Collective subscriptions: a novel funding tool for crowdsourced network infrastructures

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***Abstract*—Community networks (CNs) are initiatives led by communities of people, who collectively contribute time, effort and resources to their purpose. Over the last two decades, they have proven their capacity to provide affordable connectivity in areas not attracting the interest of commercial operators, but also strengthen local community bonds. Nowadays, the realization of ambitious broadband connectivity agendas, the desire to bring online another billion of people in developing countries, but also concerns about concentration in the telecom market, motivate a more integral role of CNs in the global networking infrastructure. Prerequisites for this role are funding models that ensure their sustainable operation.**

In our paper, we study *collective subscriptions*, a novel subscription model that can be used to fund the CN activities. With collective subscriptions, a fixed subscription fee is charged per CN *node* and is shared between all individuals or households subscribing to the node. Maximizing the revenue out of the collective subscriptions while respecting the requirements for community inclusion turns out to be a complex problem with a non-trivial objective function. Hence, we look closer into particular scenarios of interest and devise both exact and approximate algorithmic solutions for them. The evaluation of the scheme against both real and synthetic data shows that it combines higher subscription revenue with higher community inclusion when compared to the default fixed price individual subscription scheme. On a practical note, our analysis helps the CN operator understand and optimize this funding tool for sustainably engaging the community into the CN activities. The scheme itself could find more general use as a subscription model for other shared community resources such as computational power and storage space.

***Index Terms*—community networks, network economics, pricing, economic sustainability, crowdsourced infrastructures, maxmin fair resource allocation**

I. Introduction

Community networks (CNs) are crowdsourced initiatives typically inspired, built and managed by communities of people, who collectively contribute time, effort and resources to their purpose. They originally emerged in the late 90s and have taken many forms and shapes ever since [1]. The experience with what could be called the first generation of CNs is a mixed bag of success and failure stories. Some CNs have become obsolete due to the rise of commercial high speed broadband networks in the areas they operated. Others have flourished; not only have they responded to the need for affordable connectivity but they have also strengthened the community links in the areas they cover, acquiring an added value that is hard to assess in financial terms. The Catalan CN guifi.net in Spain[1], the B4RN network in Lancashire, UK [2], and the Freifunk initiative in Germany[2], are CN instances each counting several years of activity and tens of thousands of network access nodes. Nevertheless, CNs have not yet managed to unleash their full potential and establish themselves as integral parts of the global telecommunications infrastructure. They are rather branded so far as alternative non-profit networks filling in the coverage gaps of commercial operators in areas they consider it cost-inefficient to invest in.

Currently there are at least three compelling reasons for reiterating on the possible role of CNs in the overall telecommunications landscape. First, broadband Internet connectivity is promoted as core priority in political agendas throughout the world. In Europe, for example, the European Commission has set ambitious policy objectives for the years to come[3] that demand huge investment costs. Grassroots initiatives such as CNs are acknowledged as one way to diffuse these costs towards more stakeholders such as municipalities and citizens [3].

Secondly, there are several efforts, also involving large Internet corporations such as Google, Microsoft and Facebook, to connect another billion users in developing areas worldwide [4] [5]. Given provisions for access to unlicensed spectrum and cheap fiber, small crowdfunded community operators that generate local value for the local people may be the obvious approach towards realizing this vision, circumventing the need for complex and centralized systems.

Thirdly, the CN model facilitates the separation of the network infrastructure from the service provision layer and generates opportunities for sharing the related costs between multiple actors, including commercial entities such as local/regional ISPs. This stands in stark contrast with vertically integrated models, where all the network layers belong to one entity and end users are left with limited options when it comes to choosing an operator. Such models typically give rise to monopoly- or oligopoly-driven market distortions [6].

To live up to these expectations, CNs need first to ensure their economic sustainability. This is pursued through a mix of funding tools that is CN-specific and weighs differently regular member subscriptions, donations from non-members, grants

[1] guifi.net, Internet in community networks, https://guifi.net/
[2] Freifunk, initiative for free wireless networks, https://freifunk.net/en/
[3] https://ec.europa.eu/digital-single-market/en/broadband-europe

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or subsidies from public or private non-profit entities, and, more rarely, contributions from private for-profit sector [7]. The experience with different CN instances and funding tools suggests that the most favorable and reliable funding source are the regular subscriptions of their own members. These are paid by individual members on a monthly or annual basis and are typically cheaper than what commercial ISPs charge for Internet access.

However, there are two issues with individual member subscriptions. First, more often than not, these subscriptions are optional for the community members. As a result, CNs experience high levels of free riding: many members use the network without entering any subscription relationship with it, thus not contributing at all to its operational costs. Secondly, in CNs it is of highest priority to ensure affordable network access to *all* community members, including the financially weakest ones. With the default fixed price subscription scheme, this is only possible with very low fees that end up shrinking what CNs can reap from their main funding source.

In this paper, we propose and analyze the *collective subscriptions*, a novel subscription scheme that responds to both issues faced by individual fixed price subscriptions. Collective subscriptions are currently under consideration in several CNs, as a scheme than can improve their economic sustainability. According to them, the elementary subscribed unit is a *CN node*, rather than an individual or household using the CN services, and the *node subscription fee* is shared between those users who subscribe to it.

Besides matching well the participatory and sharing ideals of CNs, collective subscriptions are a tool that can better mobilize and sensitize the community to the sustainable funding of the CN activities. We show in this work, that they outperform the default model with respect to both objectives, combining higher subscription revenues with increased community participation. The scheme essentially serves as a countermeasure against free riding, motivating existing CN users to actively recruit new subscribers as a means to reduce their own share of the subscription fee.

