Joint Dynamic Wireless Edge Caching and User Association: A Stochastic Optimization Approach

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***Abstract*—Wireless edge caching alleviates the capacity burden imposed on the backhaul of dense cellular networks while reducing the user-perceived latency. In this paper, we consider the joint optimization of caching and user association (JCA) policies with respect to the total cache hit ratio, taking into account the limited radio and storage resources, the service requirements, and the quality of the wireless channels. In order to effectively capture the dynamic nature and randomness of the system under study, which is reflected respectively in the mobility and content preferences of the users as well as in the time-variance of the wireless channels due to fading, we use a stochastic optimization framework based on Lyapunov optimization. After formulating the corresponding dynamic JCA problem, we solve this NP-hard task by applying a low-complexity heuristic algorithm that alternates at each timeslot between the caching and user association problems. Numerical simulations highlight the performance gains of the proposed dynamic JCA scheme over its static and decoupled caching/user association counterparts and shed light on the effect of various parameters on the performance.**

***Index Terms*—Wireless caching, user association, mobility, stochastic optimization, Lyapunov optimization.**

I. Introduction

The ongoing growth of the mobile data traffic's volume, which is mainly attributed to the proliferation of terminals and the delivery of high-quality video content [1], has motivated the dense deployment of small-cell base stations (SBS) as a means to enhance the capacity of cellular networks [2]. Wireless edge caching alleviates the heavy burden that is imposed on the mobile backhaul by network densification via the storage of popular content in cache servers placed at the cell sites, such that future user requests are served locally [3].

The resulting backhaul traffic savings and latency reduction gains are commonly quantified via the achieved cache hit ratio, i.e., the fraction of user requests that are satisfied by the cache server. This caching efficiency measure, in turn, is determined by the applied caching policy, which takes caching decisions either at regular intervals (offline) or on a per-request basis (online) based on the frequency or/and recency of user requests for each file [3]. *The joint optimization of caching and user association (JCA) policies can further improve the hit ratio.*

This NP-hard problem has been solved in the literature by formulating a one-to-many matching game under the assumption of identical content preferences among the users that correspond to a global Zipf requests pattern [4]. In [5], the authors consider user clustering based on the individual content demand profiles of the users and solve the caching problem via reinforcement learning. In [6], a heuristic iterative algorithm that alternates between solving a 0-1 Knapsack problem (KP) for caching and a generalized assignment problem (GAP) for user association is presented. Another decoupling approach based on the Generalized Benders decomposition is derived in [7], taking into account as well the leasing costs for the content providers. In [8], the authors develop a near-optimal distributed algorithm that minimizes download delay, considering the quality of the radio access and backhaul links. However, this method, which involves cost function linearization via the use of McCormick envelopes and Lagrange partial relaxation, presents high computational complexity.

Different from the aforementioned studies on static JCA, this paper employs a stochastic optimization framework based on Lyapunov optimization [9] for the derivation of a dynamic JCA policy. This approach enables maximization of the long-term time-average total cache hit ratio subject to time-average constraints on network resources. It essentially maps the JCA problem into a virtual queue stability and penalty minimization problem, where the virtual queues pertain to the long-term constraints, and the penalty corresponds to the objective function. Despite its apparent simplicity, *this method effectively captures the dynamic nature and randomness of wireless communication networks, which is attributed to the mobility and content preferences of the users as well as to the time-variability of the wireless channels due to fading*. Nonetheless, while Lyapunov optimization has been applied in caching or user association problems [10], [11], *it has not been used under the JCA context*, to the best of our knowledge.

Following the Lyapunov optimization methodology, we solve the dynamic JCA problem heuristically, considering at each time slot a static JCA problem that incorporates the instantaneous user locations and virtual queue values. Hence, the overall problem is solved without any assumption on the statistics of user mobility processes, other than that of the existence of a stationary distribution. The solution is compared via numerical simulations against that of the static JCA problem as well as of a decoupled caching/user association strategy. The simulation results showcase the superiority of the proposed approach and highlight the effect of parameters such as the cache storage capacity, the cell capacity, and the user population on the performance.

