Power Control Using Game Theory in a Shared Open Spectrum

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***Abstract*—The main problem of the today's IEEE 802.11 WLANs is the small number of available channels. In this paper we focus on a novel way of maximization of the network throughput and the provision of fairness which are key challenges in IEEE WLANs, using *Game Theory*. We examine two types of power control games, namely the *non-cooperative* and the *cooperative power control game*. In the case of *non-cooperative power control game* we find the *Nash equilibrium* in a distributed way. In the case of *cooperative power control game* we assume that there exists a central entity called *coordinator* which announces the calculated *Nash bargaining solution* to the access points. Finally, we present the results of simulations implemented in MATLAB through a series of plots and tables.**

I. Introduction

To maximize network throughput while providing fairness is one of the key challenges in IEEE 802.11 WLANs. The main problem of today's IEEE 802.11 WLANs is the small number of available channels. Specifically, an IEEE 802.11 WLAN is comprised of 14 channels. Two channels are not overlapped when they are separated by 4 channels. Thus, considering the case of deployment of three access points in a given area, we conclude that the only assignment which satisfies the requirements for non overlapped channels is the combinations of channels 1, 6, 11 according to [1]. In this topology, if we add one more access point, we will have the problem of the overlapped channels. Obviously, interference management is a critical research area which should be bloomed in order to enhance IEEE 802.11 WLANs' performance. Research works such as [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13] apply power control in order to give a solution to this arisen problem.

In this paper we focus on how to control the transmission power of access points' pilot signals using *game theory*. First, we consider a *non-cooperative power control game (NPG)* between competitive operators. In this case we compute the power transmission level of each access point as the *Nash equilibrium (NE)* of NPG. Second, we assume that operators are cooperative namely we examine the case of a *cooperative power control game (CPG)*. In this type of game, we assume the existence of a central authority called *coordinator*. In such a game there exists a *Nash bargaining solution (NBS)*. Moreover, we prove the uniqueness and the feasibility of NBS. Furthermore, we apply the well known bisection method in order to derive NBS. Finally, we present a punishment strategy enforced by the coordinator in order to punish selfish access points.

The remainder of the paper is organized as follows. In Section II we discuss related work. In Section III we describe the proposed methodologies while in Section IV we present the simulation results. We present our conclusions and plans for future work in Section V.

II. Related Work

Nowadays, a lot of researchers have argued in favor of a more flexible and more efficient management of the wireless spectrum, leading to the possible coexistence of various network operators in a shared spectrum area.

In [14], authors suppose that mobile nodes can freely roam[1] among various operators. They model the behavior of different network operators in a game theoretic setting. According to this methodology each operator decides the power of the pilot signal of its base stations. First, they compute possible *NE* in an theoretical setting when all base stations are located on the vertices of a two-dimensional lattice. Afterwards, they show that in a more general case, computing *NE* is an NP-complete problem. In addition, they prove that a socially optimal *NE* exists and it can be applied and enforced by using punishment tactics.

In [9] authors show that a cross-layer approach is required when is needed to perform starvation free power control in IEEE 802.11 WLANs. Specifically, they say that transmitting power levels and carrier sensing parameters of MAC layer should be jointly tuned. In addition, they present a framework which identifies optimum settings for the carrier sensing parameters aiming to maximize the network throughput for elastic traffic. In fact, they apply a distributed power control algorithm which uses a Gibbs sampler.

In [15] authors highlight that interference in coexisting wireless local area networks can be viewed as a layered space-time (LST) structure, in which the number of access points is equal to the number of transmitting antennas. Thus, interference that is caused by access points of different vendors is equivalent to interference between transmitting antennas in LST architecture. This analogy can be further extended to IEEE 802.11 WLANs receiver strategies, so that receiver

[1] the decision of connection to a base station is taken considering the strength of the pilot signal of each base station. Every mobile node attaches to the station with the strongest pilot signal.

structures derived from LST architectures can be directly applied to mitigate interference between vendors. To improve bit error rate further, a cross-layer design in both PHY and MAC layers is proposed. It is shown that the proposed receivers demonstrate superior performance to standard receivers for IEEE 802.11 WLANs.

Whilst Yates [16] treats distributed power control as a general fixed point problem, Goodman [3], [4], [7], [13] considers distributed power control as a distributed interactive objective maximization problem. In fact, he treats this problem as a game.

III. Proposed Methodology

We suppose that two network operators have employed their access points in a given area alike to the scenario described in [14]. Their access points operate within the same unlicensed frequency band and they can adjust the power level of their pilot signals in order to increase their *utility functions*[2]. Thus, a two player game is emerging. Obviously, a problem of co-channel interference is caused when clients associated with these access points are within the overlapped area of transmissions. In figures 1, 2 we depict the two different kind of networks in NPG and CPG, respectively. Specifically, our proposed methodology in the case of NPG implements the following steps:

- Definition of *utility function* for each access point.
- Definition of non cooperative power control game.
- Derivation of Nash equilibrium.

For CPG we follow the steps which are listed below using the same *utility function* we defined in NPG:

- Definition of cooperative power control game.
- Implementation of an algorithm to determine Nash bargaining solution.
- Derivation of Nash bargaining solution in cooperative power control game.
- Implementation of a method to enforce Nash bargaining solution.
- Implementation of an algorithm to determine Nash bargaining solution.

Each access point runs the CSMA/CA algortihm. Due to the fact that the access points meet the hidden node problem in both NPG and CPG each of them cannot sense the transmission of the competitor as we can see in figures 1, 2. As a result two transmissions to the associated mobile clients are taking place concurrently causing a *collision*.

Assuming no RTS/CTS mechanism each access point can never be informed about the concurrent transmission of the other. This situation results to the degradation of the *Signal-to-Interference Ratio* (*SIR*) because it actually increases the interference seen by each client. As a result, every access point has to adjust its transmission power level in a way that maximizes: (i) its *mean utility* and (ii) the *mean SIR*.