Our contributions: To the best of our knowledge, this is the first study of the collective subscriptions scheme in CNs. Our key contributions are made at three levels:

- *Novel problem formulations:* We formulate the theoretical problems that emerge when applying collective subscriptions to CNs. With collective subscriptions, the CNO simultaneously assigns CN users to node subscriptions and determines the uniform CN node subscription fee to maximize its revenue while ensuring maximum participation of the community members in the network. The resulting optimization problem (*collective subscription problem*) has a non-trivial max-min-min objective function, which has independent theoretical interest.
- *Algorithmic solutions for the optimization problem:* In light of the computational complexity of the problem in the general case, we look closer into particular scenarios of interest and devise both exact and approximate algorithmic solutions for them.
- *Practical implications:* Our analysis and comparative evaluation of the scheme demonstrates its main properties and helps CN operators gain valuable insights to this novel funding tool. It provides them with guidelines on how to optimally tune it for their own CN (*e.g.*, assign users to subscriptions and set the node subscription fee) so as to balance their aspirations about maximum revenue and community inclusion.

The rest of the paper is structured as follows. We present the system model in Section II and summarize the fixed price individual subscription scheme, the current de facto scheme in CNs, in section III. We then proceed with formulating the problem of determining optimal collective subscriptions that maximize the subscription revenue under requirements for high community participation in Section IV. We numerically assess properties and the efficiency of the collective subscriptions scheme in Section V, also considering a variant of the scheme. Related work is reviewed in Section VI and directions for further work are discussed in Section VII.

II. System model and assumptions

We consider a community network (CN) providing network connectivity to a community of users $\mathcal{U}$, with $U \doteq |\mathcal{U}|$, through a set of wireless network nodes. Practically, and despite the highly decentralized process through which the CN evolves, these nodes can be "organized" into three distinct layers. The top layer consists of the CN core nodes. These nodes connect in mesh mode network parts that physically lie far away the one from another; it could be different neighborhoods in a town or city or different villages in a rural area. This *core* layer provides access to the Internet. The bottom layer, called the *access* layer, includes Access Points through which end users access the CN. Inbetween the core and access layers, there are additional nodes forming the distribution layer. These nodes distribute the wireless signal from the core nodes to the access layer and move traffic from/to CN users to/from the core nodes, and eventually the Internet.

More often that not, the purchase and deployment cost of core- and distribution-layer CN nodes is undertaken by the team of volunteers that initiate the CN and may be subsidized by public agencies. On the other hand, the access nodes are purchased and maintained by the individual users who decide to join the CN. What remains to be secured are the operational expenses (opex) of the CN, involving costs related to the nodes at the core and distribution layer, electricity consumption, personnel costs, and the cost of leased lines that connect the CN to other ISPs providing Internet transit service to the CN. We assume in this paper that these expenses are covered by subscriptions that are paid by community members on a monthly basis. This is a plausible assumption in line with the findings in [7] about the funding sources of (successful) CN initiatives throughout the world. The less mainstream assumption in our work, is that these subscriptions

Figure 1: Example physical system layout and options available to CN users under the collective subscriptions' scheme. Each user may join the collective subscription of one node out of a subset of the N CN nodes or abstain (*i.e.*, "join node n_0").

are collective and organized around the set $\mathcal{N}$ of CN core and distribution nodes, with $N \doteq |\mathcal{N}|$.

A. Model actors

The main actors in this setting are:

The CN operator (CNO): The term denotes the small group of people who have initiated and typically operate the CN, ensuring its sustainable funding. The CNO leases network access capacity from commercial upstream ISPs. It then determines the monthly node-level subscription fee f_s with two things in mind. On the one hand, the revenues from subscription fees should make up for the CN operational expenses and help fund the further growth of the CN. On the other hand, the fee should not serve as a barrier for the engagement of community members in the CN. In fact, the maximum possible participation of the community in the CN is a prerequisite for the CN sustainability, both in economic and socio-ethical terms [8] [9].

The CN users: These are community members who can potentially use the CN after subscribing to it[4]. Users appreciate differently the value of the CN and Internet connectivity and this is directly reflected in the maximum monthly price r_u, $u \in \mathcal{U}$ they are willing to pay for participating in it. Hereafter, we refer to these user-specific prices as *price ceilings*. Without loss of generality, we subsequently index CN users in order of decreasing price ceilings so that

$$\forall u, v \in \; [1..N] \quad u < v \Longleftrightarrow \; r_u \geq r_v \tag{1}$$

B. Charging for CN access

The CN charges network nodes rather than individuals and each node-level subscription fee, f_s, is collectively paid by those community members who are assigned to the respective node. The assignment of users to node subscriptions is carried out by the CNO after taking into account their price ceilings. The process may also consider stated preferences of users as to which nodes' subscriptions they would like to join. Formally, for each CN user $u \in \mathcal{U}$, we can define a subset $\mathcal{N}_u \subseteq \mathcal{N}$ of CN nodes, including those nodes the user tends to use more often such as nodes in the vicinity of her residence or workplace (ref. Fig. 1).

[4]Hereafter, the term *user* denotes the holder of an individual subscription, *i.e.*, a user may correspond to a household, as with typical subscriptions to commercial network operators.

C. Assumptions

We make the following assumptions in this work:

A1) The individual *price ceilings* are known to the CNO. In standard commercial networks, this is a challenging task for network operators, who invest money and effort to get insights to what users are willing to pay for network access. In community networks, due to the stronger social links and participatory processes in place, this task is easier, even though the possibility of strategic behavior on behalf of community members (*e.g.*, understate what they are willing to pay for CN access) cannot be excluded.

A2) Under collective subscriptions, *the pricing at node level is uniform across nodes*, *i.e.*, the CNO does not discriminate between different CN nodes when setting f_s. This is a rather plausible assumption since it forms the node-level counterpart of non-discriminatory pricing when charging individuals.

A3) Users subscribed to a given node share equally the subscription fee. Hence, the subscription share f_s/k of an individual community member is not uniform *across* nodes but rather depends on the number k of users who join a node's subscription. Since the individual fee decreases with the number of subscribers to a given node, existing CN users are given a tangible incentive to actively recruit new users to the subscription of a CN node, and eventually, to the CN itself.

The assumption A2 holds throughout the paper. We relax assumption A3 in section V-D and further discuss assumption A1 in section VII. Note that the assignment of users to CN node subscriptions should not be confused with the actual patterns of user-to-CN node associations and possible network congestion effects. CN users roam across the CN and access it from several nodes, besides the one they subscribe to. The nodes they use to access the network, their network activity, and any network congestion these generate are independent of the way the collective subscriptions are organized. In fact, the CN copes with congestion through distinct network dimensioning and resource allocation processes such as the addition of new nodes/links and the upgrading of existing ones.

