

A Stochastic Optimization Framework for Personalized Location-Based Mobile Advertising

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Abstract—Mobile location-based advertising has seen a lot of progress recently. We study the problem of optimal user targeting and monetization through advertising, from the point of view of the owner of a venue such as a shopping mall, an urban shopping district or an airport. The fundamental distinguishing characteristic of advertising in this setup is that the probability that the user will respond to an ad depends on timeliness of ad projection, hence it is important to target a mobile user with an appropriate ad or offer at the right time.

A set of mobile users roam around the venue. Each user is profiled in terms of preferences based on prior visits. The system knows estimated instantaneous locations of users in the venue, e.g. through WiFi access point connectivity. A machine-learning model is used to derive a per-user time-varying probability of response to an ad, which depends on the relevance of the ad (store) to the user profile and on the time-varying physical proximity of the user to the store. Each store has a set of available ads, and each time the user responds to a projected ad, an amount is paid by the store to the venue owner. We use a stochastic-optimization framework based on Lyapunov optimization to address the problem of advertisement selection and allocation for maximizing the long-term average revenue of the venue owner subject to: (i) a constraint on maximum average ad projection rate per user for preventing user saturation, and (ii) a long-term average budget constraint for each store. We derive an algorithm that operates on a time slot basis by solving a simple assignment problem with instantaneous user locations while being agnostic to user mobility statistics. We test our algorithm with a real dataset of check-ins from Foursquare, complemented with data from user questionnaires. Our approach results in substantial improvement in revenue compared to approaches that are location- or relevance-agnostic.

I. INTRODUCTION

In recent years mobile advertising has evolved into a prime market segment in the advertising ecosystem, and its spend is forecast to exceed \$35 billion in 2017. Along with the proliferation of mobile apps and social media, novel forms of mobile advertising have emerged, such as location-based advertising [1], advertising within mobile apps [2], and native advertisements in post feeds in social media [3], [4].

In location-based mobile advertising, the system first detects the presence of a mobile user close to the location of an advertiser through some localization technology which may involve GPS or WiFi signal strength, or it may use the WiFi access point that the user is connected to as a means for localization.

The latter technology seems less restricted and less privacy-intrusive for users and is widely applied. For instance, Skyhook Wireless uses WiFi hotspots to determine device locations so as to offer location-based services, apps and advertising with accuracy of 20 meters. The situation when a user is detected to enter a region around the advertiser premises is referred to as *geofencing*. When geofencing occurs, user location information is sent to the central server where advertisements reside, and an appropriate mobile advertisement or discount coupon is pushed to the mobile device [1].

The adoption of WiFi hotspots in venues such as shopping districts, shopping malls or airports has created the need for their owners to monetize their network through advertising. Given that users usually connect to the hotspot through their social media account (e.g. Facebook, LinkedIn) or through an account created with the hotspot, the opportunity for personalized advertising arises through user profiling based on prior user behavior in the venue. For instance, the WiFi platform may deduce that a user oftentimes visits toy stores in the airport or electronics stores in a mall and may project targeted ads to her.

A fundamental distinguishing characteristic of location-based advertising compared to other forms of advertising is that the likelihood of user response to the ad or offer depends on the *timeliness* of the offer. Since users usually spend a limited amount of time in the venue or may be busy with preplanned activities, it is more likely for the user to respond to a store offer if this is made when the user is close to the store. This ad selection process raises several questions. How can the platform effectively target users with ads at certain times while avoiding user saturation because of showing too many ads? How can the platform use wisely the limited budget for each store by targeting ads to users that are more likely to respond to the ad, without knowledge on statistics or future user mobility patterns? Is it preferable at a certain time to project an ad to a user that pays visits to a store more often but is still at some distance from it, or to a user that visits the store less often but is closer?

The different user profiles and dynamic user mobility patterns give rise to an advertisement selection and allocation problem that is fundamentally different from the one in web-search or native advertising. In both web-search and native advertising, context relevance is static: in the former, the user keyword search propels personalized ad selections for users based on

context relevance to search; in the latter, ads are placed close to relevant posts in the post feed. In location-based mobile advertising, relevance still arises in the form of similarity of a user and a store. However, the ultimate decision of whether the user will respond to the offer depends also on the relative proximity of the user and the store at the time when the ad is shown to the user. This proximity changes with time as users roam in the venue.

A. Our Contribution

A set of ads or offer coupons for stores emerge out of a bidding and an auction process. We study the problem of personalized advertisement selection and allocation in a dynamic setup with users moving around the venue. The contributions of our work to the literature are as follows.