[2]we will define below utility functions for each access point.

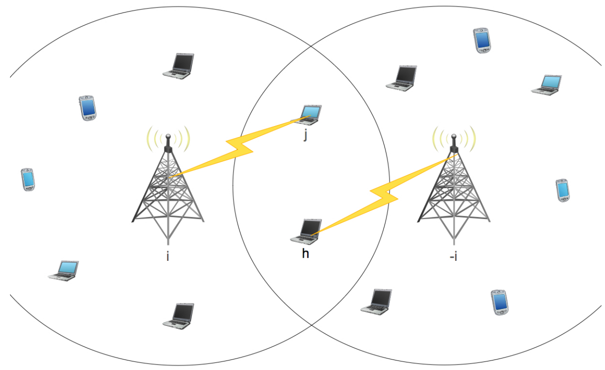

Fig. 1. An example of the wireless environment in the case of *NPG* .

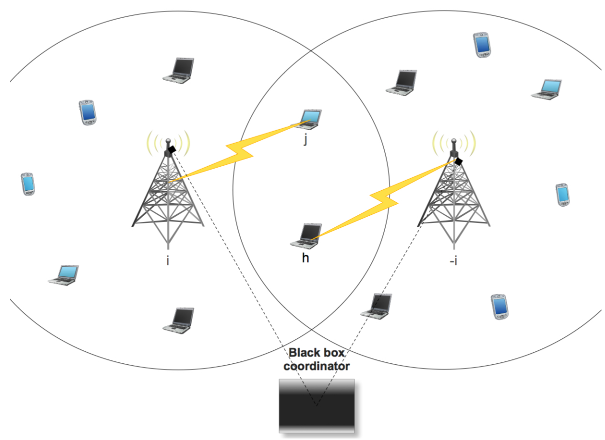

Fig. 2. An example of the wireless environment in the case of *CPG* .

A. Non-cooperative power control game

Let $G = [N, \{P_k\}, \{u_k(\cdot)\}]$ denote the two player NPG where $N = \{A_i, A_{-i}\}$ is the index set of access points in a given area, P_k is the strategy set and $u_k(\cdot)$ is the *utility function* of access point k. Each access point selects a power level p_k such that $p_k \in P_k$. Let the power vector $\mathbf{p} = (p_{A_i}, p_{A_{-i}}) \in P$ denote the outcome of NPG in terms of selected power levels where P is the set of all power vectors. The strategy space satisfies $P = P_{A_i} \times P_{A_{-i}}$.

1) Utility Function: Let $i, -i$ be the set of access points who share the downlink bandwidth of the IEEE 802.11b cell. We assume that access point i controls its transmitted power p_i chosen from a set of strategies $P_i = [0, +\infty)$. We assume that access points' preferences are expressed through the utility function u_i which quantifies the level of satisfaction for each access point from using the wireless resources. According to [10] we express the utility function as the number of bits that are successfully received per unit of consumed energy as:

$$u_i(p_i, \gamma_j) = \frac{R}{p_i}(1 - 2BER(\gamma_j))^L \quad bits/Joule \tag{1}$$

, with the following terminology to be considered:

- R: rate of access point's transmitted information in bits/second[3].
- p_i: access point's i transmitted power.
- γ_j: *SIR* seen by client j which receives data from i.
- L: the number of bits per packet.
- BER: the bit error rate, which is the ratio between the number of incorrect bits transmitted to the total number of bits.

The level of utility that each access point gets depends on its own power level and on the strategy chosen by the competitive access point.

Supposing that the modulation scheme is DBPSK[4] we will have that:

$$BER = \frac{1}{2} e^{-\gamma_j} \tag{2}$$

In addition, according to [10] the value of *SIR* of client j associated with access point i is equal to:

$$SIR_j = \gamma_j = \frac{W}{R} \frac{g_{ij} p_i}{g_{-ij} p_{-i}} \tag{3}$$

, with the following terminology to be considered:

- W: the bandwidth in Hertz.
- g_{ij}: link gain between access point i and associated client j.
- g_{-ij}: link gain between the competitive access point $-i$ and client j.
- p_{-i}: transmitted power of the competitive access point $-i$.

Finally, assuming a client j associated with access point i and combining (1), (2), we have that the utility of access point i obtained by serving client j, is equal to:

$$u_i(p_i, \gamma_j) = \frac{R}{p_i} (1 - e^{-\gamma_j})^L \quad bits/Joule \tag{4}$$

In the two player NPG, each access point maximizes its own utility in a distributed fashion. We assume two clients j, h which are associated with access points $i, -i$, respectively. In our implementation we will follow the following steps:

- We will reduce the power level of each access point from the initial value P_{max}[5] until the achievement of NE.
- Every time the power level is reduced we check if the current power strategies of access points comprise a NE. If this happens, we will stop the power levels' reduction.

Formally, NPG is expressed as:

$$(NPG)\ max_{p_i \in P_i} u_i(p_i, p_{-i}),\ \forall\ i \in I \tag{5}$$

, where u_i is the utility of any access point i, given in (4), P_i is the strategy space of i and p_{-i} is the power level of the competitive access point. It is necessary to characterize a set of powers when an access point is satisfied with the utility it receives given the power selection of the other access point. Such an operating point is called an *equilibrium*[6]. At NE, given the power level of the competitive access point, i can not improve its utility level by making individual changes in its power level.

Definition: A power vector $p = (p_{A_i}, p_{A_{-i}})$ is NE of $G = [N, \{P_k\}, \{u_k(\cdot)\}]$ if, for every access point $k \in I = i, -i$, $u_i(p_i, p_{-i}) \geq u_i(p'_i, p_{-i}) \forall\ p'_i \in P_i$.

Specifically, the power level chosen by a rational self-optimizing access point constitutes a best response to the choice of the competitive access point.

2) Existence and Uniqueness of NE: In the problem we examine there is one and only one NE, as shown in the following theorem, according to [7].

