PROMISE: A Framework for Truthful and Profit Maximizing Spectrum Double Auctions

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***Abstract*—Auctions provide a platform for licensed spectrum users to trade their underutilized spectrum with unlicensed users. Existing spectrum auctions either do not apply to the scenarios where multiple sellers and buyers both make offers, or assume the knowledge of the users' valuation distribution for maximizing the profit of the auction. To fill this void, we design PROMISE, a framework for spectrum double auctions, which jointly considers spectrum reusability, truthfulness, and profit maximization without the distribution knowledge. We propose a novel technique, called *cross extraction*, to compute the bid representing a group of secondary users, who can share a common channel. We prove that PROMISE is computationally efficient, individual-rational, and truthful. In addition, PROMISE is guaranteed to achieve an approximate profit of the optimal auction.**

1. Introduction

Spectrum is a very scarce resource. On the one hand, the exploding spectrum-based services and devices make the available unlicensed spectrum become extremely scarce. On the other hand, large blocks of licensed spectrum, e.g. TV channels, are underutilized [15]. With the advances in the innovative cognitive radio technology [24], Dynamic Spectrum Access techniques enable unlicensed users (secondary users) to take advantage of the idle spectrum from the licensed users (primary users) and make more productive use of limited spectrum resources. One of the main barriers lying between the fast growing bandwidth demand and the efficient utilization of the licensed spectrum is the lack of good trading forms. A fair and efficient trading form provides incentives to licensed users and encourages them to trade their unused spectrum with unlicensed users for achieving win-win situations.

Auction has been used as one of the trading forms throughout history and adopted by the Federal Communications Commission (FCC) to distribute spectrum for long term leases. It allows competitive price discovery, and fair and efficient resource allocation [11]. In this paper, we model the spectrum auction involving both primary users and secondary users as a *double auction*. In this auction, each primary user names a price for leasing its channel, and each secondary user gives price that it is willing to pay for using a channel. The objective is to design a double auction that allocates the secondary users to the channels, and computes the payment to each primary user and the payment from each secondary user. In addition, it is desirable for the auction to satisfy the following properties: 1) *computationally efficient*: the outcome of the auction can be computed in polynomial time, 2) *individual-rational*: each agent can expect a non-negative utility by reporting its true valuation, 3) *truthful*: no agent can improve its utility by reporting its valuation dishonestly, and 4) *profit maximizing*: the difference between the payment from all the secondary users and the payment to all the primary users is maximized. Different from the existing spectrum auctions [1, 4, 8, 9, 12, 20–23, 27, 28], we jointly consider the above four properties in the *prior-free* situation where the knowledge of valuation distribution is not available. Some existing profit maximizing single-sided auctions, where only buyers bid for channels, assume that the bidders' valuations are independent and drawn from a known prior distribution [1, 8, 9]. In some cases, it is not easy or practical to determine the valuation distribution [5].

Challenges arise while designing an auction satisfying all the desirable properties, especially when the *spatial reusability* of the spectrum needs to be considered. A group of secondary users can share one common channel as long as they do not interfere with each other. Therefore, these users should be considered together to bid one channel. However, different group formations may result in different profits for the auction. Even when the group formation is given, it is not clear how to compute one bid, referred to as group bid, based on the bids of the secondary users in the group to represent the group. A trade-off must be made between profit maximization and truthfulness. To find this trade-off, we propose a novel technique, called *cross extraction*. It preserves as much profit as possible while still guaranteeing the truthfulness of the auction. To measure the performance of an auction in terms of the achieved profit, we use the notion of *competitive auction* [5]. An auction is β-competitive if it achieves at least $\frac{1}{\beta}$ of the optimum.

The main contributions of this paper are as follows:

- To the best of our knowledge, we are the *first* to design a spectrum double auction, which jointly considers the truthfulness and profit maximization, without requiring the knowledge of the valuation distribution.
- We design a novel technique, called *cross extraction*, to decide the bid representing a group of secondary users who can share a common channel.
- We propose PROMISE, a general framework for truthful and profit maximizing spectrum double auction. PROMISE jointly considers the spatial reusability of the spectrum and the profit maximization.
- We prove that PROMISE is computationally efficient, individual-rational, truthful, and competitive.