- We use a machine-learning logistic-regression model that builds on the assumption that users can be profiled based on prior visits. The output of the model is a per-user probability of response to a projected ad, which depends on the relevance of the ad (store) to the user profile, and on the (time-varying) physical proximity of the user to the store.
- We formulate the problem of advertisement selection and allocation for maximizing the long-term average revenue for the venue owner, subject to (i) a constraint on maximum average projection rate per user that prevents user saturation and disengagement, and (ii) a constraint on a long-term average budget to be spent for each store. If the latter constraint is not satisfied, this leads to free advertising service provided by the platform and therefore to revenue losses.
- We use a stochastic-optimization framework based on Lyapunov optimization [6] to derive a dynamic policy for the problem above. The problem is mapped to a virtual queue stability one, where virtual queues pertain to constraints. To the best of the authors' knowledge, the problem and formulation, although simple and intuitive, have not appeared before in the literature.
- We show that the problem reduces to solving a simple assignment one at each slot, based on instantaneous proximities of users to stores, while being agnostic to mobility statistics.
- We test our approach using real user trajectories derived from a Foursquare check-in dataset, complemented with data we collected through questionnaires. Our approach results in substantial improvement in revenue compared to corresponding approaches that are location- or ad relevance-agnostic.

Most of prior work on location-based advertising is data-driven e.g. [1]. To the best of our knowledge, our work is the first to apply a stochastic optimization framework in this setting. Stochastic optimization has been applied previously in web-search advertising [5], albeit in a different problem,

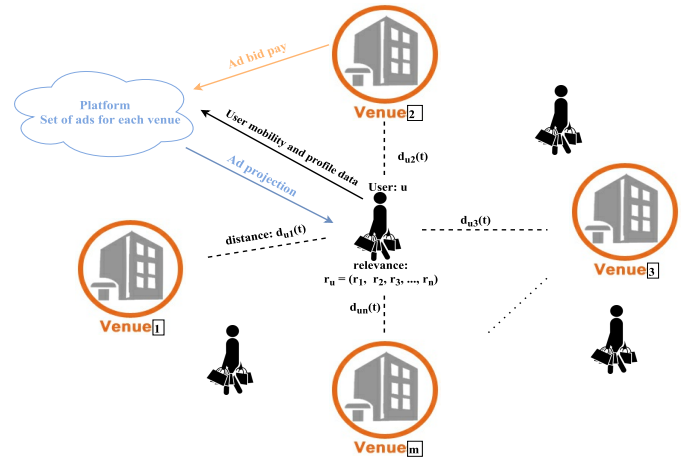


Fig. 1. A pictorial view of the setup.

that of advertisement selection and allocation to ad slots by a web-search service provider given the click-through ratios and dynamic keyword query arrivals. Our problem differs from the one faced in web-search advertising. First, user mobility makes user response probability to ads time-varying, thus the timeliness of ad allocation to users is important. Second, the realistic constraint of a maximum projection rate of ads is included. Third, we also cater explicitly for user profiling using machine-learning methods, and we use the profiles to compute user response probability to ads.

The organization of the paper is as follows. In section II we present the model and problem formulation, and in section III we present the solution. Numerical results are presented in section IV. Related work is briefly surveyed in section V, and the paper is concluded in section VI. We use the terms ad and advertisement interchangeably.

II. MODEL AND PROBLEM FORMULATION

A. System Model

1) *Setup*: We consider a venue (e.g. a shopping mall, urban shopping district or airport) with a set $\mathcal{S}$ of m stores/advertisers in the venue area. There is also a set $\mathcal{U}$ of n mobile users in the area. Users may connect to the platform through a webpage or app. They either enter credentials of an account they maintain with the venue, or they log in by using a social media account e.g. Facebook, LinkedIn or Google+. There exists a set of ads, offer coupons or discount coupons associated with a product or a service of each store. A coupon may also be an offer that expires after a while.

Time is slotted. Fix attention to a specific time t . The location vector of a user u is denoted as $\mathbf{d}_u(t) = (d_{us}(t) : s \in \mathcal{S})$ where $d_{us}(t)$ is the instantaneous distance of user u from store s , $s = 1, \dots, m$ at time t . The ensemble location vector of all users at time t is denoted as $\mathbf{d}(t) = (\mathbf{d}_u(t) : u \in \mathcal{U})$. Locations of all users are assumed to be known at all times e.g. through WiFi access point connectivity. The *trajectory* of each user u

is a sequence of pairs of timestamps and location vectors for user u , i.e. $\{t, \mathbf{d}_u(t)\}_{t=1,2,\dots}$.

2) *Relevance of user and store:* For each store $s \in \mathcal{S}$ and each user $u \in \mathcal{U}$, let $r_{us} \in [0, 1]$ be the *relevance* i.e. the similarity between store s and user u . This similarity emerges from the collective profile of a user in terms of past purchases, frequency of visits to the store or other factors that denote preference. For example, a store that sells athletics products is more appealing to a user that usually makes sports related purchases than to one that makes toy purchases. Relevance may be computed through collection of user-related data about her past activity in the venue or in social media. Then, cosine similarity [7, Ch.9] or other metrics on vectors of values of attributes that are deemed representative of the store and the user profile may be applied to quantify similarity.