Theorem 1: There exists a unique NE in NPG..

Proof: The proof follows from Debreu's Theorem [17], due to the fact that utility given in (4) is defined over the convex set Γ and is quasi concave in p_i ([7], [18]). ■

3) Derivation of NE: The first derivative of the utility with respect to p_i is equal to:

$$\frac{du_i}{dp_i} = -\frac{R}{p_i^2}(1 - e^{-\gamma_j})^L + \frac{R}{p_i} L (1 - e^{-\gamma_j})^{L-1} e^{-\gamma_j} \frac{g_{ij}}{g_{-ij} p_{-i}} \tag{6}$$

Aim of each access point is to maximize its utility function. At the point of maximization the first derivative of the utility with respect to p_i should be zero. Thus:

$$\frac{du_i}{dp_i} = 0 \tag{7}$$

We can easily see that for $p_i = 0$ the utility is maximized, but this power level cannot be a maximizer. Thus, according to (6), (7), the solution of γ^* is derived from the equation:

$$e^{\gamma^*} = 1 + L\gamma^* \tag{8}$$

The (8) can be solved numerically, and according to (4) we have that:

$$u_i^* = \frac{R}{p_i^*}(1 - e^{-\gamma^*})^L \tag{9}$$

In addition, we observe that at NE both clients j, h, enjoy equal non-zero SIR γ^*. Also, we suppose that:

$$v(q_{ij}) = (1/g_{ij}) u_i,\ \forall\ i \in I \tag{10}$$

, where q_{ij} is the received power by client j, namely:

$$q_{ij} = g_{ij} p_i \tag{11}$$

Moreover, from (9) and (10) we will have that:

$$u_i^* = g_{ij} v(q^*) \tag{12}$$

The derived equilibrium is fair, as both clients achieve the same SIR and throughput. According to [7], this NE is not *Pareto optimal*.

[3]we assume that R is equal for all access points.

[4]modulation scheme used by IEEE 802.11 WLANs for transmission at 1Mbps.

[5]in the section of simulation results we will discuss this value.

[6]this is the NE.

B. Cooperative power control game

Except from NPG we will examine the case of CPG. We provide a fair and efficient solution to the power control game similar with the one that is proposed in [19] for CDMA wireless data networks. We will show that if there exists a *coordinator* then it is possible for access points to achieve a Pareto optimal solution.

In this scenario there exists a central authority, as it is depicted in figure 2, which plays the role of *coordinator* between access points. To be specific, *coordinator* enforces cooperation resulting in derivation of a more efficient point than NE. This point is called NBS [20]. NBS is a *Pareto optimal* point and as a result it maximizes the social welfare.

An other concept, generally different from NE, is *Pareto efficiency* or *Pareto optimality*. A strategy profile is Pareto optimal or Pareto efficient if there is no way to improve the performance of one player without harming the performance of another one. Formally a strategy profile σ^* is said to be Pareto optimal if only if there exists no other strategy profile σ', such that:

$$\text{if for some } k,\ u_k(\sigma') > u_k(\sigma^*) \Rightarrow u_i(\sigma') > u_i(\sigma^*) \quad (13)$$
$$,\ \forall\ i \in\ set\ of\ other\ players$$

Obviously, a strategy profile that constitutes NE may not be Pareto efficient. Pareto efficient is a cooperative dominating solution. In cooperative games, users are able to make enforceable outcomes through centralized authorities. Thus, for cooperative games, the interests lie in how good the game outcome can be, namely how to define and choose the optimality criteria in cooperative scenarios.

Further, it is worth mentioning that NBS plays an important role in cooperative games. NBS which is a unique Pareto optimal solution to the game modeling bargaining interactions based on six intuitive axioms. Nash gave four axioms that any NBS should satisfy which are: (i) invariant to affine transformations, (ii) Pareto optimality, (iii) independence from irrelevant alternatives and (iv) symmetry. To be specific, in a transaction when the seller and the buyer value a product differently, a surplus is created. A bargaining solution is then a way in which buyers and sellers agree to divide the surplus. A definition of *NBS* is the following:

Definition: A mapping $F : G \rightarrow \Re^N$ is said to be NBS, where G denotes the set of achievable utilities with respect to the status quo utility u^0, if:

- *a.1:* $F(U, u^0) \in U_0$, where U_0 is the set of achievable utilities which are superior to u^0.
- *a.2:* $F(U, u^0)$ is Pareto optimal.
- *a.3:* F satisfies the linearity axiom, thus if ϕ: $\Re^N \rightarrow \Re^N$, $\phi(u) = u'$ with $u'_i = a_i u_i + b_i,\ a_i > 0,\ i = 1, ..., N$ then $F(\phi(u), \phi(u^0)) = \phi(F(u, u^0))$.
- *a.4:* F satisfies the irrelevant alternatives axiom, thus if $V \subset U$, $(v, U^0) \in G$ and $F(U, u^0) \in V$, then $F(U, u^0) = F(V, u^0)$.
- *a.5:* F satisfies the symmetry axiom, namely if U is symmetric with respect to a subject $J \subseteq 1, ..., N$ of indices. More specific if $u \in U$ and $i, j \in J$, then if $u_i^0 = u_j^0$ then $F(U, u^0)_i = F(U, u^0)_j$.

The solution of Nash, which satisfies all the above axioms, is achieved at the point where the product of utility functions of users, with respect to the status quo utilities of the game is maximized. At *NBS*, the product of utilities of involved users is maximized, subject to the constraint that the SIR of every user must be within the respective bounds and that the utility of each user must be superior to his status quo utility, thus:

$$max_p\{\prod_{j=1}^{N}(u_j(p) - u_j^0)\},\ p \in X,\ X = \{r \in \Gamma\ \ u(r) > u^0\} \quad (14)$$

1) NBS in CPG: Consider a linear function $\phi : \Re^n \rightarrow \Re^n$, where $\phi(u) = v$ and $v(q_i) = (1/g_i)u_i,\ \forall\ i \in I$. The transformed function v_i can be expressed as:

$$v_i(q_{ij}) = \frac{R}{q_{ij}}(1 - e^{-\gamma_j})^L \quad (15)$$

, where $q_{ij} = g_{ij}p_i$.