In section IV, we analyze the optimization problem that is faced by the CNO when organizing the collective subscriptions. But earlier, in section III, we briefly summarize the de facto subscription scheme in community networks, *i.e.*, individual subscriptions with a fixed uniform fee.

III. Individual subscriptions with non-discriminatory subscription fees

Individual subscriptions with a fixed fee are maybe the simplest possible subscription scheme. Not least because of this simplicity, they represent the current practice in many successful instances of CNs [7]. With this scheme, the single control parameter at the hands of the CNO for jointly addressing the revenue and community engagement objectives is the choice of the individual subscription fee f_{is}. In general, however, these two objectives are not aligned.

Example: Consider a toy-example with four users, one CN node, and price ceilings 8, 12, 13, and 15, respectively. The engagement of the users is maximum for subscription fees in the range [0,8], setting an upper bound of 32 for the CNO revenue. Yet, the CNO could increase the fee to 12 and achieve a revenue of 36 from the three users who would be willing to pay it, at the expense of one user who cannot afford it. If a fifth user were added with price ceiling $r_5 = 8$, then there is a fee value ($f_{is} = 8$) that simultaneously maximizes the CNO revenue and the number of CN subscribers.

Formally, if f_{is}^R is the CNO revenue-maximizing fee and f_{is}^U the fee that maximizes the individual subscriptions of users in the CN, the two fees coincide, $f_{is}^R = f_{is}^U = r_U$ when

$$\max_{k \in [1..U]} k r_k = U r_U \tag{2}$$

In the general case, the CNO revenue under individual subscriptions equals

$$R_{CNO} = f_{is} \sum_{u \in \mathcal{U}} \mathbb{1}_{r_u \geq f_{is}} \tag{3}$$

whereas the number of users who cannot afford a subscription (*abstainers*) is

$$U_{abs} = \sum_{u \in \mathcal{U}} \mathbb{1}_{r_u < f_{is}}. \tag{4}$$

The aggregate user surplus is due to community members who afford to join a subscription and can be written as

$$W_U = \sum_{u: r_u \geq f_{is}} (r_u - f_{is}) \tag{5}$$

so that the total (social) welfare, including both the CNO and the users, equals

$$W_T = W_U + R_{CNO} \tag{6}$$

IV. Optimizing collective subscriptions

Consider a feasible partition $p = (p_0, p_1, p_2, .., p_N)$ of CN users to CN node subscriptions, $k_n = |p_n|$ being the number of users who share the subscription of node $n \in \mathcal{N}$, and k_0 the number of users who abstain from the CN. To ease notation, we introduce a virtual node n_0 so that $p_0 (k_0)$ is the subset (number) of users who "join" n_0 (*i.e.*, abstain from the CN), and we can define the extended node sets $\mathcal{N}_u^+ = \mathcal{N}_u \cup \ n_0$, for each CN user, and $\mathcal{N}^+ = \mathcal{N} \cup n_0$, for the overall CN (see Fig. 1).

Given this partition, the maximum fee the CN operator can collect from node n is (ref. assumption A3)

$$fee(n) = k_n \cdot \min_{u \in p_n} r_u, \quad n \in \mathcal{N} \tag{7}$$

that is, for a given subset of users that share a node's subscription fee, the per user fee share cannot exceed what the user with the minimum price ceiling is willing to pay.

At network level, since the subscription fee is uniform across all CN nodes that attract subscribers (assumption A2), the CNO revenue becomes

$$R_{CNO}(p) = \min_{\substack{n \in \mathcal{N} \\ k_n > 0}} fee(n) \cdot \sum_{n \in \mathcal{N}} \mathbb{1}_{\mathbb{Z}^+}(k_n) \tag{8}$$

where $\mathbb{1}_{\mathcal{A}}(x) = 1$, for $x \in \mathcal{A}$, is the indicator function and $\mathbb{Z}^+$ is the set of positive integers.

Hence, the partition of CN users that maximizes $R_{CNO}(p)$ is the solution to the optimization problem

$$\max_{p} \quad R_{CNO}(p) \qquad (OPT)$$

$$s.t. \quad k_n = \sum_{u: n \in \mathcal{N}_u} x_{un} \quad \forall n \in \mathcal{N}^+ \tag{9}$$

$$\sum_{n \in \mathcal{N}_u^+} x_{un} = 1 \quad \forall u \in \mathcal{U} \tag{10}$$

$$k_0 \leq \alpha \quad \forall u \in \mathcal{U} \tag{11}$$

$$x_{un} \in \{0, 1\} \quad u \in \mathcal{U}, n \in \mathcal{N}^+ \tag{12}$$

where $x_{un} = 1$ when user u shares the subscription fee for node n and $x_{un} = 0$ otherwise. The input constant α in constraint (11), hereafter called the *exclusion-tolerance constraint*, sets an upper bound on the number of community users ($k_0 \equiv U_{abs}$) who may be left out of the collective subscription scheme[5].

Overall, the revenue the CNO can collect is subject to a $double\ min\ effect$: first, what can be collected at node level is determined by the user with the minimum price ceiling, in line with (7); then, at network level, the common collective subscription fee is the minimum fee that can be collected across all nodes, in line with (8). If the per node collected fee in (7) equaled the *sum* of subscribers' price ceilings, we would face an instance of the restricted max-min fair allocation problem (see, for example, [10]). It is the *min* operator in (7) that renders the objective function in (OPT) non-trivial.

The complexity of the generic optimization problem can be relaxed in two cases. More interesting and relevant is the case, where the user subscription assignment preferences are symmetric, *i.e.*, when $\mathcal{N}_u = \mathcal{N}, \forall u \in \mathcal{U}$. This symmetry implies that users are flexible as to which CN node's collective subscription to join, which is a reasonable hypothesis, at least in small rural communities. The other case, which is less realistic but discussed for the sake of completeness, is when users exhibit identical price ceilings, *i.e.*, $r_u = r_v \ \forall u, v \in \mathcal{U}$.

