Efficient Peer-to-Peer Energy Trading Mechanisms with Unreliable Prosumers

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ABSTRACT

We model and analyze a peer–to–peer (p2p) energy trading market under uncertainty in the traded energy, in a setting with multiple sellers and buyers. A set of prosumers sell in the market their energy surplus units, which are subject to uncertainty for being actually available, while other prosumers buy energy to cover their deficit units, which are subject to uncertainty for being actually needed. Given the different levels of uncertainty of different prosumers and different energy units, the p2p trading problem is to match energy demand and supply and to specify the payments of buyers and compensations to sellers.

We propose an innovative variant of the Vickrey–Clarke–Groves (VCG) auction customized to our setting, motivated by the properties of the standard form of the VCG auction, namely maximizing social welfare while ensuring participants' truthfulness. We determine the bidding profiles of players by considering the uncertainty in the declared amounts of energy surplus or deficit. Moreover, we develop a low-complexity allocation rule, that provably leads to maximization of the expected social welfare, where the expectation is with respect to uncertainties of energy units. We also derive closed-form expressions for winners' payments. We compare our mechanism to a double auction, which is currently used as a p2p trading mechanism. The results reveal that our mechanism outperforms the double auction one, and lead to interesting intuitions and guidelines that shed light into p2p energy market design.

CCS CONCEPTS

• **Theory of computation → Computational pricing and auctions**.

KEYWORDS

Peer-to-peer energy trading, demand and supply uncertainty, Vickrey-Clark-Groves auction, double auction.

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1 INTRODUCTION

Advances in the liberalization of energy markets, together with the increasing penetration of renewable energy sources (RESs), and the upcoming integration of electric vehicles (EVs) in the smart grid have given rise to new forms of prosumer interaction with the grid. The existing so-called feed-in-tariffs (FITs) offer compensation to renewable energy producers, while providing price certainty and long-term contracts that help finance renewable energy investments, and they constitute a new form of prosumer interaction with the smart grid.

However, FITs still involve the interaction of prosumers with the central utility operator that keeps prices under its control. FITs merely aim to convince more consumers to contribute their energy. However, FITs constitute a rather static procedure that cannot actively prevent potential over-provisioning of renewable energy supply that challenges the stability of the grid. Also, the massive introduction of RESs in the smart grid will likely lead to diminishing FIT prices, exhaustion of funding for RES owner remuneration, and a clear decline in FIT systems and their effectiveness, hence leaving the challenge of balancing dynamic supply and demand largely unresolved. These issues are discussed in [19], [11], and reveal the inherent limitations of FITs.

Recently, *peer-to-peer energy trading mechanisms* have emerged as a viable alternative towards making optimal use of the generated energy from distributed RESs; indeed, they aim at balancing supply and demand at very small-time scales, while at the same time providing the freedom to prosumers to manage their own energy and make the most out of it. In p2p energy trading, prosumers (peers) engage in energy trading markets. Prosumers with energy surplus can declare and sell their surpluses amount back to the system so that they can be used to cover the energy needs of other prosumers that declare energy deficits; the latter in turn would buy the amount of energy needed from the system [21].

However, a challenge to overcome before p2p energy trading markets become widely adopted, is the *uncertainty in the amounts of traded energy.* This uncertainty is predominantly attributed to inherent prediction errors both in the generated energy from renewable sources (e.g., due to unpredictable weather conditions) and in energy needed due to consumer demand (e.g., due to occasional unpredictable demand surges and unpredictable consumer behavior). Even if the market runs very close to production/consumption,

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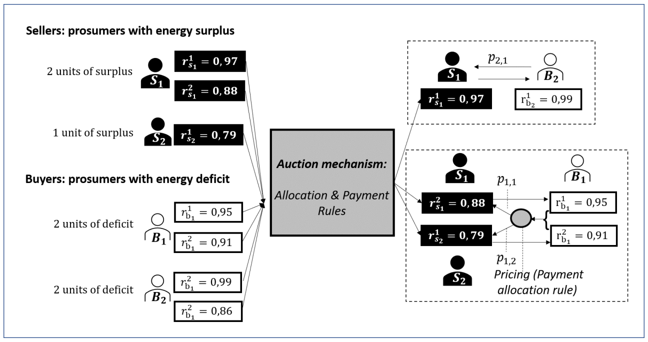


Figure 1: An example of the proposed mechanism with two sellers offering multiple units of surplus with different reliability probabilities, and two buyers wishing to cover multiple units of deficit with different reliability probabilities.

where the amounts of energy surplus deficit can be declared with some confidence, this uncertainty is not fully eliminated. The amount of surplus or deficit declared in the market by a prosumer depends on her energy generation and her demand. This uncertainty in the units of energy surplus/deficit traded, requires a novel viewpoint in modeling prosumers and in designing the trading mechanism. Designing a viable peer-to-peer energy trading market in the presence of such uncertainties is precisely the goal of our work.

In this work, we propose an innovative variant of a VCG auction with multiple sellers and multiple buyers as a p2p trading mechanism, where we take into account the uncertainties of energy units, which influence the utilities of the prosumers and as a result their bidding strategies. Using monotonicity properties, we show that the optimal prosumer matching as a result of the auction can be derived with little complexity. The auction achieves maximization of the expected social welfare, where expectation is with respect to the uncertainties of the energy units involved in the trading.

It should be noted that such welfare maximization problems in the presence of multiple sellers are in general of combinatorial nature since they require the analysis of an exponentially high number of allocation combinations [10]. However, in our work we derive closed-form expressions for total payments of the winners. Nevertheless, since we deal with a multi-seller auction, computing the total payment for each winner is not sufficient.

Therefore, we define the p2p pricing scheme for each energy unit traded as the outcome of a proposed allocation rule that distributes the total VCG payments of winning buyers among respective sellers. This rule satisfies the participation constraints of both buyers and sellers and attains fairness for sellers. To the best of our knowledge, the allocation of VCG payment of a winning buyer among respective sellers in a multi-seller VCG auction mechanism has not been addressed so far. Also, the auction mechanism per se in the presence of uncertainty in both the energy demand and supply constitutes an other important novelty of this work. A sketch of the proposed mechanism is provided in Figure 1.

This work's contributions to p2p energy trading literature are:

- We introduce uncertainty in the declared amount of supply and demand of peers that participate in the energy exchanges, i.e., whether the declared amount of energy surplus or deficit will be produced or consumed. This uncertainty is captured by means of Bernoulli random variables that are characterized by the probabilities for each unit of declared energy surplus and deficit being actually available and needed, respectively. The model addresses heterogeneous prosumers in terms of uncertainty as well as the different uncertainty for each unit of energy surplus or deficit.
- We define a prosumer model with utility functions that capture the expected benefit from prosumers' participation in the market and the uncertainty in energy surplus or deficit.
- We propose an innovative and non-trivial variant of a multi-seller VCG auction as the p2p trading mechanism, taking into account the uncertainties of energy units, which influence the utilities of the involved prosumers and as a result their bidding strategies. Based on monotonicity properties, we show that the optimal prosumer matching as a result of the auction can be derived with low complexity, together with closed-form expressions for the total payments of the winners. The auction achieves maximization of expected social welfare, where expectation is with respect to uncertainties of the energy units involved in the trading.
- We define the p2p pricing scheme (for each unit traded) as the outcome of a proposed allocation rule that distributes the total VCG payments of winning buyers among respective sellers. The rule satisfies the participation constraints of prosumers and attains fairness in VCG payments' allocation.
- We also design a suitable double auction to capture uncertainty of buyers and sellers, and we compare its outcome in terms of expected net benefit of prosumers to that of our mechanism. We derive interesting intuitions and guidelines that shed light into p2p energy market design.