3) *Probability of user response to an ad:* Prior to ad projection to users, it is important to quantify the probability of user response to an ad. User response probability to an ad depends on three types of attributes: (i) Attributes that are inherent to the ad/store, e.g. ad or store quality. These are reflected onto ad design format, nature of the advertised product, accompanying text, and store reputation. (ii) Attributes that are related to user profile and activity. Thus, relevance of the ad to the user profile captures the similarity of an ad to user preferences, past purchase activity, past trajectories, store visits, and so on. For example an ad about a restaurant seems more appealing to a user that usually visits a restaurant in an airport than it is to a user that never visits one. (iii) Attributes that are related to the spatio-temporal placement of the ad within the user trajectory. These concern the projection of ads to the user at certain locations and times. Evidence from real data on online platforms [1] seems to suggest that user response probability depends on all the factors above. For instance, an offer about a discount in a certain store may be better received by a user if it is projected when she is close to the store than when she is further away; in the latter case, she may not have time or may simply forget about it by the time she passes from the store.

In our model, user response events to ads are represented through Bernoulli random variables. We consider two main determinants for the probability of user response to an ad: (i) relevance of the store to the user, and (ii) distance (in multiples of some distance unit) of the user from the store. We assume that we can learn from historical data the probability $p_u(r, d)$ that a user u responds to an ad from a store, if the store is at distance d from the user, and if it is of relevance r to her. Logistic or softmax regression or other machine-learning tools may be used to train a model based on past responses (or non-responses) of users to ads and to learn these probabilities for each user u and store s . User response to an ad may depend on other attributes stated above, such as ad quality or its temporal appearance (early/late) in the user trajectory. However, we choose not to include them here for clarity and because we wish to focus on attributes that are peculiar to location-based advertising where dynamic decisions on ad selection and

allocation need to be taken.

The way each user u weighs the attributes associated with an ad of store s (namely, relevance r_{us} to her profile and distance d_{us} to be traversed to the store) so as to reach a decision to respond or not is modeled through a soft-max regression model [8, Ch.11.3]. Each user u and store s is characterized by a vector of weights $\mathbf{w}_{us} = (w_{us}^r, w_{us}^d)$ that capture the significance that user u places on ad relevance and distance when deciding about store s ; this vector is learned from historical data. Let $\mathbf{x}_{us} = (r_{us}, d_{us})$ be the vector of attribute values for user u and store s . Given the ensemble attribute vector $\mathbf{x}_u = (\mathbf{x}_{us} : s = 1, \dots, m)$, the probability that a user responds to an ad from store s is

$$\Pr(\text{user } u \text{ visits } s | \mathbf{x}_u) = \frac{\exp(\mathbf{w}_{us}^T \cdot \mathbf{x}_{us})}{\sum_{j=1}^m \exp(\mathbf{w}_{uj}^T \cdot \mathbf{x}_{uj})}. \quad (1)$$

4) *User visits and mobility dynamics:* We consider the pay-per-visit model, which is similar to the prevalent pay-per-click one in web-search advertising. Each time a user responds to the ad and visits the store, a given amount is paid by the store to the platform. Through localization technologies, the platform may detect whether a user visits a store after an ad of the store is shown to her. Each time a user responds to an ad of store s , for example when she visits the store, a given amount b_s is paid by the store to the platform. This amount is the outcome of some auction process which we assume has already taken place. The product of user response probability and revenue incurred per user visit to the store is the expected revenue that comes from the ad.

Each store has a total budget that may be spent on having its ads displayed to users. The budget of each store is renewed after a certain time interval, and we define B_s as its long-term average value. Without loss of generality, we assume that $\tilde{B}_s = \frac{B_s}{b_s}$ is integer.

When an ad about store s is projected to user u at time slot t' with a distance vector $\mathbf{d}_u(t')$, the user may or may not respond to the ad. If the user does not respond to the ad, there is no impact on the user mobility pattern. If the user responds to the ad and goes to store s , the assumption is that at the end of the time slot she returns to her location with distance vector $\mathbf{d}_u(t')$. The rest of her mobility pattern $\{\mathbf{d}_u(t)\}_{t>t'}$ is assumed to be the same as the one if the ad were not projected to her. This assumption is important so that mobility processes $\{\mathbf{d}_u(t)\}_{t=1,2,\dots}$ for each user u are either i.i.d. or Markovian across time slots and are not affected by ad projection decisions.

5) *User saturation:* In order to avoid user saturation and fatigue from ad and offer projection, we define a minimum average elapsed interval β between consecutive projection of ads to a user. Typical values for β are of the order 5 – 20 minutes. Then $1/\beta$ is the maximum average ad projection rate in the sense that no more than $1/\beta$ ads per unit of time should be pushed to each user on average, or equivalently, ads may

be pushed to a user less often than once every β time units on average.