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The remainder of this paper is organized as follows. In Section 2, we briefly review the spectrum auctions in the current literature. In Section 3, we introduce the system model and describe desirable properties of an auction. In Section 4, we discuss the challenges for designing a spectrum double auction satisfying the desirable properties. We present our spectrum double auction in details in Section 5. In Section 6, we analyze PROMISE and prove its properties. We evaluate the performance of PROMISE and compare it with existing auctions in Section 7. We conclude this paper in Section 8.

2. Related Work

As pioneers in spectrum auction design, Zhou *et al.* [27] designed VERITAS, the first truthful auction considering the spectrum reusability and computation efficiency. In [8], based on the concept of virtual valuation and assuming the knowledge of the bidders' valuation distribution, Jia *et al.* designed an exponential time VCG-based auction to maximize the expected revenue, and a polynomial time auction without the revenue guarantee. Along this line, Al-Ayyoub and Gupta [1] designed a polynomial time spectrum auction that yields approximated expected revenue. Kakhbod *et al.* [9] also considered spread spectrum, where the primary user allocates transmission power to secondary users, and interpreted the solutions to the revenue-maximization auction design. In [23], Wu and Vaidya studied the scenario where the owner of the spectrum has a reserved price for each of the channels. They designed a truthful auction, called SMALL, to guarantee that the owner's utility is non-negative. Following the same design methodology, Wei *et al.* [22] designed SHIELD that improves spectrum utilization and buyer satisfaction compared with VERITAS and SMALL. Inspired by the group-buying service on the Internet, Lin *et al.* [12] designed a three-state auction, called TASG, to enable group-buying among secondary users. TASG allows a leader in each group to conduct an outer auction for aggregating the bids within the group. In [6], Gopinathan and Li studied spectrum auctions with prior-free setting and designed a truthful auction to approximately maximize the revenue.

TRUST [28] is the first truthful double auction designed for spectrum trading. It follows the same methodology of McAfee [14] and was proved to be individual-rational, budget-balanced, and truthful. Feng *et al.* [4] extended to heterogeneous spectrum auctions and designed TAHES. In [20], a double truthful auction, called DOTA, was proposed to allow each user to bid for more than one channel. Considering the fact that secondary users may join the network in an online fashion, Wang *et al.* [21] designed TODA, a general framework for truthful online double auction for spectrum allocation. In [25], Xu *et al.* modeled the spectrum trading as a multi-unit double auction, and designed a truthful auction to maximize the total utility gains obtained by all participating users.

Table 1 summarizes the differences between the most related works and the auction designed in this paper. Other related auctions can be found in [18, 29]. In [18], Subramanian *et al.* proposed efficient spectrum allocation auctions to maximize the revenue without considering the truthfulness. In [29], Zhu *et al.* extended the spectrum auction to multi-hop secondary networks with the objective of maximizing the social welfare, i.e., the total valuation of all the secondary users. They first presented a simple heuristic auction without providing any guarantee on social welfare, and then designed a linear programming based auction with proven approximation.

TABLE 1
COMPARISON WITH EXISTING AUCTIONS

	Double auction	Prior-free	Profit/Revenue
VERITAS [27]	✗	✓	✗
JZZ [8]	✗	✗	✓
AG [1]	✗	✗	✓
KNT [9]	✗	✗	✓
SMALL [23]	✗	✓	✗
SHIELD [22]	✗	✓	✗
TASG [12]	✗	✓	✗
GL [6]	✗	✓	✓
TRUST [28]	✓	✓	✗
TAHES [4]	✓	✓	✗
DOTA [20]	✓	✓	✗
TODA [21]	✓	✓	✗
XJL [25]	✓	✓	✗
PROMISE[this paper]	✓	✓	✓

3. System Model

We consider a cognitive radio network consisting of m primary users $\mathcal{P} = \{P_1, P_2, \ldots, P_m\}$ and n secondary users $\mathcal{S} = \{S_1, S_2, \ldots, S_n\}$. Each primary user provides one channel for lease at a certain price, and each secondary user is interested in obtaining a channel by paying a price. The secondary users do not differentiate one channel from another. Each primary user P_i has a valuation c_i on its channel, representing the cost of possessing the channel, e.g., the cost of the spectrum license. Each secondary user S_i has a valuation ν_i on any channel. The valuation is private to the user itself and not public to others.