From (15) we can see that the utility of two access points are symmetric. In addition, $v_{A_1}^0 = v_{A_2}^0 = v^*$. Thus, according to *a.5* we have that at NBS:

$$v_{A_1}(q) = v_{A_2}(q) \quad (16)$$

Formally, CPG is expressed as:

$$(CPG)max_q\{(v_{A_1}(q) - v_{A_1}^0)(v_{A_2}(q) - v_{A_2}^0)\}, \quad (17)$$
$$q \in \{r \in S : v(r) > v^0,\ v_{A_1}^0 = v_{A_2}^0 = v^*\}$$

Due to (16), the optimization problem becomes:

$$(CPG)\ max_q v(q) \Rightarrow max_q\{\frac{R}{q}(1-e^{-\gamma})^L\},\ q \in \{r \in S_i : (r) > v^*\} \quad (18)$$

2) Uniqueness of NBS: Lemma: There is a unique positive power q_{nbs} that maximizes function $v(q)$.

Proof: At the point where function $v(q)$ is maximized the first-order optimality condition must hold:

$$\frac{dv(q)}{dq} = 0 \Rightarrow \frac{d(\frac{R}{q}(1 - e^{-\gamma})^L)}{dq} = 0 \quad (19)$$

$$\frac{dv(q)}{dq} = \frac{R}{q}(1 - e^{-\gamma})^{L-1}\{Le^{-\gamma}\frac{\sigma^2}{(q + \sigma^2)^2} - \frac{1}{q}(1 - e^{-\gamma})\} \quad (20)$$

For $q = 0$, from (20) the first order optimality is satisfied. Furthermore, $v(0) = 0$ whilst v^* violates the first axiom of NBS. Therefore, we can derive the following condition for the first derivative:

$$\frac{dv(q)}{dq} = 0 \Rightarrow e^{-\gamma}L\frac{\sigma^2 q}{(\sigma^2 + q)^2} - (1 - e^{-\gamma}) = 0 \quad (21)$$

$$\Rightarrow L\frac{\sigma^2 q}{(\sigma^2 + q)^2} = e^{\gamma} - 1 \quad (22)$$

We define $r(q) = L\frac{\sigma^2 q}{(\sigma^2+q)^2} - e^{\gamma} + 1$. We will prove that the function $r(q)$ has a unique root in the interval $(0, +\infty)$.

Towards this proof we have to find that the interval within the left-hand side of (21), let it be $k(q)$, is increasing, checking the monotonicity of this clause. We have that:

$$k(q) = 0 \Rightarrow \frac{d\{\frac{L\sigma^2 q}{(\sigma^2+q)^2}\}}{dq} = 0 \Rightarrow q = \sigma^2 \ or \ q = -\sigma^2 \quad (23)$$

From the above solutions we ignore $q = -\sigma^2$ because we cannot have a negative received power. The only solution therefore of (23) is $q = \sigma^2$ and $k(q) = L/4$. Thus, $(\sigma^2, \frac{L}{4})$ is a minimum or maximum point of $k(q)$ because at this point the first derivative of the clause is equal to zero. Now, we have to check if the $(\sigma^2, \frac{L}{4})$ is minimum or maximum. Observing that $k(0) = 0$ and $k(\sigma^2) = \frac{L}{4}$ we derive that k is increasing in the interval $[0, \sigma^2]$ and decreasing in the interval $[\sigma^2, +\infty)$. Accordingly, the point $(\sigma^2, \frac{L}{4})$ is a maximum of k function. Moreover, the right-hand side of (21), is increasing in $[0, +\infty)$. For the first derivative of $r(q)$ we have that:

$$\frac{dr(q)}{dq} = \frac{L\sigma^2(\sigma^2 - q)}{(\sigma^2+q)^3} - \frac{e^\gamma \sigma^2}{(\sigma^2+q)^2} \quad (24)$$

At $q = 0$:

$$\frac{dr(q)}{dq}\, |_{q=0} = \frac{L-1}{\sigma^2} \quad (25)$$

For the length of data packet L, we know that $L > 1$ supposing header information and data information. So, $\frac{dr(q)}{dq}|_{q=0} > 0$. This implies that r is increasing at $q = 0$. Moreover, $r(0) = 0$, hence, there is a sufficiently small positive scalar δ for which $r(\delta) > 0$ which means that at δ, k dominates the right-hand side of (21). Moreover:

$$r(q) < 0 \Rightarrow \frac{L\sigma^2 q}{(\sigma^2+q)^2} < 1 - e^\gamma \quad (26)$$

From the well-known inequality $e^\gamma - 1 > \gamma, \ \forall \ \gamma > 0 \Leftrightarrow 1 - e^\gamma < -\gamma$, we have that:

$$\frac{L\sigma^2 q}{(\sigma^2+q)^2)} < -\gamma \Rightarrow q > \sigma^2(L+1), \ r(q) < 0 \quad (27)$$

Thereupon, for values $q \in [\sigma^2(L+1), +\infty)$ the right-hand side of (21) dominates the left-hand side. We summarize the above facts in the following:

- The left-hand side dominates the right-hand side in (21) for $0 < q < \sigma^2(L+1)$.
- The right-hand side dominates the left-hand side in (21) for $q > \sigma^2(L+1)$.

So, there is one point of intersection of left-hand side and right-hand side quantities for $q > 0$. As a result, $r(q)$ has a single positive root for $q > 0$. We assume that q_{nbs} is root of $r(q)$. We will have that $q_{nbs} \in [\delta, \sigma^2(L+1)]$. Due to the fact that $r(q)$ is the first derivative of $v(q)$, q_{nbs} is a point where the first-order optimality condition of CPG problem is satisfied. As well as, we have that:

- for a small scalar δ, $r(\delta) > 0$ implies that $v(q)$ is increasing.
- for $q > \sigma^2(L+1)$, $r(q) < 0$ implies that $v(q)$ is decreasing.