B. Problem Statement and Formulation

In the presence of roaming mobile users around the venue area, it is important to target the appropriate users at appropriate times with ads of stores. Users differ from each other in the way they respond to offers, depending on the relevance of the offer to their profile and on the distance they have to traverse to go to the store. This heterogeneity is reflected on the different trained softmax regression models for users that generate the probability of response for each user. These probabilities of user response to ads of different stores are time-varying because of user mobility.

The budget constraints for each store suggest that each ad can be pushed to users for a certain maximum amount of times. Thus, judicious targeting of users is needed, so that each ad is projected to those users at times when they have higher chances to visit the store. On the other hand, the constraints on maximum ad projection rate to each user to avoid saturation dictate that a limited number of stores may be advertised to each user within a time window, and again, at times when user locations lead to maximum probability of visit to the stores.

In the presence of user mobility and in the absence of information about future user locations, various dilemmas arise. Should an ad for a store be pushed now to a user that is in certain proximity to the store, or should it be withheld for some time later when another user location would arise with a possibly larger probability of response? Which ads to show to a user and when, given the maximum ad projection rate constraints? The long-run perspective we adopt offers valuable insights, it circumvents these types of questions and demonstrates that they actually do not affect long-term performance.

We are interested in a dynamic policy for ad selection and allocation to users that maximizes the long-term average revenue for the venue owner. For user u , store s and time t , we define the decision variable $y_{us}(t)$, which is 1 if an ad from store s is pushed to user u at time t , and 0 if it is not. Let $\mathbf{y}(t)$ denote the $0-1$ $n \times m$ matrix whose entries are $y_{us}(t)$; this matrix denotes the global advertisement selection and allocation at time t . A policy $\mathbf{y}$ is a sequence of ad selections and allocations, $\{\mathbf{y}(t)\}_{t=1,2,\dots}$. Let T denote the length of a time horizon. The problem above can be formulated as follows:

$$\max_{\mathbf{y}} \limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{u=1}^n \sum_{s=1}^m b_s p_u(r_{us}, d_{us}(t)) y_{us}(t) \quad (2)$$

subject to the constraints:

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{s=1}^m y_{us}(t) \leq \frac{1}{\beta}, \quad \forall u \in \mathcal{U}, \quad (3)$$

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{u=1}^n y_{us}(t) \leq \tilde{B}_s \quad \forall s \in \mathcal{S} \quad (4)$$

and

$$\sum_{s=1}^m y_{us}(t) \leq 1, \forall t = 1, \dots, T, \quad \forall u \in \mathcal{U}. \quad (5)$$

The objective in (2) is the long-term average revenue for the platform. Constraint (3) says that the long-term average projection rate of ads to each user should not exceed a threshold that signifies user saturation, while constraint (4) stems from the budget constraint for each store and implies that each store can display on average a limited number of ads to users. Finally, constraint (5) means that at most one offer should be made to each user at any given time.

III. SOLUTION

A. Virtual Queues and Lyapunov optimization

We use Lyapunov optimization to tackle the problem. We start by mapping constraints (3) and (4) to queue stability problems, so that the problem is mapped to one of optimal control of a dynamic queueing system. For each of the n constraints in (3), we define a virtual queue $Q_u(t)$, $u = 1, \dots, n$. In order to make the size of the queue take integer values, we multiply both sides of (4) with β , so that $Q_u(t)$ evolves as follows:

$$Q_u(t+1) = \max\{Q_u(t) + \beta \sum_{s=1}^m y_{us}(t) - 1, 0\}. \quad (6)$$

This queue builds up when an ad is pushed to user u . Clearly the assignment of an ad to user u at time t is equivalent to increase of the size of the queue. Due to constraint (5), the size of the queue at each time slot may increase at most by β . On the other hand, if no ad is assigned to user u , the queue size decreases by 1. The queue empties when no ad is assigned to the user for some time. Let $\mathbf{Q}(t) = (Q_1(t), \dots, Q_n(t))$ be the vector of sizes of virtual queues for all users.

Further, for each of the m constraints in (4), we define a virtual queue $Z_s(t)$, $s = 1, \dots, m$, which evolves as follows:

$$Z_s(t+1) = \max\{Z_s(t) + \sum_{u=1}^n y_{us}(t) - \tilde{B}_s, 0\}. \quad (7)$$

Here, the queue builds up when an ad from store s is pushed to users. The assignment of an ad from store s to user u at time t is equivalent to increase of the size of the queue. An ad may be pushed to more than one users at the same time, and the queue size at each time increases according to the number of users to which the ad is pushed. If no ad from store s is pushed to a user, the queue size decreases by $\tilde{B}_s$. The queue empties when an ad from store s is not assigned to any user for some time. Let $\mathbf{Z}(t) = (Z_1(t), \dots, Z_m(t))$ be the vector of sizes of virtual queues for all stores. Let $\Theta(t) = (\mathbf{Q}(t), \mathbf{Z}(t))$.