In section 2 we present the model, and in section 3 we gradually develop the VCG auction for multiple sellers offering multiple units of energy surplus, and multiple buyers wishing to cover multiple units of deficit, starting from simpler cases. In section 4 we numerically evaluate our mechanism and compare it to a double auction one. In section 5 we discuss the related work, and we conclude in section 6. We use the terms "peer" and "prosumer" interchangeably.

2 MODEL

We consider a set $\mathcal{N}$ of N prosumers that participate in a p2p energy market. Each prosumer may generate and also consume energy. The energy surplus or deficit for each prosumer is the net result of her generation and consumption. For example, if at a certain time prosumer i generates 2 units of energy and consumes 1, then she has a surplus of 1 unit; if she consumes 2 units of energy and generates 1, then she has a deficit of 1 unit. Considering the inherent uncertainty in both the generated energy from RES and in the energy needed, the probabilities that each of the units of energy surplus or deficit of prosumer i will actually occur, is the net result of these uncertainties.

We define a set $\mathcal{S}(t)$ of S prosumers to be suppliers (have energy surplus) at time t and a set $\mathcal{D}(t)$ of D prosumers to be the set of peers that demand energy (have energy deficit) at time t, where $\mathcal{S}(t) \cup \mathcal{D}(t) = \mathcal{N}$. These sets may change with time.

Each prosumer $i \in \mathcal{D}(t)$ is characterized by the vector of probabilities of having each of its units of deficit truly needed at time t, and each prosumer $j \in \mathcal{S}(t)$ is characterized by a vector of probabilities of having each of its units of surplus truly available at time t. For instance, for prosumer $j \in \mathcal{S}(t)$, we denote as $r_{s_j}^k$ the probability of actually having the k^{th} unit of energy surplus available, where k takes values in $[1, K_j]$ and K_j is an upper bound that denotes the maximum number of surplus units of prosumer j that can exist. The maximum total number of surplus units is $\sum_{i=1}^{S} K_j$. Similarly, for each prosumer $i \in \mathcal{D}(t)$, we denote as $r_{d_i}^l$ the probability of actually consuming the l^{th} unit of energy, where l takes values in $[1, L_i]$ and L_i is an upper bound that denotes the maximum number of deficit units of prosumer i that can exist. The maximum total number of energy deficit units is $\sum_{i=1}^{D} L_i$. We assume that these probabilities are known and fixed for the duration of the mechanism. Prosumers with surplus participate in the market to sell their excess energy and, we call them *sellers*, while prosumers with deficit participate in the market to purchase energy to cover this deficit and, we call them *buyers*. It is possible and desirable that a prosumer may switch between buyer and seller at different times-slots.

3 MECHANISM

We restrict attention to a specific time instant (time units are omitted), and we study p2p energy trading under the scenarios: (*i*) one seller with one unit of energy surplus and multiple buyers wishing to cover one unit of energy deficit each, (*ii*) one seller with multiple units of surplus and multiple buyers wishing to cover possibly multiple units of deficit each and (*iii*) multiple sellers offering possibly multiple units of surplus each and multiple buyers each wishing to cover possibly multiple units of deficit. We take these steps progressively to demonstrate the different aspects of the mechanism and derive the general case by building on the previous two.

3.1 One seller with one unit of energy surplus and $N-1$ buyers, each with one unit of energy deficit

We first consider the scenario where one seller expects to have one unit of energy surplus with reliability probability r_s; this is the probability that she actually has an energy surplus. In other words, the surplus of the seller is a Bernoulli random variable that takes values 1 or 0, with probability r_s and $1 - r_s$ respectively. The remaining $N-1$ prosumers have one unit of energy deficit, each with probability $r_{d_1}, \cdots, r_{d_{N-1}}$; namely, r_{d_1} is the probability that the energy needs of prosumer i will actually exceed by one unit the available self-produced energy. We assume without loss of generality that these probabilities are ordered as follows: $r_{d_1} > \cdots > r_{d_{N-1}}$. Again, the deficit of prosumer i is a Bernoulli random variable that takes values 1 or 0, with probability r_{d_i} and $1 - r_{d_i}$. Modeling the surplus or deficit of a prosumer as a Bernoulli random variable is a simple way of modeling uncertainty, that allows us to start investigating our problem. Moreover the Bernoulli probabilities can be estimated easily by means of historical data. We consider the class of mechanisms that operate as follows:

- Prosumer i with a deficit will pay for this unit of energy in advance, even if she finally does not consume the unit; the probability of this event is $(1 - r_{d_i})$.
- Prosumer i will be able to resell the unit she obtained directly to the grid at price p_{FIT}, in case the unit is actually produced but not consumed, because the deficit of prosumer i does not arise; the probability of this event is $r_s(1 - r_{d_i})$.
- Prosumer i will have to cover her energy deficit from the grid at price p_{retail}, if the unit is not produced but the deficit of prosumer i arises; the probability of this event is $(1 - r_s)r_{d_i}$.
- If the unit of energy surplus is not produced and the deficit of prosumer i does not arise, then no transaction with the main grid will take place; the probability of this event is $(1 - r_s)(1 - r_{d_i})$.

Note: $p_{retail} > p_{FIT}$ under the plausible assumption that FIT prices are low enough, due to the massive introduction of RES [11].

Thus, each buyer's willingness to pay (valuation) for an unreliable unit of energy surplus consists of: (*i*) the buyer's willingness to pay for the unreliable unit, (*ii*) plus the anticipated revenue she will obtain if the unit is actually produced but not consumed, (*iii*) minus the amount she will pay if the unit is not produced by the seller, but a unit needs to be consumed by the buyer and needs to be purchased from the grid. The buyer's value for the unit sold is:

$$v_i = r_s r_{d_i} p_{retail} + r_s(1 - r_{d_i}) p_{FIT} - (1 - r_s) r_{d_i} p_{retail} \,. \quad (1)$$

3.1.1 *Vickrey auction.*

In order to achieve incentive compatibility in the presence of prosumer uncertainty, we employ a variant of a Vickrey auction mechanism with reserve price. Vickrey auction is known to maximize the social welfare, which in a single-item auction means allocating the item to the bidder with the highest offer. The seller announces the reliability probability r_s of the unit of energy surplus sold, and each buyer i submits her bid b_i to obtain this unit. The prosumer with the highest bid wins and pays a price equal to the highest losing bid. The seller's minimum accepted bid (reserve price) is equal to the anticipated amount of money she will receive if the energy surplus indeed arises and she sells it directly to the grid. This amount equals to $s = r_s \cdot p_{FIT} + (1 - r_s) \cdot 0 = r_s \cdot p_{FIT}$. Note that the seller is assumed to have no cost and is only characterized by its reliability. If there were a cost, this would be included in her participation constraint, i.e. in the reserve price.

In a Vickrey auction, truth-telling is a dominant strategy, and thus $b_i = v_i$. In order to avoid the seller's possible attempt to manipulate the auction, the reliability of energy surplus sold via the auction is verified by a central controller based on historical data, context information and, if needed, it is amended accordingly based on grid monitoring data. Each buyer extracts her own reliability probability based on historical data and weather forecast.