So, q_{nbs} is a maximum of v_q in the interval $(0, +\infty)$. Combining this with (25) and the fact that q_{nbs} is the single root of $r(q)$ we have proved the statement of lemma. Thus, "*there is a unique positive power q_{nbs} that maximizes function $v(q)$*". ■

We believe that q_{nbs} is Pareto efficient point due to the fact whereas the clients receive the same power q_{nbs}.

3) Feasibility of NBS: Lemma: The positive received power q_{nbs} that maximizes function $v(q)$ is a feasible solution for *CPG*.

Proof: Due to the fact that $v_A^0 = v_B^0 = v^*$, in order to prove the lemma we need to show that $v(q_{nbs}) > v(q^*)$. The root of $r(q)$ is a global maximum of $v(q)$. Thus, $v(q^*) \leq v(q_{nbs})$. So, we have to prove that the $v(q^*) = v(q_{nbs})$ does not hold. In other words we have to prove that the q^* is not a maximizer of $v(q)$. For this purpose we will use the method of contradiction. Assuming that q^* maximizes $v(q)$ we know that q^* is root of $v(q)$. Moreover:

$$r(q^*) = \frac{-Lq^{*2}}{(q^* + \sigma^2)^2} = -L\gamma^{*2} \quad (28)$$

We know that $q^* > 0$. So, from (28), we have that $r(q^*) < 0$. It is obvious that q^* cannot be root of $r(q)$ and cannot be maximizer of $v(q)$. As a result $v(q^*) \leq v(q_{nbs})$ ■

4) Algorithm to Determine NBS: In order to determine the discussed *NBS*, *coordinator* runs an iterative algorithm. After the implementation of the algorithm, the *coordinator* announces to access points the value of the received power q_{nbs} when NBS is achieved. Each of them has to adjust its transmission power level p_{nbs} according to the equation $p_{nbs} = \frac{q_{nbs}}{g_{ij}}$, namely they have to adjust their power levels of their pilot signal in order to achieve the announced value q_{nbs}.

As we proved in the previous sections NBS in CPG coincides with $q_{nbs} \in [\delta, \sigma^2(L+1)]$ and function $r(q)$ is continuous in this interval. As we have located the interval where root is belonged to, we have to apply a root-finding algorithm in this interval for the determination of NBS. According to [21] we can use the *bisection method* [22] in order to find the NBS. Actually, bisection method iteratively divides in half an interval and then it selects the subinterval in which a root exists. Therefore, we set the limits q_{inf}, q_{sup} towards the derivation of NBS. We implement the following algorithm:

5) Enforcement of NBS: NBS is a point where the utilities of two cooperative access points are maximized, and it is announced by the *coordinator* to the access points. This point is the threshold value of received power by any client in the overlapped area. In addition, NBS is a point where social welfare is maximized although may not be adopted by one or more players. For example, a non compliant player may desire to change its transmission power violating the maximization of the social welfare in order to achieve larger utility. This violation, in the most of cases, causes significant degradation to the performance of the competitors. Thus, it is essential to propose a mechanism to enforce NBS and conform the selfish access points.

Algorithm 1 Algorithm for the derivation of *NBS*

set $q_{inf} = 0,\ q_{sup} = \sigma^2(L+1)$
while $\mid q_{sup} - q_{inf} \mid > 2 * \epsilon$ **do**
  $\diamond$ *termination criterion* $2 * \epsilon$ *is a positive small scalar*
  set $q_{midpoint} = \frac{q_{inf}+q_{sup}}{2}$
  if $r(q_{midpoint}) = 0$ **then**
    set $q_{nbs} = q_{midpoint}$
    return q_{nbs}
    exit running
  else
    if $r(q_{inf})r(q_{midpoint}) > 0$ **then**
      $q_{inf} = q_{midpoint}$
    else
      $q_{sup} = q_{midpoint}$
    end if
  end if
end while
set $q_{nbs} = q_{midpoint}$
return q_{nbs}
exit running

To be more specific, as we have discussed, at NBS, two clients associated with competitive access points receive the same power. As a result, a selfish access point is an access point whose associated client receives a more powerful signal than the one indicated by the coordinator. On the other hand, this deviation is not intentional. For example suppose an access point which underestimate the path link gain and it transmits with higher than the threshold transmission power.

Assume that a selfish access point i transmits with power p_i'. Let q_i' be the received power at the associated client. Supposing that q_i' is χ Watts larger than q_{nbs}, then client's *SIR* will be:

$$\gamma_i' = \frac{W}{R}\frac{q_i'}{q_{nbs}+\sigma^2} = \frac{W}{R}\frac{\chi + q_{nbs}}{q_{nbs}+\sigma^2} \Rightarrow \Delta\gamma_i = \frac{W}{R}\frac{\chi}{q_{nbs}+\sigma^2} \tag{29}$$

The bit error rate is equal to $\frac{1}{2}e^{-\gamma_i}$, hence according to (29):

$$\Delta BER_i = BER_i e^{-\Delta\gamma_i} \tag{30}$$

As we have discussed one role of *coordinator* is to derive and announce the NBS to the access points. Another role is to punish the selfish players. A mechanism for the latter purpose is proposed in [10]. In order to punish an access point for improving his *BER* to the harm of other users, the *coordinator* should increase the errant user's *BER* by randomly inverting bits in the client's packet with a certain probability. Supposing that BER_{nbs} is the bit error rate at NBS, the aim of the punishment is to give the client a *BER* equal to BER_{nbs}. As a result the utility of the access point is smaller than the utility obtained at NBS because the consumption of energy is larger in the case of a selfish behavior. Although transmitted power increases, *BER* remains the same due to the punishment procedure. The procedure implemented by the coordinator is summarized at the following steps:

- *coordinator* calculates the NBS considering the systems parameters, namely the gain links between the access points and their associated clients.
- *coordinator* announces the NBS to the access points, namely the required level of received power at their associated clients.
- *coordinator* monitors for selfish users and punishes each of them reducing their *BER* to BER_{nbs}.

