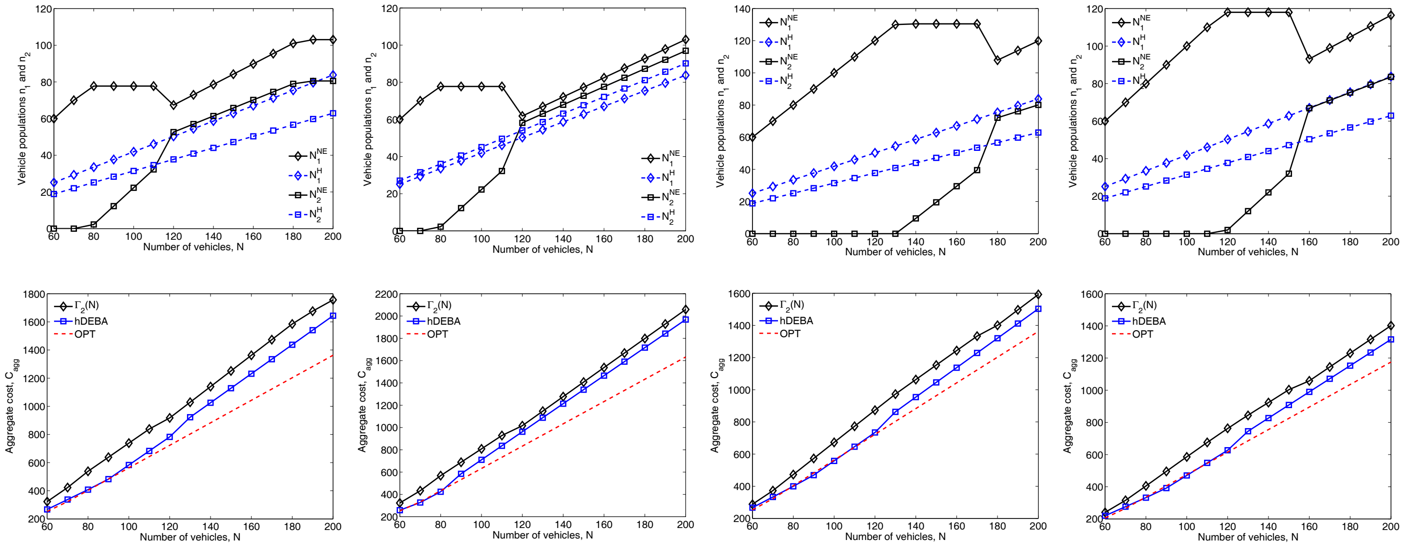
However the decision is made, the aggregate cost that drivers will have to pay is a function of their partitioning to the three groups $\{N_1, N_2, N_3\}$. Its most general expression is given by (20).

The analytical comparison of the two decision-making models along the lines of Theorem 1 is not straightforward. Instead, we plot in Fig. 5 the partitions of the vehicles' population and compare the aggregate cost generated by the NE of the $\Gamma_2(N)$ game and the operation of the hDEBA heuristic for typical parameter values. In all four scenarios, and many more that are not shown here due to space constraints, the trend is similar: the number of vehicles that select the on-street parking space is consistently higher under strategic decision-making than under hDEBA, whereas the situation is reversed for the vehicle population that selects the cheaper distant parking lot, at least for low and moderate demand/supply ratios. The excess cruising cost due to the first group of vehicles is higher than the excess cost paid by vehicles that do not manage to get a parking space in the distant parking lot and perforce end up in the expensive parking lot. As a result, the aggregate cost under hDEBA turns out to be smaller than that induced by the NE of $\Gamma_2(N)$ and sometimes even very close to the minimum possible (OPT).

5. DISCUSSION-CONCLUSIONS

We have carried out a performance analysis study that acknowledges the role of cognitive heuristics in drivers' decision-making and assesses their impact on the efficiency of the parking search process. These heuristics cater for humans' bounded rationality and present a radical departure from the normative model of strategic decision-maker who systematically aims at maximizing (minimizing) its gains (losses). Two parking search scenarios have been considered and treated as instances of multi-attribute choice problems with two (three) alternatives, respectively. The proposed lexicographic heuristics draw on research in the field of cognitive psychology and we show analytically that their use reduces the aggregate cost paid for satisfying the parking demand when compared to the prescriptions of game-theoretic models as-

$$\begin{aligned} C_{agg} &= min(R_1, N_1)(c_1 + f_r(r_1)) + min(max(N_1 - R_1, 0), max(N_2 - R_2, 0))(c_2 + c_{exc} + f_r(r_2)) \\ &+ max(N_1 - R_1 - max(R_2 - N_2, 0), 0)(c_3 + c_{exc}) + min(R_2, N_2)(c_2 + f_r(r_2)) + max(N_2 - R_2, 0)(c_3 + c_{exc}) + N_3 c_3 \end{aligned} \quad (20)$$