The seller has the incentive to *participate in the market if and only if* $b_i \geq r_s \cdot p_{FIT}$ at least for some of the bids. However, from equation (1), it follows that it is necessary and sufficient to have $r_s \geq p_{retail}/(2 \cdot p_{retail} - p_{FIT})$. In fact, this condition applies for all bids, regardless of the value of r_{d_i}. In meaningful scenarios, i.e. when $p_{retail} > p_{FIT}$, we have that $p_{retail}/(2 \cdot p_{retail} - p_{FIT}) < 1$, which implies that for sufficiently high values of r_s the seller always has the incentive to participate. If $p_{FIT} = a \cdot p_{retail}$ with $a \in [0, 1]$, then the value of the r_s threshold is $r_s \geq 1/(2 - a)$. In some cases, the FIT price values are as low as one fifth of retail prices [3], i.e. $a = 1/5$. In this case, it suffices that $r_s \geq 0.56$, while for higher values of p_{FIT} this threshold increases. However, its value always remains within the feasible region, i.e. below 1, since $a < 1$.

Since $r_{d_N} < \cdots < r_{d_1}$ the winner is buyer 1, i.e. the most reliable one, and pays the highest losing bid, i.e. $b_2 = r_s r_{d_2} p_{retail} + r_s(1 - r_{d_2}) p_{FIT} - (1 - r_s) r_{d_2} p_{retail}$. The *revenue of the seller* is,

$$\begin{aligned} R &= r_s r_{d_2} p_{retail} + r_s(1 - r_{d_2}) p_{FIT} - (1 - r_s) r_{d_2} p_{retail} \\ &= (2r_s - 1) r_{d_2} p_{retail} + r_s(1 - r_{d_2}) p_{FIT} \, . \end{aligned} \tag{2}$$

Since $r_s < 1$ it follows that: $R < r_s \cdot r_{d_2} \cdot p_{retail} + r_s(1 - r_{d_2}) \cdot p_{FIT}$.

In our setting, due to the uncertainties, we estimate the expected net benefit of the winning buyer as a weighted average that consists of the four possible events: (i) the buyer's maximum willingness to pay in order to cover her expected unit energy deficit, which will arise with probability r_{d_1} (which equals to the amount she would pay if she had to buy one unit of energy directly form the grid, i.e. p_{retail}), (ii) minus the amount buyer 1 will pay due to winning the auction, which is given by equation (2), (iii) plus the amount of money she will receive by reselling the unit of energy to the grid if it is actually produced, i.e. p_{FIT}, (with probability r_s) but not consumed (which will happen with probability $1 - r_{d_1}$) and (iv) minus the amount of money she will pay in order to buy one unit of energy from the grid in case the unit of energy bought in the p2p energy market is not actually produced, but a unit still needs to be consumed (with probability $(1 - r_s) r_{d_1}$).

$$\begin{aligned} NB_1 &= r_{d_1} p_{retail} - (2r_s p_{retail} - r_s p_{FIT} - p_{retail}) r_{d_2} \\ &\quad - r_s p_{FIT} + r_s(1 - r_{d_1}) p_{FIT} - (1 - r_s) r_{d_1} p_{retail} \\ &= r_s(r_{d_1} - r_{d_2})(p_{retail} - p_{FIT}) + r_{d_2} p_{retail}(1 - r_s) > 0 \, . \end{aligned} \tag{3}$$

Therefore, the expected net benefit of the winning buyer is always positive despite the risk in case the energy surplus that the seller has promised does not arise. Thus, the winner's participation constraint is always satisfied.

Note that if the unit sold were certain i.e. $r_s = 1$, then the prosumer's willingness to pay would equal the amount she would spend to cover her unreliable unit energy deficit from the grid, i.e. $r_{d_i} p_{retail}$. In that case buyer's value for the unit sold is

$$v_i = r_{d_i} p_{retail} + (1 - r_{d_i}) p_{FIT} = r_{d_i}(p_{retail} - p_{FIT}) + p_{FIT} \, . \tag{4}$$

Again, the buyer with the most reliable deficit wins the auction.

3.2 One seller with multiple units of energy surplus and $N - 1$ buyers, each with multiple units of energy deficit

In this subsection, we extend our previous model to one seller with multiple units of uncertain energy surplus, and $N - 1$ buyers, with multiple units of uncertain energy deficit each.

Reliability probability vector: The seller with an energy surplus of at most K units, is characterized by a reliability probability vector $\mathbf{r}_s = [r_s^1, \cdots, r_s^K]$ and each buyer i, with an expected energy deficit up to L_i units, is characterized by a reliability vector $\mathbf{r}_{d_i} = [r_{d_i}^1, \ldots, r_{d_i}^{L_i}]$, respectively. For the seller, r_s^k is her probability for actually producing unit k conditioned on the fact that she has already produced $(k-1)$ units of surplus. Similarly, for buyer i, $r_{d_i}^l$ is her probability for actually having a deficit of unit l conditioned on the fact that she has already consumed $(l-1)$ units of energy deficit. The entries of both vectors above are assumed to be non-increasing in k and l, respectively. Intuitively, producing or consuming an extra unit of energy is no more likely than producing or consuming the previous one. Therefore, the aforementioned reliability vectors have non-increasing entries with k (number of surplus units) and l (number of deficit units) respectively. If the existence of the k^{th} unit of seller's energy surplus is independent from the previous $(k - 1)$ units, then r_s^k is equal to the product of the marginal probabilities that each unit will arise, i.e. of the conditional probabilities given that all other units prior to that have arisen.

3.2.1 *VCG auction mechanism.*

Among the different auction mechanisms we choose to build on the multi-object, sealed-bid, VCG auction, to allocate the units of energy surplus among the participating prosumers, for the following reasons: (i) the socially optimal solution is achieved, (ii) truth-telling is buyers' dominant strategy and (iii) the sealed-bid mechanism preserves bidders' anonymity.

An alternative would be to run separate parallel or sequential Vickrey auctions. However, running separate parallel auctions or a sequential auction (e.g. one for each unit of energy surplus sold) could make buyers more conservative, and it could cause inefficiencies since such auctions are strategically complicated. Therefore, it might happen that more reliable units of energy surplus are bought by prosumers with less reliable energy deficits. In addition, one–round pay–your bid is not efficient due to bid shading, i.e. placing bids below what a bidder believes that the good is worth.

Bidding formulation: The bid offer by each buyer i can be formulated as an $L_i \times K$ matrix $\mathbf{b}_i$, where the k^{th} element of the l^{th} row is the buyer's willingness to pay for the k^{th} unit of energy surplus sold by the seller in order to cover her l^{th} deficit unit.

$$\mathbf{b}_i = \begin{bmatrix} b_i^{1,1} & b_i^{1,2} & \cdots & \cdots & \cdots & b_i^{1,K} \\ 0 & b_i^{2,2} & \cdots & \cdots & \cdots & b_i^{2,K} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & b_i^{L_i,L_i} & \cdots & b_i^{L_i,K} \end{bmatrix}$$

Element $b_i^{l,k}$ is given by an equation such as (1), namely:

$$b_i^{l,k} = r_s^k r_{d_i}^l p_{retail} + r_s^k(1 - r_{d_i}^l) p_{FIT} - (1 - r_s^k) r_{d_i}^l p_{retail} \, . \tag{5}$$

Since bidding is truthful, the right–hand quantity equals the a priori value of buyer i for the k^{th} surplus unit, which will be required only if the previous $(l - 1)$ deficit units occur. To simplify these formulas, we set $2r_s^k p_{retail} - r_s^k p_{FIT} - p_{retail} = A_k$ and $r_s^k p_{FIT} = B_k$ for each prosumer k and equation (5) becomes:

$$b_i^{l,k} = A^k r_{d_i}^l + B^k \, . \tag{6}$$

The values A_k and B_k are common for all buyers and depend only on the reliability probability r_s^k of the k^{th} unit sold; thus, A_k and B_k are also non-increasing in k and thus $b_i^{l,k} \geq b_i^{l,k+1}$, i.e. the buyer places a lower bid for a less reliable unit of energy surplus. Also, $r_{d_i}^l$ is non-increasing in l and $b_i^{l,k} \geq b_i^{l+1,k}$, i.e. the bid placed by a buyer to cover a less reliable unit of her own deficit is lower.