We highlight that for the purpose of NBS's enforcement the type of games where the *coordinator* can apply the punishment procedure are the *repeated games*. Actually, a repeated game is an extensive form game which consists of some number of repetitions of a stage game. Thus, the set of players they compete each other on multiple occasions. In a single stage game the *coordinator* does not have the chance to apply the punishment in future steps. Thus, in order to achieve cooperation in the power control game we consider it as a repeated game.

IV. Simulation Results

In this section, we present the results of simulations regarding our proposed methodologies through a series of topologies, plots and bars. The assumptions of our scenarios are summarized in the following:

- Simulation of an access point-driven mechanism.
- Assumption of two access points which belong to competitive operators and they:
  - operate in the same frequency, time and location.
  - meet the hidden node problem.
  - use the IEEE 802.11b and IEEE 802.11e protocols.
  - adopt the BPSK modulation scheme.
  - set their transmission power at the maximum value serving all the clients they can at the beginning considering that the maximum permissible power is equal to +30dBm[7].
- The RTS/CTS mechanism is not used in order to avoid increased delivery delays and reduced throughput according to [15].
- Carrying out simulations considering:
  - 10, 20, 30, 50 and 100 clients.
  - clients are distributed uniformly.
  - clients are static namely they do not have mobility for the period of time we apply our methodologies in the wireless network.

More analytically, we assume an IEEE 802.11b standard implementation with BPSK modulation scheme, achieving 1 Mbit data rate. The simplest form of PSK uses two carrier waves, shifted by a half cycle relative to each other. One wave, the reference wave, is used to encode an 0; the half-cycle shifted wave is used to encode an 1. The modulation for 1 Mbit is BPSK. From [23] the *BER* becomes:

$$BER_{BPSK} = \frac{1}{2}e^{-\frac{E_b}{N_0}} \tag{31}$$

, where $\frac{E_b}{N_0}$ indicates the *signal-to-interference ratio (SIR)*.

[7]FCC regulatory standards set upper bounds on the transmitted power for IEEE 802.11 WLANs operating in the US. The maximum theoretical range of an IEEE 802.11b WLAN operating at the maximum EIRP of 30 dBm and for path loss coefficient $N = 3$, is 154 meters.

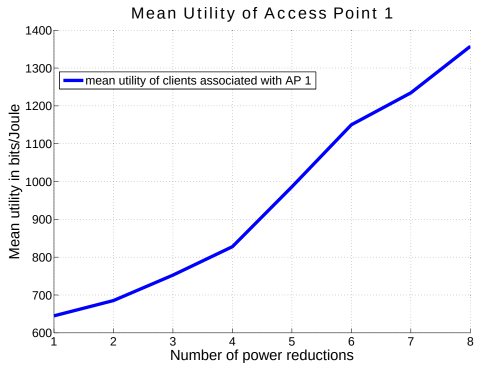

Fig. 3. The *mean utility* of access point 1 as a function of the power reduction steps in NPG.

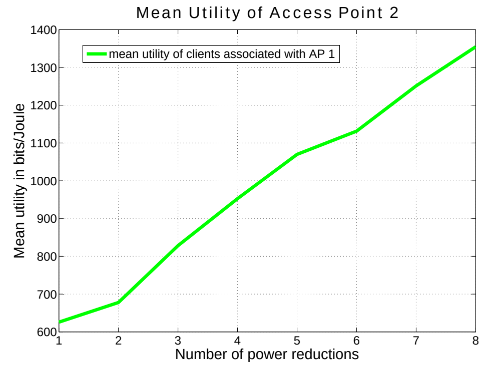

Fig. 4. The *mean utility* of access point 2 as a function of the power reduction steps in NPG.

Reducing the power level gradually (pure strategies):
Let two competitive access points be access point 1 and access point 2. We assume that 100 clients are associated with each access point. The strategy of each access point is described in the following. Specifically, each access point reduces its transmission power level gradually, until the achievement of NE. At the beginning, we assume that the access points act selfishly and greedily transmitting with the maximum power level, namely 30 dBm or 1 Watt. This value is the maximum permissible limit appointed by the FCC as we have mentioned. In every step of our simulation, each access point decreases its power level by 0.05 Watt which is considered as enough small decrement towards the achievement of NE. Thus, the process of power reduction continues until NE is achieved. The derived NE is considering as *pure*, see [24], because each access point chooses to take one action with probability 1. According to theoretical results, at NE the SIR of the nearest to access point 2 client j associated with access point 1 is equal to the SIR of the corresponding client h associated with access point 2, namely $SIR_{client_j} = SIR_{client_h}$, as we have proved. First, in figure 3, we depict the *mean utility* of access point 1 as a function of the power reduction steps. In figure 4, we depict the corresponding utility of access point 2. In figures 5 and 6 we depict the *mean SIR* observed by the clients of the network. Second, in figure 7, we depict the improvement percentage of *mean utility* of the access points at NE. We observe that for different number of clients the improvement percentage function fluctuates in the interval $[11\%, 18\%]$. Third, in figure 8 we depict the improvement of *mean SIR* observed by the clients of the network, at NE. We observe that for different number of clients the improvement percentage function fluctuates in the interval $[3\%, 6.5\%]$.