The assignment variables $\mathbf{y}(t)$ control the admission processes in the virtual queues. We define the Lyapunov function

$$L(\Theta(t)) = \frac{1}{2} \sum_{u=1}^n Q_u^2(t) + \frac{1}{2} \sum_{s=1}^m Z_s^2(t). \quad (8)$$

Then, define the one-slot conditional Lyapunov drift, $\Delta(\Theta(t))$,

$$\Delta(\Theta(t)) = \mathbb{E}[L(\Theta(t+1)) - L(\Theta(t)) | \Theta(t)], \quad (9)$$

which denotes the expected change in the Lyapunov function in one slot, conditioned on the current state, and where the expectation is with respect to the statistics of queue evolution. By using standard techniques for bounding the Lyapunov drift [6, Chap.3-4] and [6, p.33-34], we have that

$$\begin{aligned} \Delta(\Theta(t)) \leq & B + \mathbb{E}\left[\sum_{u=1}^n Q_u(t)(\beta \sum_{s=1}^m y_{us}(t) - 1) | \Theta(t)\right] \\ & + \mathbb{E}\left[\sum_{s=1}^m Z_s(t)(\sum_{u=1}^n y_{us}(t) - \tilde{B}_s) | \Theta(t)\right], \end{aligned} \quad (10)$$

where B is a positive constant that satisfies

$$B \geq \frac{1}{2}n(1 + \beta^2) + \frac{1}{2}(mn^2 + \sum_{s=1}^m \tilde{B}_s^2). \quad (11)$$

Since we aim at maximizing the objective function in (2), we employ the drift-plus-penalty function,

$$\Delta(\Theta(t)) + V\mathbb{E}[Y(t) | \Theta(t)] \quad (12)$$

where $V \geq 0$ is a fixed parameter that quantifies the significance we place at the objective function, and

$$Y(t) = - \sum_{u=1}^n \sum_{s=1}^m b_s p_u(d_{us}(t)) y_{us}(t), \quad (13)$$

for which we have

$$\begin{aligned} \Delta(\Theta(t)) + V\mathbb{E}[Y(t) | \Theta(t)] \leq & B \\ & - V\mathbb{E}\left[\sum_{u=1}^n \sum_{s=1}^m b_s p_u(d_{us}(t)) y_{us}(t) | \Theta(t)\right] \\ & + \mathbb{E}\left[\sum_{u=1}^n Q_u(t)(\beta \sum_{s=1}^m y_{us}(t) - 1) | \Theta(t)\right] \\ & + \mathbb{E}\left[\sum_{s=1}^m Z_s(t)(\sum_{u=1}^n y_{us}(t) - \tilde{B}_s) | \Theta(t)\right]. \end{aligned} \quad (14)$$

The drift-plus-penalty method aims to employ at each time slot the appropriate control so as to minimize the right-hand-size of the inequality above. In our case, this control pertains to the selection and assignment of ads subject to constraint (5) which needs to hold at each slot t . With a slight abuse of notation, let $p_{us}(t) = p_u(r_{us}, d_{us}(t))$. Then the optimization problem to be solved at each time t is

$$\max_{\mathbf{y}(t)} \sum_{u=1}^n \sum_{s=1}^m (V b_s p_{us}(t) - \beta Q_u(t) - Z_s(t)) y_{us}(t) \quad (15)$$

subject to:

$$\sum_{s=1}^m y_{us}(t) \leq 1, \forall t = 1, \dots, T, \forall u \in \mathcal{U}. \quad (16)$$

with $y_{us}(t) \in \{0, 1\}$, for all t . At each time slot, the policy observes the vectors of instantaneous sizes of virtual queues $\mathbf{Q}(t)$ and $\mathbf{Z}(t)$ and instantaneous ensemble location vector of users $\mathbf{d}(t)$ at time t and decides on the policy $\mathbf{y}(t)$ that maximizes (15) at that slot.

The solution to the problem above is found if we observe that (15) can be decomposed in separate maximization problems, one for each user u , subject to constraint (16). Let $\gamma_{us}(t) = (V b_s p_{us}(t) - \beta Q_u(t) - Z_s(t))$. Then, the solution is as follows. To each user u , we assign ad

$$s_u^*(t) = \arg \max_{s=1, \dots, m} \gamma_{us}(t), \text{ if } \gamma_{us}(t) > 0. \quad (17)$$

Otherwise, if for all s it is $\gamma_{us}(t) \leq 0$, then we do not assign any ad to user u at time t .

IV. DATASET EXPERIMENTS AND EVALUATION

A. Dataset and preprocessing

1) *Dataset*: We use a Foursquare dataset that is available online to evaluate our approach [9]. The dataset consists of check-ins in New York City from 12 April 2012 to 16 February 2013, and it contains 227,428 check-ins. Each check-in is associated with a user, a time stamp, GPS coordinates and its semantic meaning i.e. fine-grained venue categories. Each check-in contains also user id, venue id, venue category id and name, and the date and time of check-in. The dataset was originally used for studying the spatiotemporal regularity of user activity in Location Based Social Networks, but we found it relevant and applicable in our case as well.