Allocation rule: We specify an intuitive iterative procedure leading to the optimal allocation. The units of energy surplus are allocated sequentially, i.e. the mechanism proceeds with allocating the $(k + 1)^{st}$ unit of energy surplus only after the k^{th} surplus unit sold is already allocated to a buyer, which implies that all previous units $1, \cdots, k$ of surplus of the seller are already allocated. Similarly, the $(l + 1)^{st}$ unit of energy deficit of buyer i will be covered provided that her l^{th} unit of energy deficit is already covered, i.e. a unit of energy surplus is already allocated to buyer i in order to cover her l^{th} unit of energy deficit. This implies that all units $1, \cdots, l$ of deficit of this buyer are already covered. The allocation mechanism of the auction can be depicted under Algorithm 1 below.

Algorithm 1 Allocation mechanism

```
Initialization. The reliability probability vector of the seller $\mathbf{r}_s = (r_s^k : k = 1, ...K)$ is announced, i.e. the reliability probabilities of each surplus unit. Each buyer $i$ responds with a bidding strategy $\mathbf{b}_i$ taking into account her expected amount of deficit and the respective reliability probabilities. The units are allocated sequentially, and the allocation completes in $K$ steps.
for $k = 0 : K - 1$ do
    Sort competing bids of buyers claiming the $(k+1)^{st}$ unit of surplus in decreasing order, taking into account how the previous $k$ units have been allocated, i.e. that fact that each buyer $i$ has $l_i$ deficit units in the previous rounds. For each buyer $i$ the competing bids are the elements $b_i^{l_i+1,k}$ of $\mathbf{b}_i$.
    Select the highest bid $b_j^{l_j+1,k}$ of a buyer $j$ that wins the $(k+1)$ unit of energy surplus.
end for
```

VCG payment rule of our auction: Each winning bidder i has to pay the social *opportunity* cost of her winning, i.e. *the sum of bids of the auction from the second best combination of bids, i.e. when bidder i is excluded), minus what other bidders have bid in the current (optimal) combination of bids.*

In our case, the total payment of each winning bidder i for the amount of units of energy surplus allocated to her equals the total harm (i.e., loss of utility) caused by her bid to all other bidders compared to the second best combination of bids which emerges when bidder i is excluded. Thus, we have to calculate the marginal harm caused to other bidders, and then take the sum of these terms. To this end, the following two cases need to be considered:

- If buyer j (due to exclusion of bidder i) wins a certain surplus unit for the first time, then the marginal harm caused to her equals her valuation for the surplus unit she won.
- If buyer j (due to the exclusion of bidder i) wins a more reliable unit of energy surplus, then her marginal harm caused equals the difference between her valuation for the unit she obtained when bidder i is excluded, and her valuation for the unit she obtained when bidder i participates in the market.

Note that we have to compute as many terms as the number of units won by bidder i. Hence, the payment of each winning bidder i is at most as high as the sum of her original winning bids.

Property 1. *With the proposed VCG auction, the most reliable surplus units are allocated to with the most reliable deficit units.*

Proof. We provide a sketch of a proof by discussing a simple, yet illustrative example. Consider a p2p energy market with one seller, two buyers and external prices p_{FIT} and p_{retail}. The seller has an energy surplus of $K = 2$ units with reliability probability vector $\mathbf{r}_s = [r_s^1, r_s^2]$, and buyers 1 and 2 have energy deficits of $L_1 = L_2 = 2$ units with reliability probability vectors $\mathbf{r}_1 = [r_{d_1}^1, r_{d_1}^2]$ and $\mathbf{r}_2 = [r_{d_2}^1, r_{d_2}^2]$, respectively. Also, without loss of generality we assume that $r_{d_1}^1 > r_{d_2}^1 > r_{d_1}^2 > r_{d_2}^2$. Based on the allocation mechanism introduced above, the units of energy surplus are allocated as follows:

The competing bids for the first unit of energy surplus are $b_1^{1,1} = A_1 \cdot r_{d_1}^1 + B_1$, and $b_2^{1,1} = A_1 \cdot r_{d_2}^1 + B_1$. The *first unit of energy surplus* with reliability probability r_s^1 is allocated to buyer 1 to cover her first unit of energy deficit with reliability probability $r_{d_1}^1$, since $r_{d_1}^1 > r_{d_2}^1 > r_{d_1}^2 > r_{d_2}^2$ and, thus, $\max\{A_1 \cdot r_{d_1}^1 + B_1, A_1 \cdot r_{d_2}^1 + B_1\} = A_1 \cdot r_{d_1}^1 + B_1$. Note that the bids for the first unit of energy surplus differ only in the reliability probability of the unit of energy deficit $r_{d_i}^1$ that participating buyers wish to cover. Thus, the winning (i.e., the highest) bid is the one for the most reliable unit of energy deficit, which is covered by the most reliable unit of energy surplus.

Next, the active bids for the second unit of energy surplus are $b_1^{2,2} = A_2 \cdot r_{d_1}^2 + B_2$, since buyer 1 has already covered her first unit of energy deficit and $b_2^{2,1} = A_2 \cdot r_{d_2}^1 + B_2$. The *second unit of energy surplus* with reliability probability r_s^2 is allocated to buyer 2 to cover her first unit of energy deficit with reliability probability $r_{d_2}^1$, since $\max\{A_2 \cdot r_{d_1}^2 + B_2, A_2 \cdot r_{d_2}^1 + B_2\} = A_2 \cdot r_{d_2}^1 + B_2$. The bids for the second unit of energy surplus differ only in the reliability probabilities of the units of energy deficit that participating buyers wish to cover, i.e. $r_{d_1}^2$ and $r_{d_2}^1$. Since the most reliable unit of energy deficit is already covered, the winner is buyer 2 with the second most reliable unit of energy deficit, which is to be covered by the second most reliable unit of energy deficit.

For this example, we deduce that for the proposed allocation method, the attained social welfare (which equals the sum of the winning bids) is the optimal one. This is because the unit of energy surplus with the highest reliability probability, i.e. the one with highest A_k, B_k, is matched to the unit of energy deficit with the highest reliability probability. Also, the unit of surplus with the second highest reliability probability is matched to the unit of deficit with the second highest reliability probability. □

The price paid by buyer 1 for one unit of energy surplus is $p_1 = (A_1 \cdot r_{d_2}^1 + B_1 + A_2 \cdot r_{d_2}^2 + B_2) - (A_2 \cdot r_{d_2}^1 + B_2)$. The first term corresponds to the sum of bids from the second best combination of bids, i.e. when bidder 1 is excluded, and the second term corresponds to what bidder 2 has bid in the current (optimal) combination of bids. Similarly, the price paid by buyer 2 for one surplus unit is $p_2 = (A_1 \cdot r_{d_1}^1 + B_1 + A_2 \cdot r_{d_1}^2 + B_2) - (A_1 \cdot r_{d_1}^1 + B_1)$. Again, the first term corresponds to the sum of bids from the second best combination of bids (bidder 2 is excluded), and the second term corresponds to what bidder 1 has bid in the current combination of bids.

3.3 $S > 1$ sellers, each with multiple units of energy surplus, and $N - S$ buyers, each with multiple units of energy deficit

Now we study the most general case that may arise in p2p energy trading, where S sellers offer multiple units of energy surplus each, and $(N - S)$ buyers wish to cover multiple deficit units each.