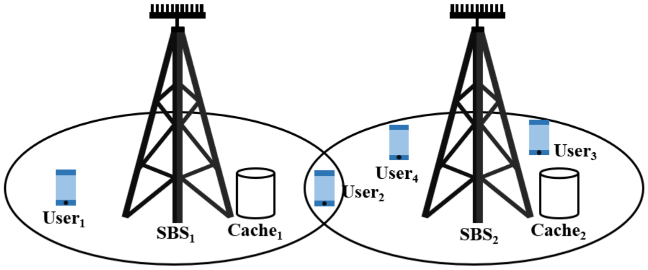


Fig. 1. Example system.

II. System Model

The considered system consists of a set $\mathcal{L} \triangleq \{1, \ldots, L\}$ of cells, a set $\mathcal{U} \triangleq \{1, \ldots, U\}$ of user terminals equipped with a single antenna each, and a catalog $\mathcal{N} \triangleq \{1, \ldots, N\}$ of files, which we assume for convenience and without loss of generality that they have equal size. A SBS equipped with a set $\mathcal{M}_l \triangleq \{1, \ldots, M_l\}$ of antennas and a cache server with storage capacity of C_l files are placed at cell $l \in \mathcal{L}$. The cache stores at each timeslot $t \in \mathcal{T} \triangleq \{1, \ldots, T\}$ a set $\mathcal{F}_l(t)$ of $|\mathcal{F}_l(t)| \leq C_l$ files from the content catalog, where T is the length of the considered time horizon.

The location vector of each user $u \in \mathcal{U}$ is denoted as $\mathbf{d}_u(t) = (d_{l,u}(t) : \ l \in \mathcal{L})$, where $d_{l,u}(t)$ refers to the distance of user u from SBS l at timeslot t. The ensemble location vector of all users at time t is denoted as $\mathbf{d}(t) = (\mathbf{d}_u(t) : \ u \in \mathcal{U})$. The trajectory of user u due to mobility is described by a sequence of pairs of timestamps and location vectors $\{t, \mathbf{d}_u(t)\}$. At each given time, user u is located within range of a set $\mathcal{L}_u(t) \triangleq \{1, \ldots, L_u(t)\} \subseteq \mathcal{L}$ of $1 \leq L_u(t) \leq L$ cells, but she is associated with only one of these cells. For example, in Fig. 1, User 2 is within range of both SBS$_1$ and SBS$_2$, but she is associated with only one of them. User u has a content preference distribution $p_u(n)$, $n \in \mathcal{N}$, $\sum_{n \in \mathcal{N}} p_u(n) = 1$.

SBS l serves at time t a set $\mathcal{U}_l(t) \triangleq \{1, \ldots, U_l(t)\} \subseteq \mathcal{U}$ of $1 \leq U_l(t) \leq U$ users, where $U_l(t) \leq M_l$, on a single time-frequency resource over the resulting multiple-input single-output (MISO) link in the single-user case ($U_l(t) = 1$) or MISO broadcast channel in the multi-user scenario ($U_l(t) > 1$). The SBSs utilize maximum ratio (MR) precoding to boost the received power in the former case or zero-forcing (ZF) precoding to eliminate the intra-cell inter-user interference (IUI) in the latter one. Furthermore, appropriate frequency planning is applied, to null the inter-cell interference (ICI).