As we have described in the CPG, an entity called coordinator computes the NBS and it announces it to the competitive access points. In the following we appose a series of diagrams to indicate the effectiveness in terms of utility and mean SIR in the case of CPG for both access points and the associated clients. First, in figures 9 and 10 we depict the *mean utility* of access point 1 and 2, at NBS. In figures 11 and 12 we

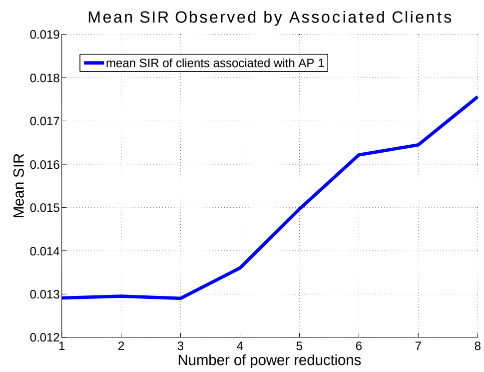

Fig. 5. The *mean SIR* of access point 1 as a function of the power reduction steps in NPG.

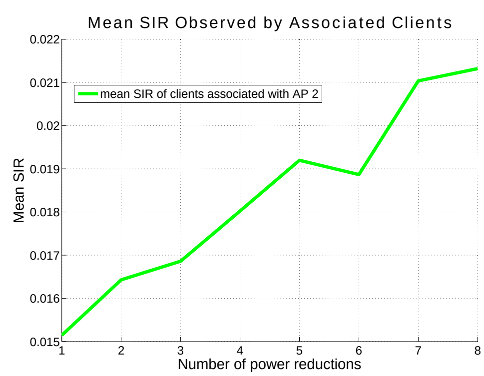

Fig. 6. The *mean SIR* of access point 2 as a function of the power reduction steps in NPG.

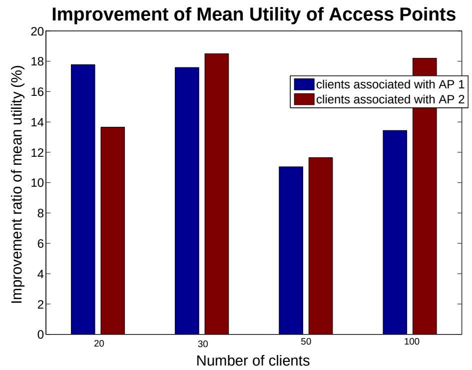

Fig. 7. The improvement of *mean utility* of the access points, at pure NE, as a function of the number of clients, let it be 20, 30, 50 and 100.

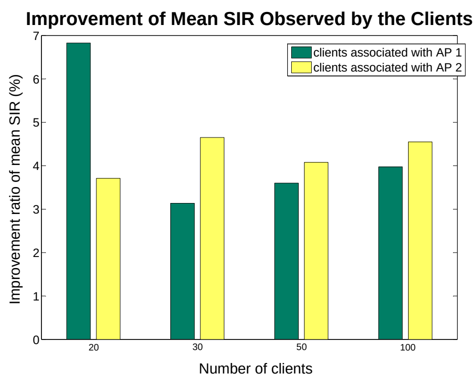

Fig. 8. The improvement of *mean SIR*, at pure NE, observed by the clients associated with access points as a function of the number of clients, let it be 20, 30, 50 and 100.

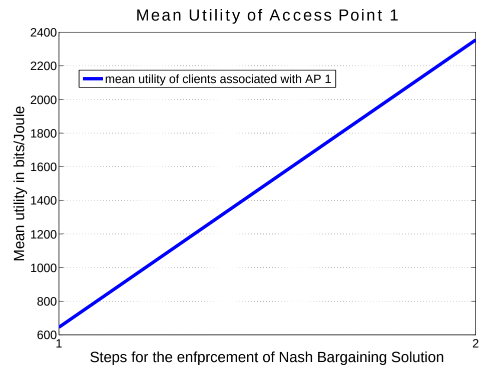

Fig. 9. The *mean utility* of access point 1 as a function of the steps until the enforcement of NBS in CPG.

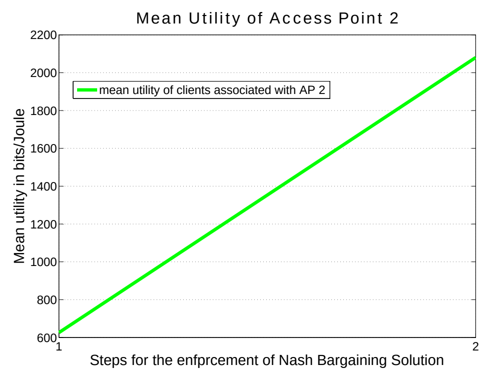

Fig. 10. The *mean utility* of access point 2 as a function of the steps until the enforcement of NBS in CPG.

depict the corresponding plot of *mean SIR* observed by the clients. Second, in figure 13 we depict the improvement of the utilities of access points $1, 2$ which fluctuates within the interval $[13\%, 40\%]$. Moreover, in figure 14 we depict the improvement of the *mean SIR* observed by the clients at NBS. We observe that for different number of clients the improvement percentage function fluctuates in the interval $[2.5\%, 8.5\%]$.

We observe that in CPG the mean utility of the access points resembles a linear function due to the fact that we need only one reduction step assuming that all the entities are not cheaters and they reduce their power to the value announced by the coordinator. Actually, the coordinator computes and announces the proper transmission level power for the achievement of NBS as we have discussed. On the other hand, in NPG, the number of power reduction steps until the achievement of NE is larger than the corresponding in CPG. As a result, the convergence of NBS is quicker than the convergence of NE. In addition, the simulation concludes that the final mean utility in CPG is higher than the mean utility in NPG. The same trends are observed for the mean SIR of the clients associated with both access points. In table I we present the results for the mean utility and the mean SIR in order two compare the efficiency of the different type of games. Due to space limitations we present the results for access point 1 mentioning that the same trends are observed for access point 2. According to the improvement of the mean utility, we observe in figures 7, 13 that in the most cases the improvement is larger in the case of CPG, as we expected. The same trends are observed for the improvement of the mean SIR though the percentage differences are smaller. Moreover, in tables II, III we present

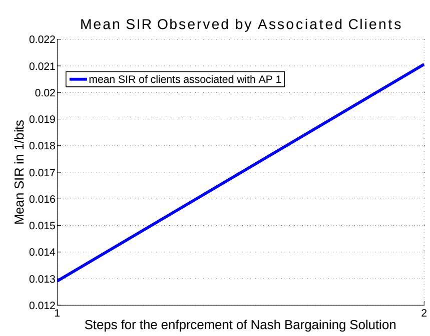

Fig. 11. The *mean SIR* of access point 1 as a function of the steps until the enforcement of NBS in CPG.