Reliability probability vector: Each seller $j \in S$, with an energy surplus up to K_j units, is characterized by a reliability probability vector $\mathbf{r}_{s_j} = [r_{s_j}^1, \ldots, r_{s_j}^{K_j}]$, and each buyer i, with an energy deficit up to L_i units, is characterized by a reliability vector

$\mathbf{r}_{d_i} = [r^1_{d_i}, \dots, r^{L_i}_{d_i}]$. The maximum total number of surplus units is $M = \sum_{j=1}^{S} K_j$ and for the energy deficit is $D = \sum_{i=1}^{N-S} L_i$.

We proceed with adjusting the multi-object sealed-bid VCG auction mechanism to this scenario with uncertain prosumers, in order to allocate the units of energy surplus among participating prosumers. The units are sold in M steps. The reliability probability vector of each seller j is announced, and then these units of energy surplus are sorted in decreasing order of reliability. The reliability probability vector $\mathbf{r}_S = [r^1, \dots, r^M]$ includes the surplus units of all prosumers, where units of different prosumers are sorted as discussed above. Therefore, each buyer i responds with a her bidding strategy $\mathbf{b}_i$, taking into account her amount of energy deficit and the respective reliability probabilities.

Bidding formulation: The bid offer by each participating buyer i is a $L_i \times M$ matrix, where the m^{th} element of the l^{th} row is the buyer's willingness to pay in order to buy the m^{th} unit of energy surplus sold to cover her l^{th} unit of energy deficit. Again, each element $b_i^{m,l}$ is given by equation (1),

$$b_i^{l,m} = r_m r^l_{d_i} p_{retail} + r_m (1 - r^l_{d_i}) p_{FIT} - (1 - r^m) r^l_{d_i} p_{retail} \cdot \tag{7}$$

This amount equals the a priori value of buyer i for the m^{th} surplus unit sold, taking into account the fact that this unit of deficit will actually be required only if the previous $(l-1)$ units of energy deficit are required as well. Similarly to section 3.2.1, for each unit of energy surplus offered, A_m and B_m are common for all buyers, since they depend only on the reliability of the m^{th} unit sold. Also, A_m and B_m are non-increasing in m and thus, $b_i^{l,m} \geq b_i^{l,m+1}$. Similarly, $r^l_{d_i}$ is non-increasing in l and thus, $b_i^{l,m} \geq b_i^{l+1,m}$.

Allocation Rule: The units of energy surplus that are sorted from most reliable to least reliable are allocated sequentially. The mechanism proceeds with the $(m+1)^{st}$ unit of energy surplus only if the m^{th} unit of energy surplus sold is already allocated to one of the buyers, which in fact implies that all units $1, \cdots, m$ of surplus of this seller are already allocated. Similarly, the $(l+1)^{st}$ unit of energy deficit of buyer i will be covered provided that her l^{th} unit of energy deficit is already covered, i.e. one unit of energy surplus is already allocated to buyer i to cover her l^{th} unit of energy deficit, which in fact implies that all units $1, \cdots, l$ of deficit of this seller are already covered. The competing bids of all participating buyers competing for the $(m+1)^{st}$ unit of energy surplus are sorted, taking into account how the previous m units have been allocated, i.e. that fact that each buyer i has secured up to that stage already l_i units of energy surplus. Thus, competing bids are the elements $b_i^{l_i+1,m}$. Then, the highest bid $b_i^{l_i+1,m}$ of buyer i is selected, and she wins the $(m+1)th$ unit of energy surplus. Following the rationale explained under Property 1, it can be proved that *the allocation mechanism proposed is the optimal one.* Note that the payment rule is the same as the one in subsection 3.2. Therefore, the total payment per winner can be computed as explained therein.

P2p pricing scheme: As already explained, from the VCG payment rule presented above, we obtain the total amount that is paid by a winning buyer for the set of units obtained. Since we have a multi-seller setting, this payment should to be allocated among respective sellers whose units were bought from the winning buyer, so that each one covers a certain unit of her energy deficit.

Let $p^{l,k}_{i,j}$ be the price to be paid to seller j for her k^{th} unit of energy surplus which is bought by buyer i to cover her l^{th} unit of energy deficit. This price should conform to the following constraints: (i) $p^{l,k}_{i,j} \geq r^k_{s_j} \cdot p_{FIT}$, which is this seller's reserve price for this unit of energy surplus and (ii) $p^{l,k}_{i,j} \leq b^{l,k}_i$, which is this buyer's maximum willingness to pay for this unit of energy surplus to cover her l^{th} unit of energy deficit. As discussed above, these constraints are always compatible if and only if $r^k_{s_j} \geq 1/(2-a)$, where $a = p_{FIT}/p_{retail}$. In particular, our p2p pricing rule allocates to each deficit unit bought the respective reserve price plus a fraction of what remains from the total VCG payment after extracting the reserve prices of all units bought. For each unit, this fraction is proportional to the "margin" between the maximum and minimum acceptable prices. Instead of giving the general formula, we provide a simple example, so as to introduce the payment allocation rule that leads to the proposed p2p pricing scheme.

Example: Assume that the winning buyer 1 won two units of energy surplus to cover her first and second unit of energy deficit, one unit being the first surplus unit of seller m with reliability probability $r^1_{s_m}$ and the other unit being the first deficit unit of seller n with reliability probability $r^1_{s_n}$. Then the total VCG payment p_1 of winning buyer 1 will be split among the sellers m and n as follows:

$$p^{1,1}_{1,m} = \tilde{P}^1_m + \frac{b^{1,1}_1 - \tilde{P}^1_m}{b^{1,1}_1 - \tilde{P}^1_m + b^{2,1}_1 - \tilde{P}^1_n} \cdot (p_1 - \tilde{P}^1_m - \tilde{P}^1_n) \tag{8}$$

$$p^{2,1}_{1,n} = \tilde{P}^1_n + \frac{b^{2,1}_1 - \tilde{P}^1_n}{b^{1,1}_1 - \tilde{P}^1_m + b^{2,1}_1 - \tilde{P}^1_n} \cdot (p_1 - \tilde{P}^1_m - \tilde{P}^1_n) \tag{9}$$

where $\tilde{P}^k_j = r^k_{s_j} \cdot p_{FIT}$. Since $b^{1,1}_1 \geq \tilde{P}^1_m$, $b^{2,1}_1 \geq \tilde{P}^1_n$ and $p_1 \leq b^{1,1}_1 + b^{2,1}_1$, it follows that the proposed allocation rule satisfies the aforementioned constraints. Also, the proposed rule attains fairness among sellers that are to be paid, since each seller will receive a price that is increasing in the buyer's willingness to pay, which in turn is increasing to the seller's reliability. Due to this rule, we argue that our mechanism is a p2p energy trading mechanism under which a certain unit of energy surplus is matched to a certain deficit unit and the trading is carried out under an individual p2p price.

Remark I: In a real system the probabilities of the number of units of deficit and surplus of each participating prosumer would be determined on the basis of historical information regarding the demand and the production of a prosumer.

Remark II: The setting above would also apply in the presence of a set of aggregators, each one representing a set of prosumers. Each aggregator could divide the total energy surplus/deficit into several units and participate in the market as a single "prosumer".

4 NUMERICAL EVALUATION

We compare the proposed VCG auction to an appropriately designed double auction mechanism, which is used as a p2p energy trading mechanism, taking into account the uncertainty in the declared surplus and deficit defined in section 3.