2) *Dataset preprocessing*: First, we plotted the data and then we chose an area that contained many shopping-related check-ins. The area we choose extends to a radius of 500 meters from Soho in New York City. It contains a large number of stores of various types, e.g. about clothing, cosmetics, athletic apparel, books, electronics, restaurants and other food related venues. The area and types of stores were suitable for our experiments as they could imitate the size and stores that one encounters in a mall or an airport.

Since each check-in contains a user id and a time stamp, we are able to extract the trajectories for each user. The only assumption we make is that, if the temporal distance between two check-ins is larger than 24 hours, a new trajectory begins. Each user has a number of trajectories, and each trajectory is formed by a sequence of pairs $\{t, \mathbf{d}_u(t)\}_{t=1,2,\dots}$, where t is the timestamp and $\mathbf{d}_u(t) = (d_{us}(t) : s \in \mathcal{S})$ is the vector of distances of user u from venues at time t .

Suppose there are K venue categories. We assign each user a vector $\mathbf{r}_u = (r_{uk} : k = 1, \dots, K)$. These may be interpreted as relevance factors between user u and venue category k . If they are normalized in interval $[0, 1]$ they may be interpreted as probabilities of visit. Thus, r_{uk} can be calculated as the number of times user u visited a venue of category k over the total number of visits she paid. We say that a user has visited a venue if there is a check-in in that particular venue

from this user. Note that relevance of a venue category to a user is not time-dependent, i.e. preferences of a user do not change in such short time interval. Still, with additional data, our model might capture possible changes in user preferences over a longer period.

3) *User Clustering to reduce data sparseness*: An important issue is data sparseness, since some users have very few trajectories and visit a small number of venues. To deal with this, we create clusters of users when users are associated with few trajectories. Each cluster of users is then treated as a single user whose trajectories are all trajectories of users in that cluster. We measure the similarity between two users u, u' using *cosine similarity* on vectors $\mathbf{r}_u$ and $\mathbf{r}_{u'}$, as $cs(\mathbf{r}_u, \mathbf{r}_{u'}) = \sum_k r_{uk} r_{u'k}$. Finally, we use the *affinity propagation* (AP) clustering algorithm [10] to determine the number of the clusters and to group users in clusters. With AP, the number of clusters does not need to be predefined.

B. User Profiling Survey and user matching

1) *Survey*: The softmax regression model in (1) uses two features that affect user response to a store ad: relevance of user and store profiles, and distance from the store. Distance can be directly derived from the Foursquare dataset. One could argue that relevance r_{uk} of user u to store category k could also be derived through the percentage of visits to stores, as discussed in IV-A above. However, this makes the two features correlated as they are computed from the same part of the dataset. The idea is that relevance should denote the *background* preference profile of each user, regardless of visits. Thus, we decided to disengage the two features by deriving relevance through a questionnaire survey.

We distributed a questionnaire to 50 students and faculty of the university. For each major venue category in the dataset, users from the questionnaire were asked about their interests in a scale from 1 to 5. We calibrated the values that users gave in $[0, 1]$ so that they correspond to the scale of r_{uk} in the Foursquare dataset, e.g. 1, 2, 3, 4, 5 would correspond to 0, 0.2, 0.4, 0.7, 0.9 respectively. Let $\tilde{\mathbf{r}}_v = (\tilde{r}_{vk} : k = 1, \dots, K)$ be the vector of preferences of user v in the questionnaire.

2) *Matching surveyed and Foursquare users*: After profiling the users of the survey, we attempt an one-to-one matching of these users and the user clusters from the Foursquare dataset. We use *cosine similarity* of vectors $\mathbf{r}_u$ and $\tilde{\mathbf{r}}_v$, i.e. $cs(\mathbf{r}_u, \tilde{\mathbf{r}}_v)$ to measure the similarity between Foursquare user cluster u and survey user v . We create a bipartite graph $G = (\mathcal{U}, \mathcal{V}, E)$ consisting of two disjoint node sets $\mathcal{U}, \mathcal{V}$. Set $\mathcal{U}$ has one node for each user cluster u , while $\mathcal{V}$ has one node for each survey user v . The weight of an edge from $u \in \mathcal{U}$ to $v \in \mathcal{V}$ is $w_{uv} = cs(\mathbf{r}_u, \tilde{\mathbf{r}}_v)$.

To match users between sets $\mathcal{U}$ and $\mathcal{V}$, we find a maximum weighted bipartite matching of graph G , i.e. a matching with maximum sum of edge weights. The method we use to solve the problem has complexity $O(|\mathcal{U}|^3)$ and uses the blossom method

for finding augmenting paths and the primal-dual method for finding a matching of maximum weight [11].