The precoding vector assigned to user $u \in \mathcal{U}_l(t)$ by SBS l at time t, $\mathbf{w}_{l,u}(t) \in \mathbb{C}^{M_l}$, can be decomposed as $\mathbf{w}_{l,u}(t) = \sqrt{P_{l,u}(t)}\bar{\mathbf{w}}_{l,u}(t)$, where $P_{l,u}(t)$ denotes the transmit power allocated to that user and $\bar{\mathbf{w}}_{l,u}(t) = \mathbf{v}_{l,u}(t)/\left\|\mathbf{v}_{l,u}(t)\right\|$ which satisfies $\left\|\bar{\mathbf{w}}_{l,u}(t)\right\|^2 = 1$ corresponds to the normalized precoding vector (i.e., the beamforming direction). We have $\mathbf{v}^{\text{MR}}_{l,u}(t) \triangleq \mathbf{h}_{l,u}(t)$ and $\mathbf{v}^{\text{ZF}}_{l,u}(t) \triangleq \left(\mathbf{I}_{M_l} - \mathbf{H}^{\#}_{l,u}(t)\mathbf{H}_{l,u}(t)\right)\mathbf{h}_{l,u}(t)$, $l \in \mathcal{L}$, $u \in \mathcal{U}_l(t)$, where $\mathbf{h}_{l,u}(t) \in \mathbb{C}^{M_l}$ denotes the SBS l–user u channel, $\mathbf{H}_{l,u}(t) \in \mathbb{C}^{(U_l(t)-1) \times M_l}$ is defined as

$$\mathbf{H}_{l,u}(t) \triangleq \left[\mathbf{h}_{l,1}(t), \ldots, \mathbf{h}_{l,u-1}(t), \mathbf{h}_{l,u+1}(t), \ldots, \mathbf{h}_{l,U_l(t)}(t)\right],$$

and $\mathbf{H}^{\#}_{l,u}(t) \in \mathbb{C}^{M_l \times (U_l(t)-1)}$ stands for the Moore-Penrose pseudo-inverse of $\mathbf{H}_{l,u}(t)$.

We consider quasi-static, frequency-flat, independent and identically distributed (i.i.d.) Rayleigh fading channels $\mathbf{h}_{l,u}(t) = \sqrt{\beta_{l,u}(t)}\tilde{\mathbf{h}}_{l,u}(t)$, where $\beta_{l,u}(t) = C_0 \left(d_{l,u}(t)/d_0\right)^{-\alpha} \zeta_{l,u}(t)$ denotes the large-scale fading coefficient, C_0 is the path loss at a reference distance d_0 in the far-field region of SBS l, α refers to the path loss exponent, $\zeta_{l,u}(t)$ corresponds to the shadow fading, i.e., $10\log_{10}\zeta_{l,u}(t) \sim \mathcal{CN}\left(0, \sigma^2_{\text{sf}}\right)$, and $\tilde{\mathbf{h}}_{l,u}(t)$ captures the small-scale fading, i.e., its m-th element represents the $\mathcal{CN}(0,1)$ fading coefficient between the m-th antenna of SBS l and user u, $m \in \mathcal{M}_l$.

When only user u is associated with SBS l, the SBS applies MR precoding to serve that user. Under null ICI, $\mathbf{h}^{\dagger}_{l,u}(t)\mathbf{w}^{\text{MR}}_{j,k}(t) = 0$, $j \in \mathcal{L} \setminus \{l\}$, $k \in \mathcal{U} \setminus \{u\}$. Thus, the received signal-to-noise-ratio (SNR) of user u is given by:

$$\text{SNR}_u(t) = \frac{\left|\mathbf{h}^{\dagger}_{l,u}(t)\bar{\mathbf{w}}^{\text{MR}}_{l,u}(t)\right|^2 P_{l,u}(t)}{\sigma^2_u}, \ u \in \mathcal{U}_l(t), \ l \in \mathcal{L}, \quad (1)$$

where σ^2_u is the variance of the zero-mean complex additive Gaussian noise at user u. When $U_l(t) > 1$, in turn. the SBS l applies ZF precoding to serve the users associated with it at timeslot t. Since this precoding strategy completely suppresses the intra-cell IUI and the ICI is null, i.e., $\mathbf{h}^{\dagger}_{l,u}(t)\mathbf{w}^{\text{ZF}}_{l,i}(t) = 0$, $i \in \mathcal{U}_l(t) \setminus \{u\}$, and $\mathbf{h}^{\dagger}_{l,u}(t)\mathbf{w}^{\text{ZF}}_{j,k}(t) = 0$, $j \in \mathcal{L} \setminus \{l\}$, $k \in \mathcal{U} \setminus \mathcal{U}_l(t)$, the SNR at user u is again given by (1) by replacing $\bar{\mathbf{w}}^{\text{MR}}_{l,u}(t)$ with $\bar{\mathbf{w}}^{\text{ZF}}_{l,u}(t)$.