A. Symmetric subscription assignment preferences

In this case, the CN users do not express preferences as to which CN node(s) they are willing to subscribe to and the CNO has more flexibility in "packing" CN users together into subscriptions. In what follows, we pursue optimal user partitions through enumerating candidate solutions in an informed manner, which dramatically reduces the search space when compared to an exhaustive search.

Let $\mathcal{P}(k_0, k_1, .., k_n)$, with $k_i \geq k_{i+1}, i \in [1..N-1]$ be the set of all possible CN user partitions that feature a permutation of numbers (k_1,..,k_n) as cardinalities of sets $(p_1, .., p_N)$ and

[5]In almost all community networks, there are special provisions for financially weaker members that cannot afford the network connectivity cost, including subsidies from the rest of the community or barter arrangements.

$|p_0| = k_0$. Let also $p_{ord}(k_0, k_1, .., k_n) \in \mathcal{P}(k_0, k_1, .., k_n)$ be the single partition [6] simultaneously satisfying

- $\max_{u \in p_j} r_u \leq \min_{u \in p_{j+1}} r_u \quad j \in [0..N-1]$ (C1)
- $k_j \geq k_{j+1} \quad j \in [1..N-1]$ (C2)

We call this partition, the *r-ordered* partition. To construct this partition for a given set $\mathcal{P}(k_0, k_1, .., k_n)$, it suffices to index users in order of increasing price ceilings and denote the first k_0 of those as p_0, the next k_1 as p_1, the next $k_2 \leq k_1$ as p_2 and so on. In each set $p_j, j \geq 1$ of this partition, we call the first user who features the minimum price ceiling and determines the subscription fee share for all subscribers to node j, the *lead user*. We can show that:

Proposition 1. *Any partition $p(k_0, \sigma(k_1), .., \sigma(k_n))$, where σ is an arbitrary permutation of the set $\{k_1, k_2, .., k_n\}$, can be converted to an r-ordered partition $p_{ord}(k_0, k_1, .., k_n)$ so that*

$$R_{CNO}(p) \leq R_{CNO}(p_{ord}) \tag{13}$$

Proof. To ease the proof exposition, we refer to the partition subsets as "subsets" and to node subscribers, and their corresponding price ceiling values, as "values". Hence, the partition subsets of node subscribers are called subsets of values and the lead users in each subset are termed lead values.

Starting from an arbitrary original partition p, we first rearrange the subsets in order of decreasing cardinality so that p_j becomes the the j^{th} largest subset; p_0 remains intact. Then, we carry out a series of pairwise exchanges of values between subsets till we get the r-ordered partition.

The procedure is repeated over all the subsets, starting with p_0 and finishing with p_{N-1} and is outlined under Algorithm 1. In each inner loop iteration for given j, z denotes the highest value in p_j, w denotes the smallest value in subsets indexed in [j+1..N-1] and m the subset to which w resides. As long as $z > w$, the two values are exchanged between the corresponding subsets; otherwise, we increase j and repeat the process with the next subset. By the time we finish the iterations for subset p_j, the first j subsets of the resulting partition coincide with those of the r-ordered partition. The outer loop terminates when we parse subset p_{N-1} and carry out any feasible pairwise exchanges with subset p_N.

We claim that in each of these pairwise exchanges, the resulting revenue R_{CNO} out of the partition (ref. equation (8) either remains the same or increases. To see this, consider any of these pairwise exchanges.

Consider first the case $j = 0$, *i.e.*, when w moves to subset p_0. This implies that the new lead value in p_m will be higher, either z (the element leaving p_0) or the second smallest element in p_m. The fee that can be extracted by p_m can only increase. If this was the minimum one over all subsets, the total revenue increases; otherwise it remains the same. Hence, trivially, after the replacement of an element of subset p_0, the achievable revenue R_{CNO} is at least as high as it was before the replacement.

[6]Whenever there are users with identical price ceiling values, there will be more than one such partitions, all sharing the same distribution of price ceiling values across the CN nodes.

When w moves to subset $j > 0$, two possibilities exist:

(a) w does not become the lead value in j (because there is another smaller value). In that case, the fee that can be derived by subset j after the exchange is $fee'(j) = fee(j)$. On the other hand, $fee'(m) \geq fee(m)$ since the new lead value in subset m cannot be smaller that w. Overall, $R'_{CNO} \geq R_{CNO}$, the inequality holding when $fee'(m) > fee(m)$ and $fee(m) > fee(l) \; \forall l \neq m$.

(b) w becomes the lead value in subset j. This means that w is the minimum element over all subsets indexed in [j..N]. Since $k_m \leq k_j$ (C2),

$$\min_n fee(n) \leq fee(m) < fee(j) \tag{14}$$

After moving w to subset j, it holds

$$fee'(j) \geq fee(m) \quad and \quad fee'(m) \geq fee(m) \tag{15}$$

because $k_m \leq k_j$ and the new lead element in subset m is at least as high as w, respectively. Hence, from (14) and (15), one of the following two possibilities exist:

$$\min_n fee(n) = \min_n fee'(n) = fee(l) \qquad l \neq j, m \tag{16}$$

or

$$\begin{aligned} \min_n fee(n) &= fee(m) \leq \min_{l \notin \{m,j\}} fee(l) \\ &\leq \min\{\min_{l \notin \{m,j\}} fee'(l), fee'(m), fee'(j)\} \\ &= \min_n fee'(n) \end{aligned}$$

Hence, as the original partition transitions to the r-ordered one through a finite number of such pairwise exchanges, the revenue R_{CNO} acquires a non-decreasing sequence of values.