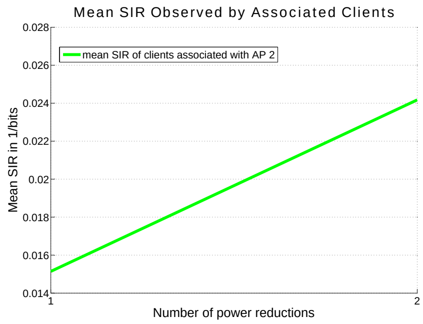

Fig. 12. The *mean SIR* of access point 2 as a function of the steps until the enforcement of NBS in CPG.

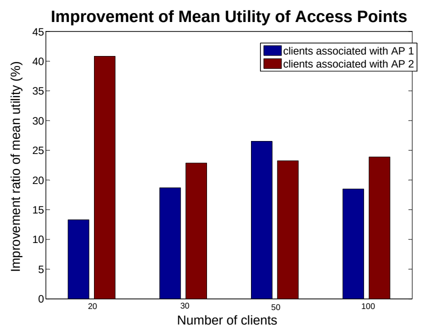

Fig. 13. The improvement of *mean utility* of the access points, at NBS, as a function of the number of clients, let it be 20, 30, 50 and 100.

the final power transmission level regarding the cases of 20, 30, 50 and 100 clients associated with both access points and the type of game played between the competitors.

Last but not least, in figure 16 we depict the total income of one access point as a function of the number of the associated clients. We assume that each client has to pay 20 euros when it is connected to an access point. According to figure 16, in the case of NPG the operators make more profit[8] due to the fact that more clients are connected with them. However,

[8]except for the case of 100 clients.

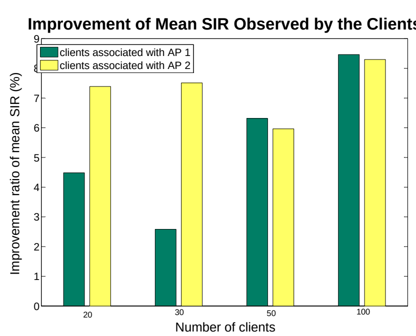

Fig. 14. The improvement of *mean SIR*, at NBS, observed by the clients associated with the access points as a function of the number of clients, let it be 20, 30, 50 and 100.

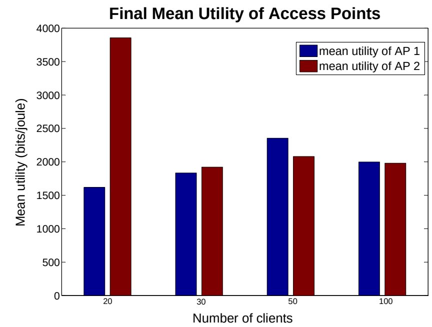

Fig. 15. The final *mean utility* of the access points, at NBS, as a function of the number of clients, let it be 20, 30, 50 and 100.

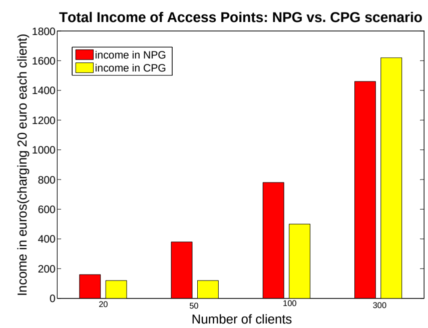

Fig. 16. The total income of the operators in euros assuming that each client is charged 20 euros to be associated with the access point of an operator.

as we proved, clients mean SIR and access point's utility are worst than in the case of CPG. Therefore, the existence of a coordinator is essential to punish any selfish operator which does not want to cooperate targeting to attract more clients and make more profit. In the case of NPG and when the number of clients is equal to 100, operators have to reduce their power in a lower level than in the case of CPG in order to achieve a NE. Thus, their incomes are higher in CPG scenario.

V. Conclusions

In this paper, we investigated the case of an open spectrum shared area using game theory. In particular, we proposed a new way of maximization of the network throughput and the provision of fairness. We calculated the Nash equilibrium and the Nash bargaining solution of a non-cooperative and of a cooperative power control game, respectively. Our future work involves experimenting with more access points in the open spectrum shared area evaluating more QoS metrics.

TABLE I
The final mean utility and mean SIR in NPG and CPG with initial number of associated clients equal to 100 for one of the two access point

Type of game	Final Mean Utility	Final Mean SIR
NPG	$\sim$ 1350 bits/joule	$\sim$ 0.0175
CPG	$\sim$ 2350 bits/joule	$\sim$ 0.021

TABLE II
FINAL TRANSMITTED POWER BY EACH ACCESS POINT IN NPG

Number of clients	**Transmitted Power of access point 1**	**Transmitted Power of access point 2**
20	0.55 Watts	0.55 Watts
30	0.5 Watts	0.5 Watts
50	0.6 Watts	0.6 Watts
100	0.5 Watts	0.5 Watts

TABLE III
FINAL TRANSMITTED POWER BY EACH ACCESS POINT IN CPG

Number of clients	**Transmitted Power of access point 1**	**Transmitted Power of access point 2**
20	0.5663 Watts	0.2583 Watts
30	0.5268 Watts	0.4892 Watts
50	0.4023 Watts	0.4445 Watts
100	0.4685 Watts	0.4788 Watts

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