The spectral efficiency (SE) of user u is given by $r_u(t) = \log_2\left(1 + \text{SNR}_u(t)\right)$, $u \in \mathcal{U}_l(t)$, $l \in \mathcal{L}$. The transmission of SBS l to its users $\mathcal{U}_l(t)$ at time t is subject to a transmit sum-power constraint $\sum_{u \in \mathcal{U}_l(t)} P_{l,u}(t) \leq P_l$ and SE constraints $r_u(t) \geq r^{\min}_u$, where P_l denotes the transmit power budget of SBS l and $r^{\min}_u$ represents the minimum required SE for user u. In practice, SBS l serves user u with its minimum required SE $r^{\min}_u$. Thus, $\log_2\left(1 + \text{SNR}_u(t)\right) = r^{\min}_u \Rightarrow \text{SNR}_u(t) = 2^{r^{\min}_u} - 1$. Substituting $\text{SNR}_u(t)$ from (1) in the last equation and solving for $P_{l,u}(t)$, we obtain

$$P_{l,u}(t) = \frac{\left(2^{r^{\min}_u} - 1\right)\sigma^2_u}{\left|\mathbf{h}^{\dagger}_{l,u}(t)\bar{\mathbf{w}}^{\text{PS}}_{l,u}(t)\right|^2}, \quad l \in \mathcal{L}, \ u \in \mathcal{U}_l(t), \quad (2)$$

where PS stands for precoding scheme and $\text{PS} \in \{\text{MR}, \text{ZF}\}$.

By virtue of user mobility, the association of user u with SBS l at timeslot t generates an association cost $b_{l,u}(t)$ which reflects the power $P_{l,u}(t)$ that this SBS allocates to that user for achieving $r_u(t) = r^{\min}_u$. Therefore, for a given precoding strategy, this cost is strongly related with the physical distance of user u from SBS l, $d_{l,u}(t)$, and the quality of the wireless channel $\mathbf{h}_{l,u}(t)$. Furthermore, the transmit power budget P_l is reflected in the upper limit of the aggregate association cost, $\sum_{u \in \mathcal{U}_l(t)} b_{l,u}(t) \leq B_l$, where B_l determines the number of users that can be served by SBS l.

III. Problem Formulation

Let $\{y_{n,l}(t)\}$ and $\{z_{l,u}(t)\}$ be two sets of binary decision variables, with $y_{n,l}(t) = 1$ if file n is stored in cache l and $y_{n,l}(t) = 0$ otherwise; and $z_{l,u}(t) = 1$ if user u is associated with SBS l and $z_{l,u}(t) = 0$ otherwise. Then, our objective is to solve the following maximization problem **(P1)** ($u \in \mathcal{U}$, $l \in \mathcal{L}$, $n \in \mathcal{N}$, $t \in \mathcal{T}$):

$$\max_{\{\mathbf{y}(t),\mathbf{z}(t)\}} \lim_{T\to\infty} \sup \frac{1}{T} \sum_{t\in\mathcal{T}} \sum_{u\in\mathcal{U}} \sum_{l\in\mathcal{L}} \sum_{n\in\mathcal{N}} y_{n,l}(t) z_{l,u}(t) p_u(n) \tag{3a}$$

$$\text{s.t.} \quad \lim_{T\to\infty} \sup \frac{1}{T} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} y_{n,l}(t) \leq C_l, \tag{3b}$$

$$\lim_{T\to\infty} \sup \frac{1}{T} \sum_{t\in\mathcal{T}} \sum_{u\in\mathcal{U}} b_{l,u}(t) z_{l,u}(t) \leq B_l, \tag{3c}$$

$$\sum_{l\in\mathcal{L}_u(t)} z_{l,u}(t) = 1, \tag{3d}$$

$$y_{n,l}(t) \in \{0,1\}, \tag{3e}$$

$$z_{l,u}(t) \in \{0,1\}. \tag{3f}$$

The reward function that we seek to maximize in (3a) is the long-term average aggregate number of cache hits. The constraints in (3b) and (3c) correspond to the long-term average cache storage and service cost constraints, respectively. Constraint (3d) captures the fact that each user is associated with a single SBS out of a subset of candidates in their neighborhood. Finally, constraints (3e) and (3f) denote the binary nature of the decision variables $y_{n,l}$ and $z_{l,u}$, respectively. The optimization variables are the caching policy $\{\mathbf{y}(t)\}$ and the user association policy $\{\mathbf{z}(t)\}$.