Exploiting *spatial reusability* can increase the efficiency of spectrum utilization. As wireless transmissions take place over a common medium, two users transmitting on the same channel may interfere with each other. To characterize this interfering relationship among the secondary users, we assume that the *conflict graph* is available. Take the protocol interference model as an example. The conflict graph is a graph $G(V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ denotes a set of vertices, and E is a set of undirected edges representing interference information. Vertex $v_i \in V$ represents secondary user S_i. There is an edge $(v_i, v_j) \in E$ if and only if secondary users S_i and S_j interfere with each other.

Given such a network, we are interested in designing a trading form such that the primary users can trade their channels with the secondary users. Auction is a widely adopted trading form, which allows competitive price discovery and fair and efficient resource allocation [11]. Following the terminology in auction theory, we call the primary users *sellers* as they provide channels to sell, and the secondary users *buyers* as they intend to purchase channels from the primary users. Throughout this paper, we use primary user and seller, and secondary user and buyer interchangeably. We call both the sellers and the buyers *agents* when referring to them in general. The government regulator, e.g. the FCC in the United States, serves as an *auctioneer* to conduct the auction. As the input to the auction, each seller P_i submits a price a_i, less than which it would not be willing to sell its channel. Each buyer S_i submits a price b_i, more than which it would not be willing to pay. We refer the price submitted by the seller and the buyer as *ask* and *bid*, respectively. We use $\mathbf{a} = (a_1, a_2, \ldots, a_m)$ and $\mathbf{b} = (b_1, b_2, \ldots, b_n)$ to denote the ask vector and bid vector submitted by all the sellers and buyers, respectively. Let $\mathbf{a}_{-i}$ denote the ask vector with seller P_i's ask removed, i.e., $\mathbf{a}_{-i} = (a_1, \ldots, a_{i-1}, ?, a_{i+1}, \ldots, a_m)$. Let $\mathbf{a}|^i a$ denote the ask vector with seller P_i's bid substituted by a, i.e., $\mathbf{a}|^i a = (a_1, \ldots, a_{i-1}, a, a_{i+1}, \ldots, a_m)$. Similarly, $\mathbf{b}_{-i}$ and $\mathbf{b}|^i b$ can be defined accordingly. In this paper, we design the spectrum auction as a *single-round sealed-bid double auction*, which is described as follows:

- Given an ask vector $\mathbf{a} = (a_1, a_2, \ldots, a_m)$ and a bid vector $\mathbf{b} = (b_1, b_2, \ldots, b_n)$, the mechanism $\mathcal{M}$ computes the outcome, consisting of the allocation vectors $\mathbf{x} \in \{0,1\}^m$ and $\mathbf{y} \in \{0,1\}^n$, and payment vectors $\mathbf{q} \in \mathbb{R}_+^m$ and $\mathbf{p} \in \mathbb{R}_+^n$ ($\mathbb{R}_+$ is the set of nonnegative real numbers), subject to the constraint that the number of winning buyers is equal to the number of winning sellers, i.e., $\sum_{i=1}^m x_i = \sum_{i=1}^n y_i$.
- The allocation vector $\mathbf{x}$ is the indicator for the sellers. If $x_i = 1$, seller i wins and receives q_i from the auctioneer. Otherwise, seller i loses and receives $q_i = 0$. The allocation vector $\mathbf{y}$ is the indicator for the buyers. If $y_i = 1$, buyer i wins and pays p_i to the auctioneer. Otherwise, buyer i loses and pays $p_i = 0$.
- The profit of the auction is $\mathcal{M}(\mathbf{a}, \mathbf{b}) = \sum_{i=1}^n p_i - \sum_{i=1}^m q_i$.