Example: Consider 12 CN users with price ceilings 3,3,5,6,7,8,10,12,14,15,15,16, and $N = 4$ CN nodes. If the users (aka. price ceilings) are assigned to the four nodes as $\{15, 12, 3\}$, $\{10, 6\}$, $\{3, 7, 16\}$, and $\{5, 8, 14, 15\}$, the r-ordered partition emerges through the steps shown in Fig. 2. □

Algorithm 1 Transformation of an arbitrary partition to its r-ordered counterpart

Input: Partition subset p_0 and subsets p_1,..,p_N, indexed in order of decreasing cardinality
Output: Subsets p_0, p_1,..,p_N of the r-ordered partition

```
for every subset j ∈ [0..N − 1] do
    z = max value in subset j, w = min value over subsets
    indexed in [j+1..N-1], m = subset hosting w
    while w < z do
        move z to the subset m and w to p_j
        z = max value in subset j, w = min value in subsets
        indexed in [j+1..N-1], m = subset hosting w
    end while
end for
```

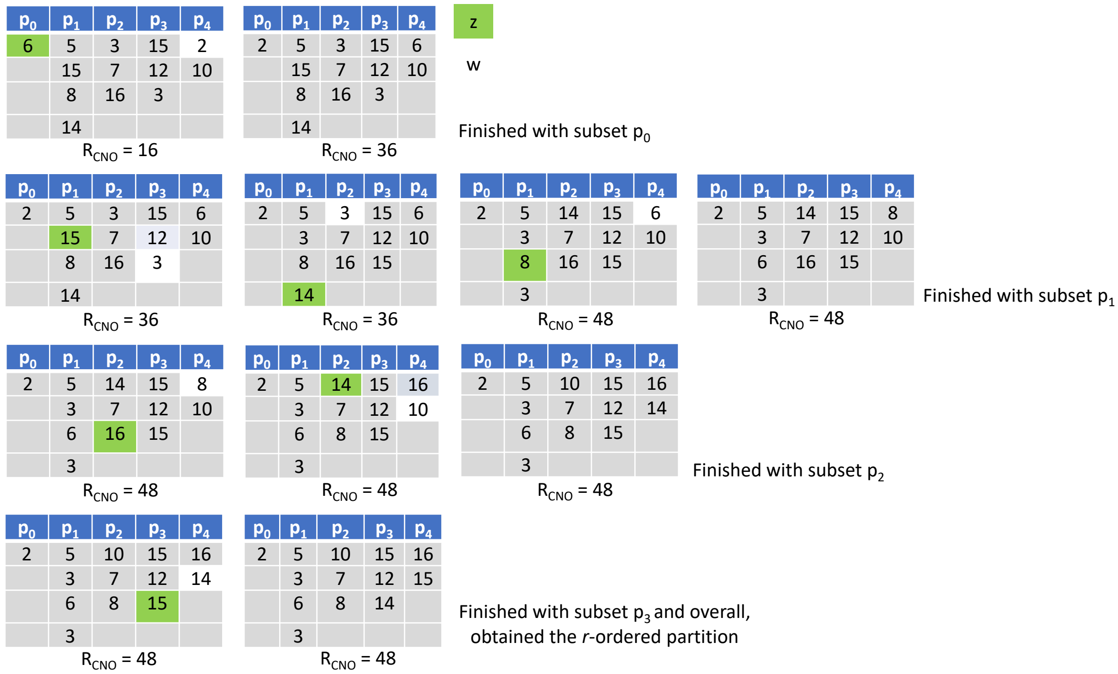


Figure 2: Example-transformation of an arbitrary partition of CN users into node subscriptions to its r-ordered counterpart. Partitions p_1, p_2, p_3, p_4 have already been indexed in order of non-increasing cardinality. R_{CNO} denotes the achievable CNO revenue.

Corollary 1. *To find the optimal partition of CN users to collective subscriptions, it suffices to search through the set of r-ordered partitions featuring $k_0 \leq \alpha$.*

Limiting the search in this set reduces the search complexity from $O(N^U)$ down to $O(U^N)$, *i.e.*, it turns the complexity from exponential to polynomial in the number of CN users. Note that typically $U >> N$.

B. Identical price ceilings

When $r_u = r_0\ \forall u \in \mathcal{U}$, the fee that can be collected at each CN node is proportional to the number of users subscribing to it. Since in this case there is no motivation to exclude some community member, *i.e.*, the revenue cannot be hurt by a user with low enough price ceiling, we can set $k_0 = 0$ (alternatively, α=0). The total fee the CN operator can collect becomes

$$R_{CNO}(p) = r_0 \cdot \min_{\substack{n \in \mathcal{N} \\ k_n > 0}} k_n \cdot \sum_{n \in \mathcal{N}} \mathbb{1}_{\mathbb{Z}^+}(k_n) \tag{17}$$

and the problem faced by the CNO can be written:

$$\begin{aligned}
\max_{p} \quad & \min_{\substack{n \in \mathcal{N} \\ k_n > 0}} \sum_{u \in \mathcal{U}} x_{un} \qquad (OPTs) \\
s.t. \quad & k_n = \sum_{u: n \in \mathcal{N}_u} x_{un} \quad \forall n \in \mathcal{N} \\
& \sum_{n \in \mathcal{N}} x_{un} = 1 \quad \forall u \in \mathcal{U} \\
& x_{un} \in \{0, 1\},\ k_n \leq U \quad u \in \mathcal{U}, n \in \mathcal{N}
\end{aligned}$$

This a special case of the restricted max-min fair allocation problem (see, for example, [10]). In the general problem typology, a set $\mathcal{C}$ of resources need to be allocated to a set $\mathcal{A}$ of players, each resource $c \in \mathcal{C}$ having a fixed non-zero value v_c and being relevant only for a subset of the players. In (OPTs), resources correspond to community members, players to CN nodes, and resource values to the identical price ceilings. There is a close relationship between (OPTs) and the problem of finding perfect b-matchings in a bipartite graph [11]. It turns out that one can construct a polynomial-time algorithm for (OPTs) by using the algorithm for the perfect b-matching problem in bipartite graphs [12] but this goes beyond the scope of this work.