C. Soft-max Regression

Given that for each user we have a sequence of pairs $\{(\mathbf{r}_u, \mathbf{d}_u(t))\}$, equal to the number of timestamps of each user trajectory, we use a machine-learning model to calculate the probability of a user visiting a venue, given the user preferences and the distance from venues. We approach user venue response behavior as a K -class probabilistic classification problem, where the K classes are venue categories. We use the softmax regression model in (1).

The training dataset for calculating the probability of a user u visiting a store s consists of triads of the form $(\mathbf{r}_u, \mathbf{d}_u, s)$, where $\mathbf{r}_u$ is the vector of relevances between user u and each store s , $\mathbf{d}_u$ is the vector of distances of user u from all the venues at any timestamp of her trajectories, and s is the venue visited by the user.

D. Results

We simulate user mobility of 50 randomly selected users from the pool of users in the area for 500 time-slots of 10 minutes each, using the random waypoint mobility model. First, we place users in the simulation area. Then, each user moves with a uniformly randomly selected speed from current point to a uniformly randomly selected point within a radius that depends on walking speed and time-slot duration. The procedure is repeated for each time-slot.

In our experiments, we use the pay-per-impression model where the advertiser pays a given amount per ad display. This choice was used since user actual visits to a venue cannot be monitored unless we run a real field experiment. We assume that the budget for each venue/advertiser is 10 and the bids are 1 money units. Finally, $V = 1$.

We run the experiment for three different policies: one that uses only the distance attribute, one that uses only the relevance attribute, and one that uses both to describe the probabilities of visit. The policies are referred to as "Only Distance", "Only Relevance" and "Relevance and Distance", respectively. All these policies rely on our machine-learning model and aim at predicting the user visit probability at a venue. Clearly, other classes of heuristics exist. For example, we can choose randomly T/β timeslots to display ads for each user among T timeslots. At these time slots, we give to each user the ad that corresponds to the venue he is closer at. Likewise, yet another policy is to choose in a similar fashion the ad that is most relevant to a user. These classes of heuristics, although practically relevant, do not lead to a computation of the probability of visit. Thus, they would not be directly comparable to the approaches above, unless we run a real field experiment.

In Figure 2, we present the long-term average revenue as a function of average elapsed interval between consecutive

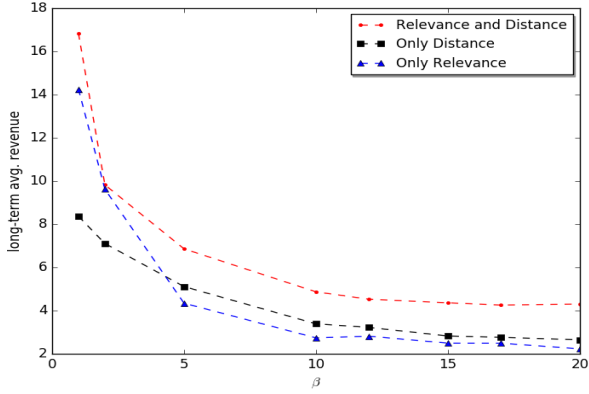


Fig. 2. Long-term avg. revenue as function of avg. elapsed interval between consecutive projected ads, β , for different policies.

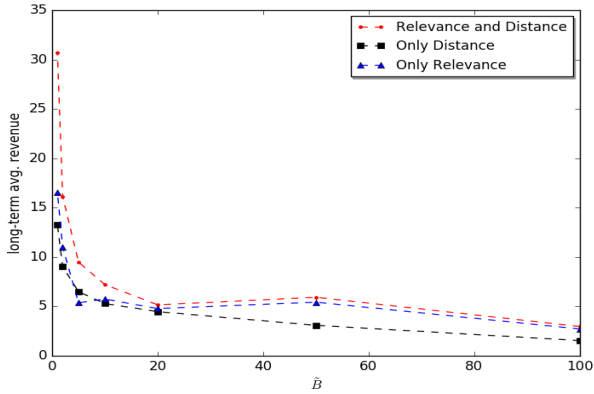


Fig. 3. Long-term avg. revenue as function of long-term average number of possible ad projections of each store/advertiser $\tilde{B}$ for different policies.

projected ads, β , for $\beta = \{1, 2, 5, 10, 12, 15, 17, 20\}$. Long-term average revenue, for all policies, is seen to be a non-increasing function of β , and for $\beta \geq 10$ it does not change significantly and seems to stabilize. We observe that using both the distance and the ad relevance attributes in predicting user visit probabilities gives the highest value of long-term average revenue. We also see that, as the time that elapses between successive ad projections gets bigger, fewer ads are shown to users and as a result long-term average revenue decreases.

In Figure 3, we depict the long-term average revenue as function of $\tilde{B}$. Since in our experiments $\tilde{B}_s$ is the same for all s , we denote as $\tilde{B}$ the long-term average number of possible ad projections to users of each store/advertiser, and we show how the long-term average revenue varies for different values of $\tilde{B} = \{1, 2, 5, 10, 20, 50, 100\}$. As expected, for all three policies, as the long-term average ad projections decreases, long-term average revenue decreases as well.