Note that the allocation vectors $\mathbf{x}$ and $\mathbf{y}$, and the payment vectors $\mathbf{q}$ and $\mathbf{p}$ are functions of $(\mathbf{a}, \mathbf{b})$, i.e., $\mathbf{x} = \mathbf{x}(\mathbf{a}, \mathbf{b})$. However, for notational simplicity, we omit the parameters when the context is clear.

To accommodate the spatial reusability of the spectrum, the allocation indicator for the buyers should be a matrix $\mathbf{z} \in \{0,1\}^{n \times m}$ instead of a vector, where $z_i^k = 1$ indicates that buyer S_i is assigned to seller P_k. The allocation indicators $\mathbf{x}$ and $\mathbf{z}$ have to satisfy three conditions: 1) $z_i^k + z_j^k \le 1$ for all k whenever $(v_i, v_j) \in E$, 2) $\sum_{k=1}^m z_i^k \le 1$ for any S_i, and 3) $x_k = 1$ if and only if $\sum_{i=1}^n z_i^k \ge 1$ for any P_k. The first condition ensures that any two secondary users assigned to the same channel are interference-free. The second condition ensures that a secondary user is assigned to no more than one channel. The third condition ensures that the number of channels assigned to winning buyers is equal to the number of winning sellers.

We make the following assumptions: 1) agents do not collude; 2) agents have full knowledge about how the auction works, but have no knowledge about the private valuations and bids of other agents; and 3) agents are selfish and interested in only maximizing their own utility functions. The *utility of primary user* P_i is

$$u_i = q_i - x_i c_i, \tag{3.1}$$

meaning that $u_i = q_i - c_i$ if P_i wins, and 0 otherwise. The *utility of secondary user* S_i is

$$\mu_i = \sum_{k=1}^m z_i^k \nu_i - p_i, \tag{3.2}$$

meaning that $\mu_i = \nu_i - p_i$ if S_i wins, and 0 otherwise. Note that although it should be $u_i = u_i(\mathbf{a}, \mathbf{b})$ and $\mu_i = \mu_i(\mathbf{a}, \mathbf{b})$, we omit the parameters for notational simplicity when the context is clear.

Desirable Properties: Since the design of an auction is heavily dependent on desired properties, we introduce four desirable properties in the following.

- **Computational Efficiency:** An auction is *computationally efficient* if the outcome can be computed in polynomial time.
- **Individual Rationality:** An auction is *individual-rational* if each agent is guaranteed to have a non-negative utility value by reporting its true valuation. That is $q_i(\mathbf{a}|^i c_i, \mathbf{b}) - c_i \ge 0$ for each primary user P_i, and $\nu_i - p_i(\mathbf{a}, \mathbf{b}|^i \nu_i) \ge 0$ for each secondary user S_i.
- **Truthfulness:** An auction is *truthful* (or *strategy-proof*) if each agent maximizes its utility by reporting its true valuation. That is for any P_i, $u_i(\mathbf{a}|^i c_i, \mathbf{b}) \ge u_i(\mathbf{a}|^i a_i, \mathbf{b})$ for any a_i, and for any S_i, $\mu_i(\mathbf{a}, \mathbf{b}|^i \nu_i) \ge \mu_i(\mathbf{a}, \mathbf{b}|^i b_i)$ for any b_i.
- **Profit Maximization:** The profit of auction $\mathcal{M}(\mathbf{a}, \mathbf{b}) = \sum_{i=1}^n p_i - \sum_{i=1}^m q_i$ is maximized.

4. Design Challenges

We first introduce the notion of *bid/ask* independence and the definition of bid/ask-independent double auction in [5].

Definition 4.1 ([5]): Let θ^s and θ^b be functions from ask vectors and bid vectors to prices, respectively. The *bid/ask-independent double auction* $\mathcal{M}_{(\theta^s, \theta^b)}$ defined by θ^s and θ^b is as follows: For each seller i, if $a_i \le \theta^s(\mathbf{a}_{-i}, \mathbf{b})$, set $x_i = 1$ and $q_i = \theta^s(\mathbf{a}_{-i}, \mathbf{b})$. For each buyer i, if $b_i \ge \theta^b(\mathbf{a}, \mathbf{b}_{-i})$, set $y_i = 1$ and $p_i = \theta^b(\mathbf{a}, \mathbf{b}_{-i})$. □

One cornerstone of double auction design is given in [3].