C. Symmetric subscription assignment constraints and identical price ceilings

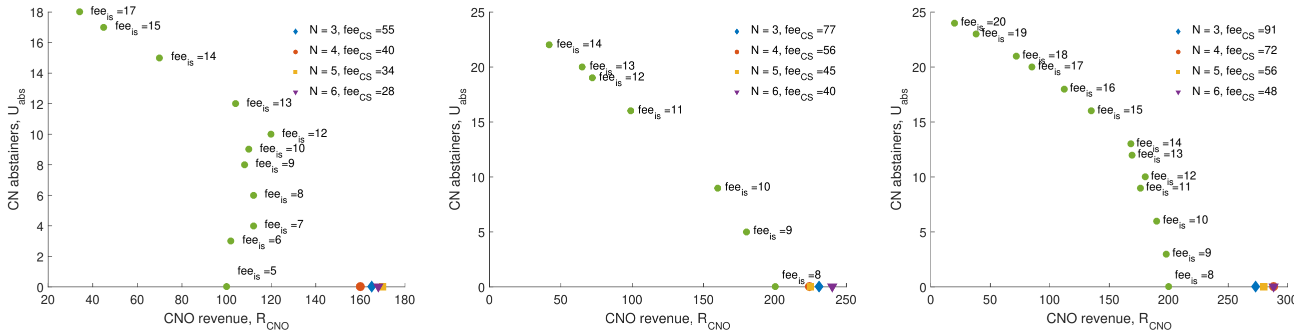
The objective function is again given by (17) and since $\mathcal{N}_u = \mathcal{N}$, CN users can be assigned to node subscriptions in round robin fashion. The total fee the CN operator can collect becomes

$$R_{CNO} = N \cdot \lfloor \frac{U}{N} \rfloor \cdot r_0 \tag{18}$$

V. Evaluation of collective subscriptions

A. Methodology

The two main performance metrics of interest regarding the collective subscriptions are the revenue, R_{CNO}, that the CNO can extract from the scheme and the resulting number of CN abstainers, U_{abs}. In general, the two metrics depend on (a) the number of CN nodes and their arrangement in physical space; (b) the number of CN users and their subscription assignment



a. $U = 20, r \sim \mathcal{U}\{5, 17\}$ b. $U = 25, r \sim \mathcal{U}\{8, 12\}$ c. $U = 25, r \sim \mathcal{U}\{8, 20\}$

Figure 3: CNO revenue and CN abstainers under fixed price individual subscriptions (subscript "is") and collective subscriptions (subscript "cs"). Some of the markers that are listed in the plot legends for collective subscriptions are not visible because they overlap with each other.

preferences; (c) the amounts of money that CN users are willing to pay for CN connectivity (*i.e.*, price ceilings). Namely, (a) and (b) determine the subscription assignment preferences of CN users and, together with (c), dictate how the scheme performs in the two metrics.

We assess the impact of these factors through simulations with both real and synthetically generated data. The real data refer to actual topologies of small CNs in a remote agricultural area in center-northern Greece. They correspond to fourteen villages in the Sarantaporo area with populations in the order of few hundred people that can be served by a few CN nodes. We use these data to inform the experimentation settings about the subscription assignment preferences of users in section V-B. On the other hand, with synthetic data, we let the numbers of CN nodes and users take arbitrary yet plausible values, while the number of node subscription alternatives per user, $(|\mathcal{N}_u|)_{u\in\mathcal{U}}$, as well as their price ceilings, $(r_u)_{u\in\mathcal{U}}$, vary stochastically. Unless otherwise stated, these two variables follow discrete random distributions, with $|\mathcal{N}_u| \sim \mathcal{U}\{1, n_{max}\}$ and $r_u \sim \mathcal{U}\{r_{min}, r_{max}\}$.

B. Collective vs. individual subscriptions

We first compare the collective subscriptions with the fixed price individual subscriptions, the scheme that is typically in use in CNs. These experiments draw on real datasets about the number of households in the communities and the CN nodes N that serve them, which range from three to six. We assume symmetric subscription assignment preferences, which is a reasonable assumption for small communities. We seek the maximum revenue under full participation of the community in the CN, *i.e.*, we solve (OPT) in section IV for $\alpha = 0$.

The plots in Fig. 3 are representative of how collective subscriptions compare with fixed price individual subscriptions. In the second case, the only way to render the CN access affordable to the whole community is by setting the fee to the minimum price ceiling across all users. Higher fees generate more abstainers who cannot afford the subscription fee, hence they hurt the community engagement, whereas their net result on revenue depends on the distribution of price ceilings across users. In Fig. 3a, for example, the maximum revenue would be achieved for $f_{is} = 12$ but only half the community would subscribe to the CN at that fee. On the contrary, in Figs. 3b,c the fee ($f_{is} = 8$) that maximizes the CNO revenue also results in maximum community engagement. Increasing the subscription fee even further is not an option since most of the community cannot afford it and the revenue collapses.

With collective subscriptions, on the other hand, the CNO can achieve (R_{CNO}, U_{abs}) values (the markers at the right bottom end of plots in Fig. 3) that Pareto-dominate those achievable with individual subscriptions. For all N values, the scheme can group properly CN users into collective subscriptions and set the per node fee so that both higher CNO revenue and community participation are achieved. In the examples that are plotted in Fig. 3, the gain in CNO revenue ranges from 12.5% to approximately 43%.

To what extent, do these results generalize? Essentially, the collective subscription scheme is a non-trivial way to combine fixed pricing (at the node level and between users assigned to the same collective subscription) with price discrimination (*across* users assigned to different collective subscriptions)[7]. For this reason, they can, intuitively, transform part of the user surplus into CNO revenue.