4.1 Double auction mechanism for uncertain supply and demand

In a double auction, each buyer i submits her bid b_i and each seller j announces her reserve price s_j. Buyers' bids are placed in descending order and sellers' reserve prices are placed in ascending order. Then, the break-even index k is found, such that $b_k \geq s_k$ and $b_{k+1} < s_{k+1}$, and the first k sellers sell the units of energy to the first k buyers. There are several types of double auctions, which differ in the payment rule. A payment rule should satisfy the following properties [14]: (i) *Individual Rationality:* no player should lose from joining the auction, (ii) *Strong Budget Balanced:* all monetary transfers must be done between buyers and sellers (iii) *Economic Efficiency:* the social welfare should be the best possible and (iv) *Truthfulness:* buyers and sellers should have no incentive to misreport their valuations. However, according to the Myerson – Satterthwaite theorem, it is not possible to achieve all these requirements in the same pricing mechanism [15].

A payment rule that has the three of the four desirable properties above for p2p market clearing is a double auction with the average pricing mechanism pricing, i.e. a uniform price $p_{p2p} = (b_k + s_k)/2$, where k is the break-even index. This double auction has the following properties: (i) *Individual Rationality,* (ii) *Strong Budget Balance* and (iii) *Economic Efficiency.* However, it is *not truthful.* Indeed, buyer k and/or seller k have an incentive to misreport, and since they are not aware of their rank until the market is cleared, all participating peers have an incentive to misreport.

Valuations: We consider again that each winning buyer will pay for each unit of energy she obtained in advance, even if she finally does not consume the unit. If the unit is not consumed eventually, then the buyer will be able to resell it directly to the grid at price p_{FIT}. Thus, the valuation (i.e., utility) of buyer i to cover her l^{th} unit of energy deficit with reliability probability $r^l_{d_i}$ is equal to: (i) buyer's willingness to pay if she had to buy that unit from the grid; plus (ii) the anticipated revenue the buyer will obtain if she finally does not consume the unit and resell it to the grid.

$$v_i = r^l_{d_i} \cdot p_{retail} + (1 - r^l_{d_i}) \cdot p_{FIT} = p_{FIT} + r^l_{d_i} \cdot (p_{retail} - p_{FIT}) \ . \quad (10)$$

Please note that, contrary to the VCG auction, this expression is independent of the seller's reliability.

Furthermore, the *expected net benefit* of a winning buyer for covering her l^{th} unit of energy deficit consists of three terms: (i) the buyer's maximum willingness to pay in order to cover her expected unit energy deficit, which will arise with probability $r^l_{d_i}$ (which equals the amount she would pay if she had to buy one unit of energy directly form the grid, i.e. p_{retail}), minus (ii) the amount she will pay for winning the auction, plus (iii) the amount of money she will receive by reselling the unit of energy to the grid if she does not consume it, i.e. p_{FIT}, (with probability $(1 - r^l_{d_i})$.

$$NB^{da}_{buyer_{i,l}} = r^l_{d_i} \cdot p_{retail} - p_{p2p} + (1 - r^l_{d_i}) \cdot p_{FIT} \ . \quad (11)$$

Similarly, each winning seller will receive a payment for each unit of energy she sold in advance, even if she finally does not produced the unit. If the unit of energy is not produced, then the seller will have to purchase the unit of energy she promised from the grid at price p_{retail}. Therefore, the value of seller j who offers her m^{th} unit of energy surplus with reliability $r^m_{s_j}$ is equal to: (i) the amount of money she would receive if she sold the unit directly to the grid and (ii) the amount of money she would spent to buy the unit of energy from the grid in case she does not produce it.

$$v_j = r^m_{s_j} \cdot p_{FIT} + (1 - r^m_{s_j}) \cdot p_{retail} = p_{retail} + r^m_{s_j} \cdot (p_{FIT} - p_{retail}) \ . \quad (12)$$

Contrary to VCG auction, this expression is independent of the buyer's reliability. Also, v_j is the reserve price of seller j.

Thus, the *expected net benefit* of a winning seller for selling her m^{th} unit of energy surplus consists of three terms: (i) the amount she will receive for winning the auction, minus (ii) the amount of money she will pay to buy a unit of energy from the grid if she does not produce it, i.e. p_{retail}, (with probability $(1 - r^m_{s_j})$), minus (iii) the amount she would spent if she had to sell one unit of energy directly to the grid, i.e. p_{FIT} (with probability $r^m_{s_j}$).

$$NB^{da}_{seller_{j,m}} = p_{p2p} - (1 - r^m_{s_j}) \cdot p_{retail} - r^m_{s_j} \cdot p_{FIT} \ . \quad (13)$$

4.2 Proposed VCG auction vs. double auction

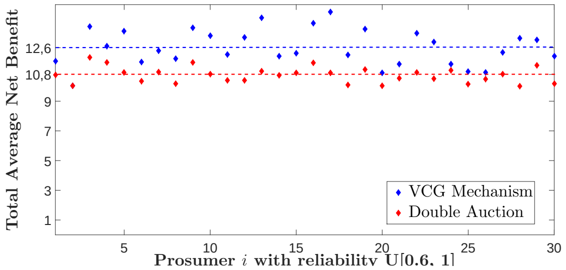
We consider meaningful cases where the retail price exceeds by far the FITs; namely, set that $p_{retail} = 24.6\ c/kWh$ and $p_{FIT} = 8\ c/kWh$. The market prices, i.e. retail prices and FIT rates, give the feasible region for the p2p price, i.e. $p_{FIT} \leq p2p \leq p_{retail}$. We have $N = 30$ prosumers that participate in the p2p energy market. Please note that the number of prosumers does not affect the qualitative outcomes of the mechanisms.

For the comparison, we calculate the *actual net benefit* for each prosumer i. To do that for the VCG auction, we take into account: (i) winning prosumers, (ii) the allocation of each unit of energy surplus of each winning seller to a certain unit of energy deficit of a winning buyer, (iii) the individual p2p pricing scheme according to the VCG payment rule and (iv) the actual energy surplus and actual energy deficit units that are eventually produced and consumed. Similarly, for the double auction we take into account: (i) winning prosumers, (ii) the uniform price that results from the average mechanism pricing rule and (iii) the actual energy surplus and actual energy deficit units that are eventually produced and consumed. These units are simulated on the basis of the reliability probabilities of the units that are cleared in the mechanisms.

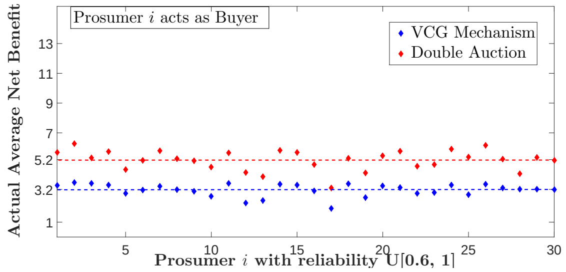
The procedure above is repeated for 100 iterations in order to estimate the *average value* of the actual net benefit obtained by each prosumer i, for both mechanisms, i.e. the net benefit after the realization of energy surplus and deficit that were actually produced and consumed. In each iteration: (i) prosumers switch between being a buyer and being a seller with probability 0.57 and 0.43, respectively and (ii) each prosumer expects to have an energy surplus or deficit of 3 units at most, if she acts as a seller or buyer, respectively. We select these values of probabilities (rather than 0.5 and 0.5) in order to investigate the more interesting case where available units of energy surplus are a bit less than those of energy deficit. Prosumers that are not selected for p2p energy trading trade directly with the grid and, thus, their net benefit is equal to zero. Under this simulation setting we study the following three cases:

4.2.1 *Prosumers with uniformly distributed reliability in* $[0.6, 1.0]$.