IV. Caching and User Association Policies

A. Virtual Queues and Lyapunov Optimization

We use Lyapunov optimization to tackle the aforementioned problem. We start by mapping constraints (3b) and (3c) to queue stability problems, so that the problem is converted to one of optimal control of a dynamic queueing system. That is, for each of the L constraints in (3b), we define a virtual queue $Q_l(t)$, $l \in \mathcal{L}$, which evolves as follows:

$$Q_l(t+1) = \left(Q_l(t) + \sum_{n\in\mathcal{N}} y_{n,l}(t) - C_l \right)^+, \tag{4}$$

where $x^+ = \max(x, 0)$. This queue builds up as items are stored in cache l and increases or decreases in size by the number of cached items above or below the cache storage capacity, respectively.

Further, for each of the L constraints in (3c), we define a virtual queue $Z_l(t)$, $l \in \mathcal{L}$, which evolves as follows:

$$Z_l(t+1) = \left(Z_l(t) + \sum_{u\in\mathcal{U}} b_{l,u}(t) z_{l,u}(t) - B_l \right)^+. \tag{5}$$

Here, the queue builds up with an amount equal to the cost to serve a user, with each user associated with SBS l, and decreases when the total service cost is less than B_l.

We should note that the amounts of increase and decrease in the queues, and hence the values $b_{l,u}(t)$, are integers. We should also mention that although the "soft" capacities C_l and B_l can be exceeded, numerical simulations indicate that the corresponding queues do not "explode".

Let $\mathbf{Q}(t) = (Q_1(t), \ldots, Q_L(t))$, $\mathbf{Z}(t) = (Z_1(t), \ldots, Z_L(t))$, and $\mathbf{\Theta}(t) = (\mathbf{Q}(t), \mathbf{Z}(t))$. The variables $\mathbf{y}(t)$ and $\mathbf{z}(t)$ control the admission processes in the virtual queues. We define the Lyapunov function $\mathbb{L}(\mathbf{\Theta}(t))$ as

$$\mathbb{L}(\mathbf{\Theta}(t)) = \frac{1}{2} \sum_{l\in\mathcal{L}} Q_l^2(t) + \frac{1}{2} \sum_{l\in\mathcal{L}} Z_l^2(t). \tag{6}$$

Then, we define the one-slot conditional Lyapunov drift as

$$\Delta(\mathbf{\Theta}(t)) = \mathbb{E}\left[\mathbb{L}(\mathbf{\Theta}(t+1)) - \mathbb{L}(\mathbf{\Theta}(t)) \,|\mathbf{\Theta}(t) \right], \tag{7}$$

which denotes the expected change in the Lyapunov function in one slot, conditioned on the current state, and where the expectation is with respect to the statistics of queue evolution. By using standard techniques for bounding the Lyapunov drift [9], we have that

$$\begin{aligned} \Delta(\mathbf{\Theta}(t)) \leq B &+ \mathbb{E}\left[\sum_{l\in\mathcal{L}} Q_l(t) \left(\sum_{n\in\mathcal{N}} y_{n,l}(t) - C_l \right) |\mathbf{\Theta}(t) \right] \\ &+ \mathbb{E}\left[\sum_{l\in\mathcal{L}} Z_l(t) \left(\sum_{u\in\mathcal{U}} b_{l,u}(t) z_{l,u}(t) - B_l \right) |\mathbf{\Theta}(t) \right], \end{aligned} \tag{8}$$

where $B > 0$. Since we aim at maximizing the objective function in (3a), we employ the drift-plus penalty function

$$\Delta(\mathbf{\Theta}(t)) + V\mathbb{E}\left[Y(t) \,|\mathbf{\Theta}(t) \right], \tag{9}$$

where $V \geq 0$ is a fixed trade-off parameter that quantifies the significance of the objective function and

$$Y(t) = -\sum_{u\in\mathcal{U}} \sum_{l\in\mathcal{L}} \sum_{n\in\mathcal{N}} y_{n,l}(t) z_{l,u}(t) p_u(n). \tag{10}$$