In fact, as far as the individual subscription scheme generates a number of subscribers that is a multiple of the number of CN nodes, $(U - U_{abs}) = \delta \cdot N, \delta \in \mathbb{Z}^+$, we can formally show that

Proposition 2. *For any given set of users and their corresponding price ceilings, the collective subscription scheme can yield* (R_{CNO}, U_{abs}) *values that Pareto-dominate those obtained with fixed price individual subscriptions.*

Proof. Assume that the CN users are indexed in order of decreasing price ceiling values, *i.e.*, $r_1 \geq r_2 \geq \ldots \geq r_U$. For a given value f_{is}, chosen under fixed price individual

[7]Note that collective subscriptions cannot directly be characterized as one of the standard three types of price discrimination, first-, second-, and third-degree [13]

subscriptions, the (R_{CNO}, U_{abs}) value achieved by the scheme is $(k \cdot f_{is}, U - k)$, where

$$k = \max\{j \mid f_{is} \leq r_j\} \tag{19}$$

For each of those outcomes, we can construct a naive collective subscription configuration that yields at least the same revenue, R_{CNO}, for the given number of abstainers, U_{abs}. It suffices to split the users into N equal groups of size k/N so that $\max_{u \in p_j} r_u \leq \min_{u \in p_{j+1}} r_u,\ 1 \leq j \leq N-1$. This way, the fee that the CNO can charge at node level sums up to at least $k \cdot r_k / N \geq k \cdot f_{is}/N$ and the revenue out of all the N nodes is $k \cdot r_k \geq k \cdot f_{is}$. Grouping the users into sets of unequal size, could further increase the revenue R_{CNO}. □

On the other hand, there are rare extreme combinations of N, U, and $\{r_u\}_{u \in \mathcal{U}}$ values, for which fixed price subscriptions outperform the collective subscriptions, *e.g.*, for U a prime number and identical price ceiling values for all users.

C. CNO revenue vs. community participation in collective subscriptions

With individual subscriptions, the subscription revenue and the number of users who are left out of the CN vary with the uniform fee, f_{is}, in the characteristic way shown in Fig. 3. In these experiments, We explore this relationship under the collective subscriptions.

We solve (OPT) under symmetric node assignment preferences and turning constraint (11) to equality. Namely, we impose a fixed number of abstainers, U_{abs}, and track the achievable CNO revenue. We do this in three scenarios for how price ceilings are distributed in [r_{min}, r_{max}]: uniformly, probability mass concentrated on the bottom half of the range of values (positive skew), and probability mass concentrated on the top half of the range of values (negative skew).

As can be seen in Fig. 4, the trend is not identical in all three plots. For uniform and positively skewed distributions of the price ceilings, the subscription revenue is maximized under maximum participation of the community in the CN. The exclusion of a few members with the lowest price ceilings tends to hurt the revenue, or, in the best case, leaves it intact. When the mix of users is such that high price ceilings are more frequent, it pays back, in terms of revenue, to exclude a few users with the lowest price ceilings.

D. Collective subscriptions with unequal node fee shares

The collective subscriptions scheme, as defined in section IV and analyzed in the previous paragraphs, apply a mild flavor of price discrimination, imposing uniform fees at CN node level (assumption A2) and equal node fee shares (assumption A3). How could the CNO boost revenue under a more aggressive price discrimination policy?

In this set of experiments, we relax assumption A3, essentially considering a possible variant of the collective subscriptions scheme. Under this variant, the users assigned to each collective subscription pay different fee shares, up to their price ceilings (first-degree price discrimination). The problem faced by the CNO with this variant, *i.e.*,

$$\begin{aligned} \max_{p} \quad & R_{CNO}(p) && (OPT2) \\ s.t. \quad & fee(n) = \sum_{u \in p_n} r_u \quad n \in \mathcal{N} && (20) \\ & \sum_{n \in \mathcal{N}} x_{un} = 1 \qquad \forall u \in \mathcal{U} && (21) \\ & (8), (9), (11), (12) \end{aligned}$$

is an instance of the *restricted max-min fair resource allocation* problem (see, for instance, [10], individual users and collective subscriptions in (OPT2) corresponding to resources and players, respectively, in [10]). When there are no constraints related to preferences of players for resources (or vice-versa), resembling the symmetric subscription assignment preferences case in (OPT), the restricted max-min fair resource allocation problem simplifies to the *multi-way number partitioning* problem [14]: divide a set or multiset of integers (in OPT2: price ceilings) into a given number of subsets (in OPT2: collective subscriptions), so that the difference between the smallest and the largest subset sums is minimized. Users assigned to the CN node(s) with the smallest subset sum (*i.e.*, the common fee for all collective subscriptions) pay exactly their price ceilings, whereas users assigned to nodes with larger subset sums share the common fee in proportion to their price ceilings, extracting non-zero surplus.

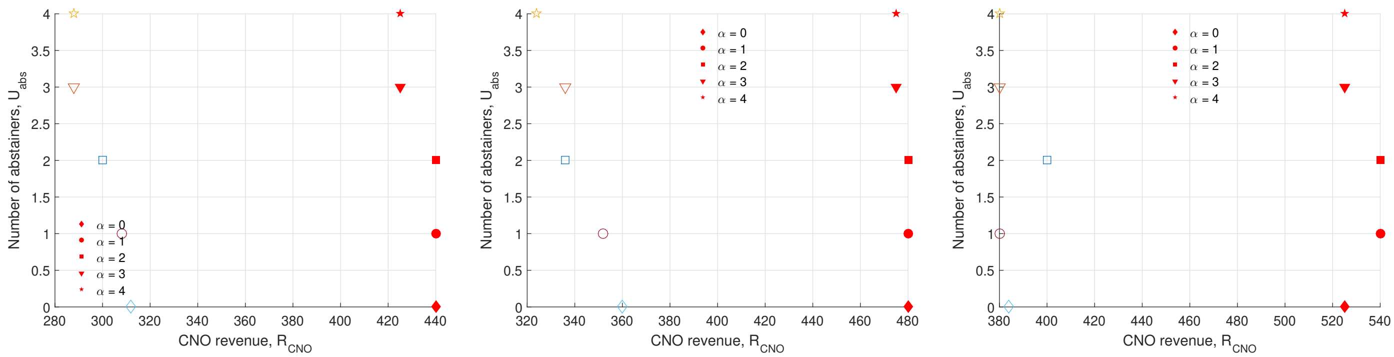
Fig. 5 compares the two collective subscription alternatives under full community participation, *i.e.*, α=0 in (OPT)[8]. As expected, the application of price discrimination to users assigned to the same collective subscription results in higher revenue. The gain in all related experiments, including those shown in Fig. 5 is fairly consistent, ranging almost anywhere from 10% to 25%. Nevertheless, this gain has to be carefully weighed against the fact that two or more community members end up paying different shares of a given node's collective subscription. The incentive to misreport the true price ceiling becomes stronger with this scheme. We further discuss this concern in section VII.