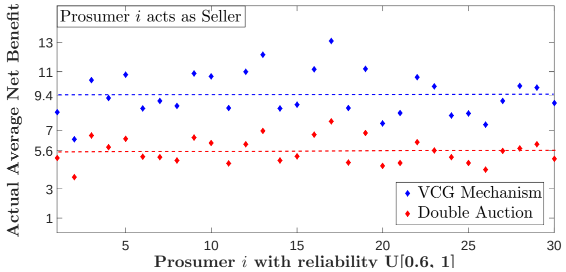
First, we assume that the reliability of the units of energy surplus or deficit of each prosumer is U$[0.6, 1.0]$. In Figure 2 we compare the proposed mechanisms in terms of average actual net benefit of



(a) Prosumer i switches between buyer and seller



(b) Prosumer i acts as buyer



(c) Prosumer i acts as seller

Figure 2: Average net benefit for prosumers with reliability $U[0.6, 1.0]$ in VCG mechanism and double auction.

each prosumer i. We observe that when prosumers act as sellers (Figure 2c) they obtain a net benefit about 40% higher in the VCG auction than in the double auction. This applies because in the VCG auction sellers face no risk, while in double auction, if a seller fails to produce the promised unit of energy, she has to purchase an equal amount from the grid at high price p_{retail}.

When prosumers act as buyers (Figure 2b), they gain a net benefit about 38% higher in the double auction. In the VCG auction buyers entail the risk to buy energy from the grid, if the unit or units they have bought to cover their deficit are not produced eventually. Finally, as each prosumer switches between being a buyer and being a seller with probabilities 0.57 and 0.43, respectively, (Figure 2a), we observe that the VCG auction outperforms the double auction. This indicates that the individual pricing scheme arising from the VCG payment rule is more efficient than the uniform pricing rule of the double auction. *Overall, the VCG auction we propose performs better in terms of average consumer net benefit.*

4.2.2 Prosumers classified according to their reliability. We assume that: (*i*) the reliability of prosumers $1, \ldots, 15$ is $\mathrm{U}[0.6 - 0.8]$ and (*ii*) the reliability prosumers $16, \cdots, 30$ is $\mathrm{U}[0.8 - 1.0]$. The reliability range characterizes the prosumer as an actor and is fixed whether she acts as a buyer or seller.

In Figure 3 we compare the proposed mechanisms in terms of average actual net benefit of each prosumer i. A general observation from Figure 3a-3c is that more reliable prosumers, i.e. the ones that are mostly selected to trade their energy surplus or deficit, obtain higher net benefits in both mechanisms proposed.

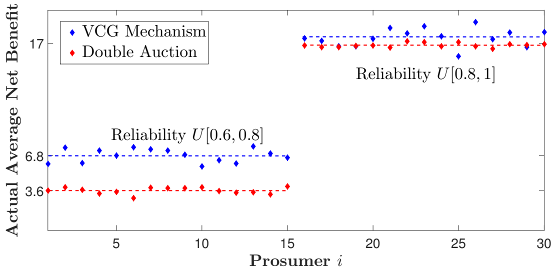
Figure 3c shows again that, when prosumers act as sellers, they obtain a higher net benefit in the VCG auction than in double auction, since sellers face no risk in the VCG auction. On the other hand, prosumers are better off in the double auction when they act as buyers (Figure 3b). However, less reliable prosumers are only slightly better off in the double auction. For this class of prosumers, the internal p2p price that results from the double auction is very close to their valuation, i.e. it is substantially high for them, and thus it negatively impacts their net benefit. As a result, as the prosumers that are less reliable switch between being buyer and seller (Figure 3a), they gain an about 53.3% higher net benefit in the VCG auction, while more reliable prosumers acquire almost the same average net benefit in both auctions, with VCG being slightly preferable. *Overall, all prosumers are better off under the VCG auction, but reliable ones are only slightly better off in the VCG auction.*

4.2.3 Misreporting prosumers in the double auction. As already mentioned, in a double auction with the average pricing mechanism, both buyers and sellers have an incentive to misreport their valuations in order to achieve higher net benefits. We examine whether misreporting in the double auction enables prosumers to achieve higher net benefits than those in our proposed VCG auction. Prosumers' reliability parameters are $\mathrm{U}[0.6, 1]$.

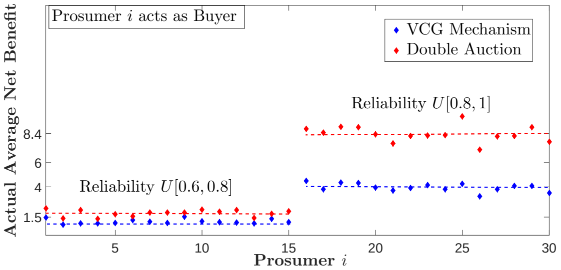
First, we assume that only sellers misreport their values in order to achieve higher net benefits. In Figure 4a we depict the average actual net benefit of prosumers as a function of sellers' increasing deviation from truthful bidding in double auction. The black horizontal lines show the prosumer average net benefit obtained from the VCG mechanism. We observe that the sellers' average net benefit increases as misreporting from truthful bidding increases up to 45%. Additional deviation does not affect the average net benefit, since the increased reserve prices prevent some sellers from being selected and thus, the sellers' average net benefit slightly decreases. On the other hand, the buyers' average net benefit decreases due to the increasing prices that occur from sellers' misreporting. As a result, prosumer average net benefit does not change when she switches between being a buyer and a seller.

In Figure 4b, the buyers deviate from truthful bidding in double auction. The buyers' average net benefit increases as misreporting from truthful bidding increases up to 30% since the clearing price decreases. For higher deviations from truthful bidding the average net benefit of buyers' decreases, since the low bidding offers prevent buyers from being selected for trading. The seller's average net benefit decreases due to the decreasing prices that occur from buyers' misreporting. As a result, prosumer average net benefit does not change when she switches between buyer and seller.

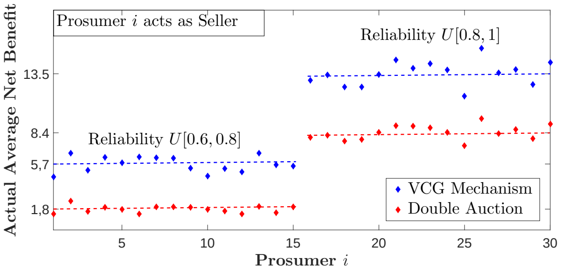
Even if buyers or sellers misreport, they do not exceed the net benefit they gain in the VCG mechanism. Of course, if both buyers and sellers misreport, i.e. buyers declare higher bids and sellers



(a) Prosumer i switches between buyer and seller



(b) Prosumer i acts as buyer



(c) Prosumer i acts as seller

Figure 3: Average net benefit for prosumers classified as more and less reliable in VCG mechanism and double auction.

lower reserve prices, the opposite effects in the clearing price will not affect prosumer's net benefit.

Main takeaways from our numerical study:

(1) The VCG auction performs better than the double auction in terms of average net consumer benefit.

(2) When prosumers are classified based on their reliability, all of them are better off under the VCG auction, but more reliable ones are only slightly better off in the VCG auction.

(3) Even if buyers or sellers misreport in the double auction, they do not attain a net benefit which is higher than what they gain in the VCG auction proposed.