For the drift-plus-penalty function we have

$$\begin{aligned} &\Delta(\mathbf{\Theta}(t)) + V\mathbb{E}\left[Y(t) \,|\mathbf{\Theta}(t) \right] \leq \\ &B - V\mathbb{E}\left[\sum_{u\in\mathcal{U}} \sum_{l\in\mathcal{L}} \sum_{n\in\mathcal{N}} y_{n,l}(t) z_{l,u}(t) p_u(n) \,|\mathbf{\Theta}(t) \right] \\ &+ \mathbb{E}\left[\sum_{l\in\mathcal{L}} Q_l(t) \left(\sum_{n\in\mathcal{N}} y_{n,l}(t) - C_l \right) |\mathbf{\Theta}(t) \right] \\ &+ \mathbb{E}\left[\sum_{l\in\mathcal{L}} Z_l(t) \left(\sum_{u\in\mathcal{U}} b_{l,u}(t) z_{l,u}(t) - B_l \right) |\mathbf{\Theta}(t) \right]. \end{aligned} \tag{11}$$

The drift-plus-penalty method aims to employ at each timeslot the appropriate control, in order to minimize the right-hand-side of the inequality above (i.e., the penalty in performance imposed by the constraints (3d)– (3f). Thus, the resulting optimization problem **(P2)** is formulated as follows:

$$\max_{\{\mathbf{y}(t),\mathbf{z}(t)\}} \sum_{u\in\mathcal{U}} \sum_{l\in\mathcal{L}} \sum_{n\in\mathcal{N}} V p_u(n) y_{n,l}(t) z_{l,u}(t) - Q_l(t) y_{n,l}(t)$$

$$- Z_l(t) b_{l,u}(t) z_{l,u}(t) \quad (12a)$$

$$\text{s.t.} \quad \sum_{l \in \mathcal{L}_u(t)} z_{l,u}(t) = 1, \; u \in \mathcal{U}, \quad (12b)$$

$$y_{n,l}(t) \in \{0,1\}, \quad n \in \mathcal{N}, \; l \in \mathcal{L}, \quad (12c)$$

$$z_{l,u}(t) \in \{0,1\}, \quad l \in \mathcal{L}, \; u \in \mathcal{U}. \quad (12d)$$

At each timeslot $t \in \mathcal{T}$, the controller observes the vectors of virtual queues $\mathbf{Q}(t)$ and $\mathbf{Z}(t)$ and the instantaneous ensemble location vector of users $\mathbf{d}(t)$, which in turn affect the dynamic service costs $\{b_{l,u}(t)\}$, and decides on the policy vectors $\mathbf{y}(t)$ and $\mathbf{z}(t)$ that maximize (12a) at that slot.

B. Heuristic Iterative JCA Algorithm

The JCA problem **(P2)** is NP-hard [8]. In order to solve it, we apply the low-complexity heuristic approach described in [6]. Specifically, we initially assume that all users within range of an SBS are associated with that SBS (with minimum distance and random selection as tie-breakers in the case of overlapping cells and equidistant SBSs, respectively). Thus, we ignore the user association constraints (12b) and (12d), set $z_{l,u} = 1$, and determine the content placement at each cache $l \in \mathcal{L}_u$ by solving independent instances of the following 0-1 KP **(P3a)**:

$$\max_{\{\mathbf{y}\}} \sum_{u \in \mathcal{U}} \sum_{l \in \mathcal{L}} \sum_{n \in \mathcal{N}} V p_u(n) y_{n,l} z_{l,u} - Q_l y_{n,l}$$
$$- Z_l b_{l,u} z_{l,u} \quad (13a)$$

$$\text{s.t.} \quad y_{n,l} \in \{0,1\}, \quad n \in \mathcal{N}, \; l \in \mathcal{L}_u. \quad (13b)$$