a. {1.5,5,8,2,6} b. {1.5,5,10,2,6} c. {1.5,5,8,1,6} d. {1,4,7,1,6}

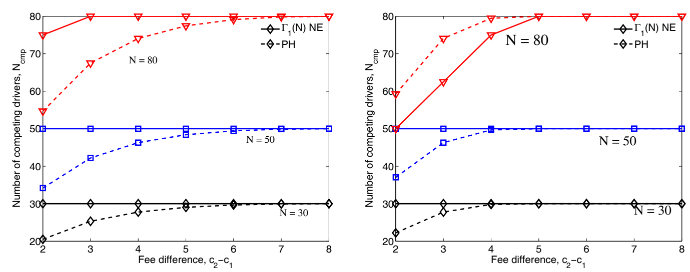
Figure 5: Distribution of vehicles to the three parking resources and respective aggregate cost for different parameter sets $\{c_1, c_2, c_3, c_{exc}, w_2\}$: R_1=25, R_2=40, $w_{min} = 3$, $w_{max} = 12$, $c_{max} = 5$.

suming fully rational decision-makers, over a broad selection of scenarios' settings.

The main implications of this work concern the software agents running over the devices that are mounted onboard the vehicles. One configuration possibility for them is to try to mimic the fully rational strategic agents. In that case, the parking assistance system infrastructure needs to provide them with detailed information about the available parking resources, the fees they charge and estimates of the parking demand. They then compute the NE of the respective game and make the respective recommendations to the driver. A simpler configuration alternative would allow for some driver personalization, possibly upon first use by her. The driver could enter very simple profile information such as whether she prefers parking lots over on-street space, what is the acceptable fee difference between the two and how far from her destination she is willing to park. The software could then just try to make recommendations that respect its owner's preferences. According to the results of the paper, in the majority of the scenarios, the second option will result in a socially more efficient parking selection process.

One of the main advantages of the heuristic decision-making models is that they directly describe how a decision is reached. Fig. 6 implies that these models may be also much more informative with respect to the sensitivity of the parking search efficiency to parameters such as, in this case, the difference in fees charged by the different parking resources. The expected number of competing vehicles is significantly more responsive to this difference under the priority heuristic model than it is under the game model $\Gamma_1(N)$.

Finally, on the methodological front, the steps taken in this paper for the performance analysis of the parking search process, *i.e.*, (a)game formulation and analysis for establishing a reference; b) adaptation and analysis of a cognitive (lexicographic) heuristic to the problem; (c) comparison of the prescriptions of the two models, could serve as a template for the analysis of many more problems. They are essentially applicable to various networking operations, whereby decisions are taken by multiple end users and jointly determine the outcome of the operation. The radio access network selection (*e.g.*, WiFi *vs.* cellular) is one such case.



a. $c_1 = 1, c_{exc} = 1, c_{max} = 0.8$ b. $c_1 = 1.5, c_{exc} = 2, c_{max} = 0.6$

Figure 6: Sensitivity of competing vehicles to the difference of parking resource fees: priority heuristic *vs.* game $\Gamma_1(N)$, $R_=25$.

On the other hand, there is an open question as to whether drivers do *actually* practice such heuristics when searching for parking space. There is compelling evidence that lexicographic heuristics are employed by humans in a wide range of very different selection tasks and can justify several well-reported empirical phenomena that result from their choices [5][6]. Yet, the ultimate validation of the heuristics' relevance calls for large-scale experimentation in real or semi-real (*i.e.*, emulator) conditions. Such experimentation is currently an open challenge for various research communities including psychologists and transportation engineers.

$$
\begin{aligned}
N_{cmp}^{NE} \quad \geq \quad & \frac{N_{13}\cdot(N_{12}+R_2)}{N_{13}+N_{23}} = \frac{\Big(c_3+c_{exc}-\big(c_1+f_r(r_1)\big)\Big)R_1\cdot\Big(\frac{c_2+f_r(r_2)+c_{exc}-\big(c_1+f_r(r_1)\big)}{c_{exc}}R_1+R_2\Big)}{\Big(c_3+c_{exc}-\big(c_1+f_r(r_1)\big)\Big)R_1+\Big(c_3+c_{exc}-\big(c_2+f_r(r_2)\big)\Big)R_2} \\
> \quad & R_1\frac{\Big(c_3+c_{exc}-\big(c_1+f_r(r_1)\big)\Big)R_1+\Big(c_3+c_{exc}-\big(c_1+f_r(r_1)\big)\Big)R_2}{\Big(c_3+c_{exc}-\big(c_1+f_r(r_1)\big)\Big)R_1+\Big(c_3+c_{exc}-\big(c_2+f_r(r_2)\big)\Big)R_2} \quad \textit{since } N_{12}/R_1>1 \\
> \quad & R_1\frac{\Big(c_3+c_{exc}-\big(c_1+f_r(r_1)\big)\Big)R_1+\Big(c_3+c_{exc}-\big(c_2+f_r(r_2)\big)\Big)R_2}{\Big(c_3+c_{exc}-\big(c_1+f_r(r_1)\big)\Big)R_1+\Big(c_3+c_{exc}-\big(c_2+f_r(r_2)\big)\Big)R_2} = R_1 \quad \textit{since } c_2+f_r(r_2)>c_1+f_r(r_1)
\end{aligned} \tag{21}
$$