Theorem 1 ([3]): A double auction is truthful if and only if it is bid/ask-independent. □

As a potential performance metric, we first present an optimal auction $\mathcal{M}_{opt}$ achieving the maximum profit.

$$\begin{aligned}
\max \quad & \textstyle\sum_{i=1}^n \sum_{k=1}^m z_i^k b_i - \sum_{k=1}^m x_k a_k \\
s.t. \quad & \textstyle\sum_{k=1}^m z_i^k \le 1, \ \forall 1 \le i \le n && (4.1)\\
& z_i^k + z_j^k \le 1, \ \forall (v_i, v_j) \in E, \forall 1 \le k \le m && (4.2)\\
& x_k = 1, \text{ if } \textstyle\sum_{i=1}^n z_i^k \ge 1 && (4.3)\\
& z_i^k \in \{0,1\}, x_k \in \{0,1\}, \ \forall 1 \le i \le n, 1 \le k \le m && (4.4)
\end{aligned}$$

In order to compare the performance of one auction with another, we adopt the concept of *competitive auction* in [5], which is motivated by competitive analysis of online algorithms.

Definition 4.2 ([5]): Let $\beta > 0$ be any given constant. A double auction $\mathcal{M}$ is *β-competitive* against auction $\mathcal{M}'$ if, for any ask vector $\mathbf{a}$ and bid vector $\mathbf{b}$, $\mathcal{M}(\mathbf{a}, \mathbf{b}) \ge \frac{\mathcal{M}'(\mathbf{a}, \mathbf{b})}{\beta}$, where $\beta \ge 1$ is called the *competitive ratio* of $\mathcal{M}$ against $\mathcal{M}'$. □

The following auction is necessary to connect the performance of any truthful deterministic auction with the optimal auction.

Definition 4.3 ([3]): The *optimal single price auction* mechanism, $\mathcal{M}_s$, is the mechanism that pays all winning sellers the same payment and charges all winning buyers the same price, such that the profit is maximized. The profit of $\mathcal{M}_s$ is

$$\mathcal{M}_s(\mathbf{a}, \mathbf{b}) = \max_i i(B^*_{(i)} - a_{(i)}), \tag{4.5}$$

where $B^*_{(i)}$ is the i-th largest group bid representing the group of buyers who are assigned to one channel, and $a_{(i)}$ is the i-th smallest ask. □

Note that we will discuss the grouping and group bid computation later in this section.

Theorem 2: For the spectrum auction, it is *impossible* to design a deterministic truthful double auction $\mathcal{M}(\mathbf{a}, \mathbf{b})$ such that $\mathcal{M}(\mathbf{a}, \mathbf{b}) \geq \frac{\mathcal{M}_{opt}(\mathbf{a},\mathbf{b})}{n \ln(n+1)}$. □

Proof: Consider a network where all the secondary users interfere with each other, i.e., the conflict graph is a complete graph. Hence no two secondary users can be assigned to the same primary user. In addition, we assume that $a_i = 0$ for all primary users and $m \geq n$. The spectrum double auction on this network is equivalent to the basic auction [5], i.e., $\mathcal{M}(\mathbf{a}, \mathbf{b}) = \mathcal{M}(\mathbf{b})$. By Theorem 4.1 of [5], we know that for any deterministic truthful auction $\mathcal{M}$ on the above network, $\mathcal{M}(\mathbf{a}, \mathbf{b}) \leq \frac{\mathcal{M}_s(\mathbf{a},\mathbf{b})}{n}$. Now assume that $b_i = \frac{1}{i}$ for $1 \leq i \leq n$. We then have $\frac{\mathcal{M}_s(\mathbf{a},\mathbf{b})}{\mathcal{M}_{opt}(\mathbf{a},\mathbf{b})} = \frac{1}{\mathcal{M}_{opt}(\mathbf{a},\mathbf{b})} = \frac{1}{\sum_{i=1}^n b_i} < \frac{1}{\ln(n+1)}$. Therefore $\mathcal{M}(\mathbf{a}, \mathbf{b}) \leq \frac{\mathcal{M}_s(\mathbf{a},\mathbf{b})}{n} < \frac{\mathcal{M}_{opt}(\mathbf{a},\mathbf{b})}{n \ln(n+1)}$. ■