Note that we have dropped the timeslot index for convenience. After this initial step, the iterative phase of the algorithm starts. That is, we first determine the user associations $\{z_{l,u}\}$ given the content placements $\{y_{n,l}\}$ obtained by solving **(P3a)**. To this end, we ignore the caching constraint (12c) and solve a single instance of the following GAP **(P3b)**:

$$\max_{\{\mathbf{z}\}} \sum_{u \in \mathcal{U}} \sum_{l \in \mathcal{L}} \sum_{n \in \mathcal{N}} V p_u(n) y_{n,l} z_{l,u} - Q_l y_{n,l}$$
$$- Z_l b_{l,u} z_{l,u} \quad (14a)$$

$$\text{s.t.} \quad \sum_{l \in \mathcal{L}_u} z_{l,u} = 1, \; u \in \mathcal{U}, \quad (14b)$$

$$z_{l,u} \in \{0,1\}, \quad l \in \mathcal{L}, \; u \in \mathcal{U}. \quad (14c)$$

With these new values for user associations $\{z_{l,u}\}$, we revisit the caching problem **(P3a)**. We continue to iteratively solve the caching and user association problems in an alternating manner until no improvement on the performance can be made, as it is quantified by the reward function in (12a).

C. Computational Complexity

Let us assume, for convenience and without loss of generality, that $C_l = C$, $\forall l \in \mathcal{L}$. Then, by using the pseudo-polynomial Dynamic Programming (DP) algorithm to solve the L independent 0-1 KP instances (e.g., see [12]), the time complexity is $\mathcal{O}(LNC)$. By using the 2-approximation algorithm in [13] to solve the GAP by decomposing it into a series of 0-1 KP problems, in turn, the time complexity is $\mathcal{O}(LNC + LN)$. Therefore, the total time complexity of the heuristic algorithm is $\mathcal{O}(I_{it} LNC)$, where I_{it} is the number of iterations. As indicated by the numerical simulations, in practice we commonly have $I_{it} < 10$. In comparison, the complexity of the sorting algorithm alone used in the decomposition method [7] is $\mathcal{O}\left(\hat{S}\hat{F} \log\left(\hat{S}\hat{F}\right)\right)$, where $\hat{S}$ denotes the largest number of regions covered by any SBS and $\hat{F}$ represents the cache capacity (in files) of any SBS.

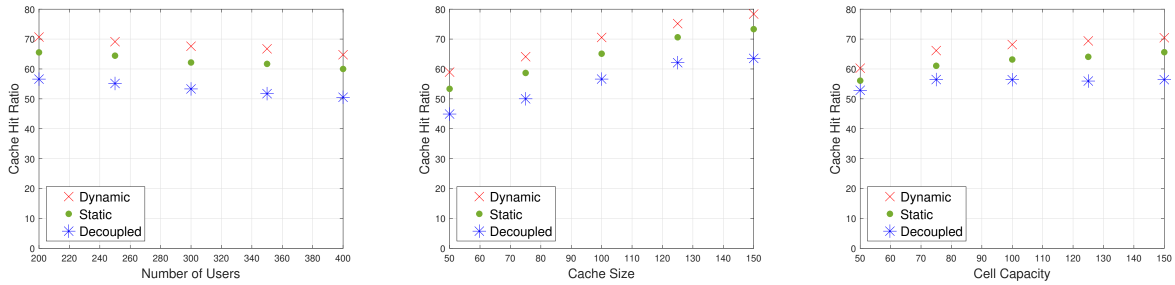
V. Numerical Simulations

In this section, we compare the performance of the proposed dynamic JCA scheme to that of its static counterpart [6], in terms of the achieved average cache hit rate, via numerical simulations. We also consider a heuristic decoupled caching/user association design, where the users are sequentially parsed and each user is associated with the cell among the candidates that incurs the minimum association cost, provided that the cell capacity constraint is met, until all users have been associated with a cell. Subsequently, the caching problem is solved as in the static JCA strategy.