VI. Related work

There are only a few studies on the economics of community networks, not least because most CNs have been promoting a non-profit operations model. Notably, most of those studies appear to be motivated by the commercial service paradigm of FON[9] and the specific context it assigns to the word "community". Nevertheless, as CNs pursue an integral role in the global telecommunications infrastructure (see section I), sustainable modes of financing their activities will become all the more crucial.

[8]We solve the multi-way number partition problem using the MULTIFIT approximation algorithm for number-partitioning problems [14]. The algorithm, first proposed in [15], combines binary search with an approximation algorithm for the bin packing problem.

[9]FON website, https://fon.com/



a. $r_u \in [8, 14]$ for 80% of users b. $r_u \sim \mathcal{U}\{8, 20\}$ for 100% of users c. $r_u \in [8, 14]$ for 20% of users

Figure 4: Optimal revenue under different levels of community exclusion. Filled markers correspond to $U = 40, N = 5$; empty ones to $U = 30, N = 4,\ r \sim \mathcal{U}\{8, 20\}$.

a. U = 40, N = 5, r_{min} = 5 b. U = 50, N = 5, r_{min} = 8 c. U = 30, N = 6, r_{min} = 8

Figure 5: Optimal revenue under full community participation under the original (OPT) and the modified (OPT2) collective subscription schemes.

Maybe the first study of pricing issues in CNs is [16]. It is assumed that the end users have the alternative of a commercial licensed based operator and the price they are willing to pay for CN connectivity grows with the coverage it achieves. Hence, the CN coverage and revenue evolve over time and, depending on the price and initial CN coverage, the result may be a competitive CN with high coverage or one that dies out. The analysis identifies the pricing strategies of the two operators at Nash equilibrium and the benefit resulting for end users due to the competition between them. In [17], the model in [16] is elaborated further to address individual user mobility patterns, and different types of network nodes to which users associate with different frequencies. The assumption is that the CNO possesses complete or partial information about the way users move and their differentiated perception about the network' coverage so that it can optimally determine the subscription fees over a number of periods ahead in time.

Afrasiabi and Guerin in [18] also propose a simple utility function to model the users' varying propensity to roam and their concern about network coverage. However, and contrary to [16], their model also accounts for negative externalities: as the users of the network grow, the roaming traffic load increases and limits what is available to them, as either home or roaming users. They find that a fixed pricing policy generally fails to align the total welfare, *i.e.*, the sum of the operator's revenue and the users' utilities, with the profit of the operator, exhibiting less flexibility than discriminatory pricing and usage-based pricing strategies, which charge the user differently if she is at home or roaming.

Finally, a study that is more directly inspired by the FON service model is presented in [19]. Three types of CN user memberships are identified therein, depending on whether users own an AP or not and whether they share and access the CN APs free of charge ("Linus" users) or for a small fee ("Bill" users). The CN users play a two-stage dynamic game involving two different decisions at different time scales: they select membership types over time intervals in the order of months and how aggressively to access the shared radio channel over time intervals in the order of a few minutes. The authors analyze the Subgame Perfect Equilibrium strategies and study when these are realized by best-response strategies.

Common to all four studies is that they price individual users. However, CN operators increasingly address communities of users rather than individuals as references for financing the network deployment and operation. B4RN [2] grows its broadband infrastructure by setting up community-wide projects at the scale of a one or more villages; and

Sarantaporo.gr[10], a Greek CN in the area of mountain Olymp, applies collective, rather that individual, subscriptions per CN node. This trend has served as main motivation for our work on the collective subscriptions' paradigm.

VII. Discussion and conclusions

In our paper, we have focused on an innovative subscription mechanism currently under consideration in community networks for self-funding their activities. Setting distinct CN nodes as subscriber entities, collective subscriptions let share the network operational cost in a way that discourages free riding practices and motivates the recruitment of new users to the CN initiatives. We have theoretically and experimentally analyzed the scheme, highlighting its performance advantage over fixed price individual subscriptions and demonstrating its main properties and alternative configurations.

Our work opens several interesting directions for future work. First of all, the formal complexity and approximability analysis of the non-trivial optimization problem in section IV has independent theoretical interest. Then, the model we have proposed in this paper could be modified in a number of ways. One possibility is to promote community inclusion as optimization objective, *e.g.*, seeking for fees satisfying

$$f_{cs}^{*} = arg \min_{f_{cs}} U_{abs}(f_{cs}) \quad (22)$$

instead of (9) in the optimization problem, with a constraint on the minimum acceptable subscription revenue. However, intuition and our evaluation results (*e.g.*, Fig. 3) suggest that the two objectives are closely interrelated: maximizing revenue demands high inclusion of community members. Hence, the additional value of such extensions would need to be weighed against the extra complexity they add to the problem.

Another model extension could address the critical assumption that the CNO possesses perfect information about the fees community members are willing to pay (assumption A1 in section II). The users could instead declare themselves their price ceilings and they could do this strategically, in an attempt to reduce their own subscription share. The CNO would then need to devise a truthful mechanism that will induce the agents to reveal their true willingness-to-pay and efficiently assign them to the CN collective subscriptions.

On a broader note, our work adds to the research thread on network infrastructure sharing. Whereas in mobile cellular networks (*e.g.*, [20]) such sharing emerges *a posteriori* as a requirement for rationalizing the cost of infrastructure deployment, infrastructure sharing in CNs is generalized practice and a necessary condition for their existence per se. Such crowdsourced initiatives are nowadays looked upon with renewed interest since they can play a major role in the realization of broadband connectivity agendas in developed countries, while contributing maximally to connecting millions of people in developing countries. Collective subscriptions could evolve to a valuable tool that will help them fund their operation in sustainable manner and respond to these challenges.

[10]Sarantaporo.gr WiFi networks, http://www.sarantaporo.gr/

Finally, collective subscriptions could be viewed as a general funding tool that stands somewhere in the middle between egalitarian fixed price subscriptions and pure price discrimination practices seeking to extract from each payer what she is willing to pay. As such, it could be adopted as subscription mechanism to other shareable resources such as computational power or storage space.

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