By Theorem 2, we know that no deterministic truthful auction can be competitive against the optimal auction with the maximum profit. In addition, $\mathcal{M}_{opt}$ is *untruthful*, since agents can rig their payments by reporting dishonestly, and thus increase their utility values. Theorem 2 forces us to focus on the design of randomized auctions. We say an auction is *randomized* if the computation of the outcome is randomized [5]. If an auction is randomized, the allocation indicators, the payment vectors, and the profit of the auction are all random variables. A randomized auction is truthful if and only if it can be described as a probability distribution over deterministic truthful auctions. Correspondingly, a randomized auction $\mathcal{M}$ is β-competitive against $\mathcal{M}'$ if $E[\mathcal{M}(\mathbf{a}, \mathbf{b})] \geq \frac{\mathcal{M}'(\mathbf{a},\mathbf{b})}{\beta}$, for any $\mathbf{a}$ and $\mathbf{b}$.

Unfortunately, by Lemma 3.5 of [5] and using the same network in the proof of Theorem 2, we also know that no truthful randomized double auction can be β-competitive against $\mathcal{M}_s$ for any constant $\beta \geq 1$.

Theorem 3: Let $\mathcal{M}$ be a randomized truthful double auction for the spectrum auction. For any constant $\beta \geq 1$, there always exists a network, an ask vector $\mathbf{a}$, and a bid vector $\mathbf{b}$, such that $E[\mathcal{M}(\mathbf{a}, \mathbf{b})] < \frac{\mathcal{M}_s(\mathbf{a},\mathbf{b})}{\beta}$. □

Therefore we focus our attention on designing randomized truthful auctions that are competitive against $\mathcal{M}^{[2]}_s(\mathbf{a}, \mathbf{b})$, which is similar to that adopted by Deshmukh *et al.* [3]. $\mathcal{M}^{[2]}_s(\mathbf{a}, \mathbf{b})$ makes trading successful between at least two pairs of buyer group and seller, i.e.,

$$\mathcal{M}^{[2]}_s(\mathbf{a}, \mathbf{b}) = \max_{i \geq 2} i(B^*_{(i)} - a_{(i)}). \tag{4.6}$$

Now the question is how to group the secondary users and compute the group bids. Combining these two operations to optimize (4.6) is likely impossible for the following reasons:

- Finding m groups of interference-free secondary users with maximum cardinality cannot be approximated within n^ϵ in general conflict graphs for some $\epsilon > 0$ [7, 13].
- Computing a single group with maximum total bids is NP-hard, even in a conflict graph modeled by unit disk graph [2].
- By Theorem 1, grouping the secondary users based on the bids or computing a bid-dependent group bid may lead to untruthful behavior from the secondary users.

Therefore, we separate the operations of grouping the secondary users and computing the group bids. Due to Theorem 1, our buyer grouping algorithm is bid-independent, which is also adopted by [28]. The optimal group bid is defined as follows for the similar reason as in Theorem 2.

$$B^*_i = \mathcal{F}(\{b_j | S_j \in \mathcal{S}_i\}), \tag{4.7}$$

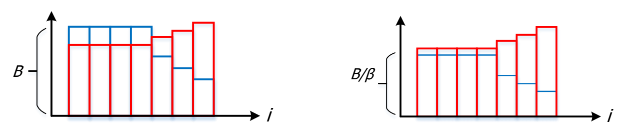
where

$$\mathcal{F}(\mathbf{b}) = \max_{1 \leq i \leq |\mathbf{b}|} i b_{(i)}, \tag{4.8}$$

$|\mathbf{b}|$ denotes the number of elements in vector $\mathbf{b}$, and $b_{(i)}$ is the i-th largest bid in $\mathbf{b}$.