V. RELATED WORK

Web and native advertising. In all forms of advertising, advertisers compete to have their ads displayed to users. Advertisers set a budget they are willing to spend within a certain period of time, and an auction process determines the ads to project and the relevant charges. Chronologically, the first form of advertising was web-search advertising with its celebrated Generalized Second-Price (GSP) auction [12]. The most common payment models are the pay-per-impression one and the pay-per-click one. The latter model, where a fee is paid to the platform after an ad click, resembles the pay-per-visit model we assume here.

In *native advertising*, ads are promoted or sponsored posts that are allocated inside the user post feeds. In [13], the problem of ad placement in a post stream is addressed. The model captures the cumulative effect of previously projected ads on user click probability which is decreasing in the number of previously shown ads. Given a set of ads, a reward and a set of candidate positions, the objective is to find an ad placement that maximizes total reward. In [4], click probability of an ad depends on the distance from the previously shown ad and on context relevance between the ad and the previous post in the feed. The goal is to find the ad selection and allocation policy that maximizes revenue for the platform, while minimizing revenue uncertainty captured by variance of consumed budget of ads.

Stochastic optimization approaches. Stochastic optimization has been used in web-search ad allocation. In [14], the optimal-auction framework is used for single-slot revenue maximization. The optimal policy is to allocate the slot to users in decreasing order of $q_i \nu_i$ where q_i, ν_i are the selling probability and valuation of user i . In [5], the authors use Lyapunov optimization for maximizing long-term average revenue for a web-search service provider by dynamically allocating ads to webpage slots in the presence of dynamic keyword query arrivals, subject to a long-term average budget constraint. The work [15] studies allocation of budget-constrained advertisers in each keyword auction round so as to maximize the likelihood of ad click or to reduce advertiser cost per click. Dynamic actions under limited budget over the entire horizon are studied in [16] through the lens of multi-armed bandit theory.

Location-based advertising (LBA). LBA leverages localization technologies to perform targeted advertising, and it has impact in the society and economy [17]. In [18], the timeliness of projecting mobile coupons is identified as a major factor that influences redemption rate. The work [1] applies a data-driven approach and a real field experiment in studying the likelihood of coupon redemption. Consumer preferences are inferred through machine-learning techniques on trajectory data and on relevant spatiotemporal and semantic information. The work [19] studies the ad broadcast scheduling problem, i.e. that of deciding which ads to send to which customers at what time, given a limited capacity of broadcast time slots, while maximizing customer response and the revenue of the marketing

company with the aid of precomputed priority weights for each ad. A challenge in location-based advertising or services is to mine user activity and mobility data for pattern discovery [20]. In [21], the authors cluster similar users based on venues they have visited, while in [22] the authors use techniques based on spatial specificity, temporal correlation and context similarity to derive and predict user activity profiles.

Finally, the work [23] proposes a WiFi monetization model for public hotspots provided by venue owners (VOs). Revenue can be generated through a premium or an ad-sponsored access. In the former, users pay venue owners for Wi-Fi usage, while in the latter users watch ads in return for free Wi-Fi. The strategic interactions among the ad platform, multiple VOs, users, and competing advertisers are studied in a three-stage Stackelberg game, whereby the ad platform computes a revenue sharing policy among VOs, VOs decide on the prices of premium access and advertising, users choose between the access modes above, and advertisers decide on the number of ad spaces to purchase from the VO.

VI. CONCLUSION

We studied the problem of ad selection and allocation and we derived the policy that maximizes long-term average revenue for the platform. To achieve that, we exploit user profiling and instantaneous user locations to come up with stores that are likely to be visited by each user. We included in the formulation a maximum ad projection rate to each user and a maximum total average budget to be spent by each store. The extent to which these constraints are satisfied or violated is mapped into virtual queues whose lengths guide ad assignment to users.

There exist several directions for future study. In this work, we assumed that the user population does not vary with time in order to expose our solution principle. However, in reality the user set varies with time since users arrive and depart from the area frequently. It would be interesting to explore new arising issues in this case. Another interesting direction is to incorporate short-term constraints in the ad projection algorithm. For example, if the user has received an ad and visited a store, it would not make sense to show the same or similar ads to that user in the near future.

The ad assignment to a user used some form of similarity of the store and the user profile. In the future we plan to use methods from recommender systems such as collaborative filtering and matrix factorization to guess which ad is likely to be preferred by a user. Another interesting direction is to mine user trajectories and derive insights that can be used in our model in order to make it more efficient through conserving advertising opportunities. For example, if a user follows a trajectory every day and almost surely visits some stores, then we may do not want to advertise these stores to her since they will be visited anyway, and instead we should try to produce revenue through other users or venues. We are also interested in applying our approach in some tens of users through a mobile app in a real field experiment.

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