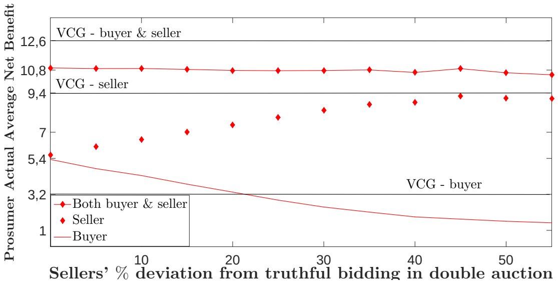
5 COMPARISON WITH RELATED WORK

VCG-based mechanisms have already been proposed in the bibliography for demand side management programs [17], [18]. Customers are requested to provide their energy demand information, which is then used by a centralized algorithm for the price calculation, for each time. Payment design ensures participants truthfulness. In our work, prosumers submit their bids and offers considering their reliability probability parameters. Our mechanism guarantees buyers' truthfulness. The authors in [16] design a combinatorial exchange in a general framework, i.e., an iterative version of a one-shot mechanism that allocates energy surplus to buyers to satisfy budget-balance and individual rationality. At each round the allocation and a price for each item are announced, giving buyers and sellers a feedback to modify their bids until convergence is met.

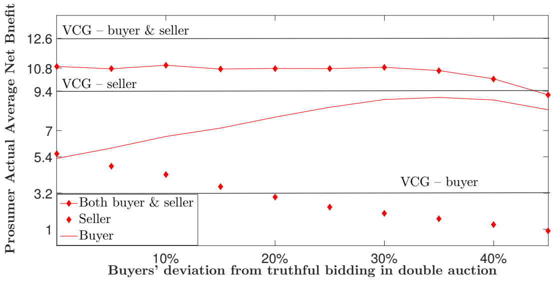
One of the main challenges in our setup is the *uncertainty in the declared amount of energy surplus or deficit*, due to unpredictable consumer demand and RES generation. Uncertainty in resource supply has been studied in a general economic framework in [5],with two suppliers one expensive but reliable, and a cheaper but unreliable. The buyer decision is on the appropriate amounts of good to purchase from each supplier. Uncertainty is modeled as a probability of carrying out the order. The authors in [13] propose bilateral contracts as a p2p market design for real-time and forward markets. In forward markets price uncertainty is treated as random variable. Through a price-adjustment process, the agents agree on contracts that none of them wishes to mutually deviate from. In [2],the authors model and analyze Nega-Watt markets taking into account the arising uncertainty due to possible non-engagement of consumers for load reduction. In this work, we take a step ahead from the bibliography discussed above, by *proposing a multi-seller variant of the VCG auction mechanism* as the p2p trading mechanism, that maximizes social welfare by ensuring participants' truthfulness, while taking into account the non-trivial assumption of uncertainty of declared amounts of surplus or deficit.

In the last few years, a significant number of *pilot projects in p2p energy trading* have been deployed. For example, Piclo (UK) and Vandebron (Netherland) are p2p energy trading platforms that enable consumers to buy electricity directly from local independent renewable energy suppliers. Trading prices are set by the producers, while consumers benefit by shaping a clean energy image and having reduced electricity fees [21]. Another p2p trading platform runs for Brooklyn microgrid and enables the coordination of distributed energy resources (DERs) to maintain continuity of supply when the microgrid is disconnected from the main grid [12]. Our mechanism is practical and applicable in realistic environments, it is low-overhead and has desirable properties such as social welfare maximization and truthfulness of participating prosumers.

The *p2p energy sharing mechanisms* allocate the revenues and cost savings among all prosumers and usually give an advantage to those that contribute more to the market. The authors in [8], [3] study different p2p energy sharing schemes such as: (*i*) bill sharing, (*ii*) Mid–Market–Rate (MMR) pricing, (*iii*) purchase costs of energy equally split and (*iv*) costs proportionally split according to each user's default cost. The authors in [20] deal with cooperative prosumer participation in p2p energy sharing and show that cooperation is sustainable if the applied internal pricing scheme lies between the FIT rate and the retail price. In [7], the authors incorporate demand response into p2p energy sharing and investigate a dynamic pricing scheme based on the ratio of supply and demand during each time-slot. In addition, the value of battery flexibility in p2p energy sharing is studied in [9] where a two-stage control method is proposed to realize p2p energy sharing in a community with photovoltaic-battery systems. These energy sharing schemes



(a) Only sellers misreport



(b) Only buyers misreport

Figure 4: Prosumer average net benefit as function of deviation from truthful bidding.

stem from top-down approaches (e.g., a central coordinator decides on the pricing scheme, or how to allocate costs and revenues), according to a reasonable yet not justified criterion, which does not guarantee socially optimal outcomes. *These works do not fully exploit the inherent advantage of p2p trading negotiations, which is to elicit the group of prosumers that can provide energy more efficiently and to discover the prices necessary to incentivize their delivery.*

In existing *p2p energy trading mechanisms*, a set of prosumers is selected to trade their energy surplus or energy deficit through the mechanism, while the internal p2p prices are elicited as outcome of the mechanism. Specifically, the authors in [6], [4] argue that a discrete-time, double-auction is an appropriate mechanism for p2p energy trading, since both buyers and sellers can submit their bids/offers. In [6], the auction is defined as a balanced one, when the total revenue of sellers is equal to the total cost savings of buyers. To achieve balance, a payment rule is proposed according to which the trading price for each buyer-seller pair is equal to the mean of the buyer's bid and the seller's reserve price. However, as explained in subsection 4.1 this payment rule does not ensure truthfulness of participating consumers. Note that this is the mechanism with which we have already compared ours (subsection 4.2). In [4] a double auction for near real-time and forward markets with uniform pricing mechanism is proposed. They use device-oriented bidding strategies (for buyers) and a probabilistic model for PV generation (for sellers). However, it is not specified what would happen if a seller does not deliver the promised amount of energy. P2p energy trading has been also studied in the EV domain [1]. The p2p trading system is designed in two steps: (i) an optimization algorithm for each driver which minimizes the electricity cost paid by each EV driver in the time and space dimensions and (ii) a p2p energy trading system among EVs parked in the same zone.

These works do not address uncertainty in energy surplus and deficit, which is a major challenge in p2p energy trading. Also, the double auctions proposed do not guarantee truthfulness of participating peers. In our work we explicitly introduce supply and demand uncertainty in the utility functions of prosumers which change their bidding strategies, and the VCG auction ensures that truthfulness and social welfare maximization are still guaranteed.

6 CONCLUSIONS

In this paper, we propose an innovative variant of a multi-seller VCG auction as a p2p energy trading mechanism. The major novelty in our setting lies in capturing the uncertainty that the declared amount of energy surplus and deficit will be eventually produced or consumed. We proposed a simple allocation rule for this auction which provably leads to maximization of the expected social welfare. Our VCG auction constitutes a p2p energy trading mechanism that allocates certain amounts of energy surplus across prosumers with certain amounts of energy deficit while individual p2p prices are elicited as outcome of the mechanism. Compared to a double auction specifically designed to capture uncertainty, our mechanism performs better in terms of average net consumer benefit. Furthermore, more reliable prosumers are better off in both mechanisms. Finally, even if buyers or sellers in a double auction may misreport their valuation, their net benefit does not exceed the one derived in the proposed VCG auction.

Independent instances of our mechanism can be run sequentially in different slots, without requiring any further modification. Furthermore, our model can be extended to include storage devices at prosumer's premises (e.g. batteries and EVs) that will affect the consumption and production of the prosumer and her available flexible loads. In this setting, the prosumers' optimal strategies include charging and discharging decisions of storage elements as well, and appropriate approximations thereof should be developed and thus constitutes a very interesting direction for future research.

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