In the simulations, we assume a content catalog of $N = 1000$ files, $L = 20$ cells, and $V = 1$. With respect to the content demand distributions of individual users, we partition the end users according to their physical location, meaning that users within range of the same SBS belong to the same subset. When two or more users are located in the overlapping region of two cells, we use a minimum distance rule or random selection, in the case of equidistant SBSs, as a tie-breaking.

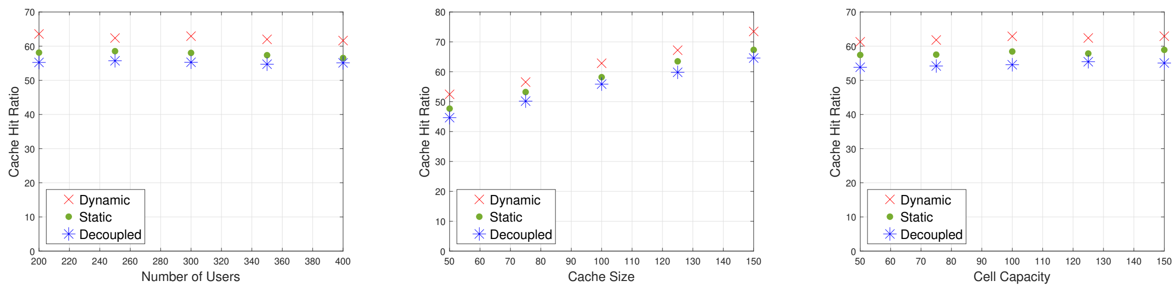
In Fig. 2(a), the storage capacity of each cache is $C = 100$ files, the capacity of each cell is $B_{\max} = 150$, and we vary the number of users as $U = 200 : 50 : 400$. In Fig. 2(b), we set $B_{\max} = 150$ and $U = 200$ and we vary the cache storage capacity as $C = 50 : 25 : 150$. Finally, in Fig. 2(c), we set $C = 100$ and $U = 200$ and we vary the cell capacity as $B_{\max} = 50 : 25 : 150$. We note that the proposed dynamic JCA strategy based on Lyapunov optimization outperforms both the static JCA approach, which focuses on the optimization of the instantaneous performance instead of the average one, and the decoupled caching/user association strategy, which does not adapt the user association decisions to the cached contents. We also notice that the performance of all schemes improves with the cache size and the cell capacity, as expected. We should mention, though, that the decoupled approach presents smaller improvement than the other strategies with the increase of the cell capacity. This is because under spatial locality regarding the content demand distributions, the content preferences of the users that are associated with a cell are more important than their number.

This becomes apparent in Fig. 3, where we repeat the above experiments assuming this time random user demands instead of spatially-clustered ones (i.e., we consider a scenario without spatial locality). Here, the number of users in a cell is more significant than in the previous test cases and the performance gap between the heuristic and decoupled schemes is much



(a) (b) (c)

Fig. 2. Average cache hit ratio vs. (a) number of users, (b) cache storage capacity, and (c) cell capacity, under spatially-clustered user demands.



(a) (b) (c)

Fig. 3. Average cache hit ratio vs. (a) number of users, (b) cache storage capacity, and (c) cell capacity, under random user demands.

smaller, although the same stands for the cache hit ratio of all methods as well, since spatial locality is not exploited.

VI. Conclusions

In this work, we proposed a dynamic, Lyapunov optimization-based JCA policy design and evaluated via numerical simulations its performance against benchmarks and prior state-of-the-art schemes. Simulation results revealed that the proposed low-complexity heuristic design is more efficient in terms of the achieved average cache hit ratio than its static counterpart or a decoupled caching/user association strategy. They also shed light on the effect of various parameters on the performance. In the future, we plan to extend this design framework by taking into account computation offloading (processing) or/and energy constraints as well.

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