6. ACKNOWLEDGMENTS

M. Karaliopoulos acknowledges the support of ERC08-RECITAL project, co-financed by Greece and the European Social Fund through the Operational Program "Education and Lifelong Learning"- NCRF 2007-2013.

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APPENDIX

A. PROOF OF THEOREM 2

We provide a sketch of the proof avoiding notation formalities that burden its comprehensibility. As long as the drivers are fewer than the on-street parking spots R_1, the NE prescribes that all of them should direct thereto. The same holds for all N values up to N_{12}, where the expected cost of choosing to search for curbside parking reaches that of the second choice (head for the distant parking lot)

$$\frac{R_1}{N_{12}}(c_1+f_r(r_1))+(1-\frac{R_1}{N_{12}})(c_2+f_r(r_2)+c_{exc}) = c_2+f_r(r_2)$$

Any additional up to R_2 drivers beyond the N_{12} ones should then head for the distant cheaper parking lot. Therefore, as N varies in $[N_{12}, N_{12}+R_2]$ the *individual* driver cost at the equilibrium remains constant and equals $c_2+f_r(r_2)$.

When N exceeds $N_{12}+R_2$, the expected *individual* cost of the first choice (on-street parking search) experiences a sudden increase since upon failure to seize an on-street spot, a driver can only drive to the expensive parking lot; the cheaper one is no longer an option. Hence, the split of (n_1, n_2) in the NE for $N = N_{12}+R_2+1$ features $n_1 < N_{12}$ and $n_2 > R_2$. The NE states (n_1, n_2) should satisfy

$$\frac{R_1}{n_1}(c_1+f_r(r_1))+(1-\frac{R_1}{n_1})(c_3+c_{exc}) = \frac{R_2}{n_2}(c_2+f_r(r_2))+(1-\frac{R_2}{n_2})(c_3+c_{exc})$$

Since $n_1+n_2 = N$, it comes out that $n_1 = \frac{N_{13}}{N_{13}+N_{23}}N$ and $n_2 = \frac{N_{23}}{N_{13}+N_{23}}N$. NE states with $n_3 > 0$ emerge only when the expected costs of the first and the second choice equal the cost (fees) of the expensive parking lot. This occurs for $n_1 = N_{13}$ and $n_2 = N_{23}$ values satisfying

$$\frac{R_1}{N_{13}}(c_1+f_r(r_1))+(1-\frac{R_1}{N_{13}})(c_3+c_{exc}) = c_3, \quad \text{and} \tag{22}$$

$$\frac{R_2}{N_{23}}(c_2+f_r(r_2))+(1-\frac{R_2}{N_{23}})(c_3+c_{exc}) = c_3 \tag{23}$$

respectively. At the NE, any amount of drivers beyond $N_{13}+N_{23}$ select the expensive parking lot so that the individual driver's cost is steady at c_3. Table 2 provides the full set of NE that emerge considering explicitly whether the aforementioned equilibria values are integers or not. Their validity check is straightforward.

B. PROOF OF PROPOSITION 1

The number of vehicles N_{cmp}^{NE} that compete for on-street parking capacity for $R_1 \leq N < N_{12}$ is $N \geq R_1$. For N in $(N_{12}, N_{12}+R_2]$, the respective number is (16)

$$N_{cmp}^{NE} = N_{12} = (1+\frac{(c_2-c_1)+(f_r(r_2)-f_r(r_1))}{c_{exc}})R_1 > R_1$$

Likewise, for $N > N_{13}+N_{23}$ the equilibrium value is

$$N_{cmp}^{NE} = N_{13} = (1+\frac{c_3-c_1-f_r(r_1)}{c_{exc}})R_1 > R_1$$

Finally, the inequalities (21) assert that the equilibrium number $N \cdot N_{13}/(N_{13}+N_{23})$ of competing vehicles also exceeds R_1 when N lies in $[N_{12}+R_2, N_{13}+N_{23}]$.