We first show that the value computed in (4.7) cannot be used as the group bid. Assume that $\mathcal{S}_1 = \{S_1, S_2\}$ with bids $b_1 = 5$, $b_2 = 2$, and $\mathcal{S}_2 = \{S_3\}$ with bid $b_3 = 3$. Only one winning group is allowed. We have $B^*_1 = 5$ and $B^*_2 = 3$. Thus $\mathcal{S}_1$ wins, and S_1 is solely responsible for the payment of $\mathcal{S}_1$. Now S_1 changes its bid from $b_1 = 5$ to $b'_1 = 3$. We have $B^*_1 = 4 > B^*_2$, and S_1 and S_2 share the payment of $\mathcal{S}_1$. S_1 decreases its payment by cheating. Hence the auction is untruthful.

Next we show that any approximation of (4.7) would not work either. Assume that there is an algorithm that can compute a β-approximation of (4.7), i.e., $B_i \geq \frac{B^*_i}{\beta}$, while guaranteeing the truthfulness of the auction. Then there exist an ask vector and a bid vector, as in Fig. 1, such that the optimal profit is positive, but the profit using the approximated group bids is 0. Thus no auction based on the approximate group bids is competitive.



(a) Optimal group bid (b) Approximated group bid

Fig. 1. Approximated group bid cannot be competitive. Red bars denote sellers' asks sorted in a non-decreasing order. Blue bars denote buyers' bids sorted in a non-increasing order.

To decide the group bid for each group, we need to make sure that the computation is independent of the bids in the group while keeping the optimal group bids as much as possible. To this end, we introduce the *cross extraction* technique. We still compute the optimal group bids as in (4.7) for all groups. Instead of using B^*_i as its group bid, group $\mathcal{S}_i$ is uniformly at random assigned the optimal bid B^*_j from another group, where $i \neq j$. We will prove in Section 5 that, based on the cross extraction technique, our designed auction is *computationally efficient*, *individual-rational*, *truthful*, and *competitive*.

5. Design of PROMISE

In this section, we design a PROfit MaxImizing SpEctrum double auction (PROMISE), which achieves computational efficiency, individual rationality, truthfulness, and competitiveness.

A. Overview and Rationale

PROMISE is based on the double auction designed by Deshmukh *et al.* [3], which does not consider the spatial reusability of the spectrum. PROMISE jointly considers the spatial reusability of the spectrum, the strategic behavior of the agents, and the profit maximization for the auctioneer. In order to incorporate the spatial reusability of the spectrum into the auction, we adopt bid-independent grouping algorithms. A key ingredient of our auction is the algorithm for computing the group bid for each group, which preserves a portion of the optimal group bid while guaranteeing the truthfulness. On a high level, our auction consists of three steps.

Step 1. Group Formation:

Without considering the truthfulness of the auction, the group formation problem is equivalent to the *Maximum Weighted K-Colorable Subgraph* (MWKCS) problem. The unweighted *Maximum K-Colorable Subgraph* (MKCS) problem is to determine the maximum number of vertices that can be colored with K colors subject to the condition that no two adjacent vertices are colored the same [26]. The MKCS problem is NP-hard, since it includes as a special case the Maximum Independent Set problem which is known as an NP-hard problem. Time complexity is not the only issue that hinders the application of the algorithms for the MWKCS problem. As proved by Theorem 1, bid-dependent grouping algorithm may make the auction vulnerable to the dishonest behavior from the buyers. Therefore we adopt bid-independent grouping algorithms as in [28]. For example, the Max-IS algorithm [19] and the greedy algorithms in [17] are all good candidates.

Step 2. Group Bid Computation:

Since the buyers in the same group bid together as a group, we treat each group as a *super buyer*. We need to compute a representative bid for each group as its group bid. As proved and discussed in Section 4, it is only possible to compete against an auction with B_g^*, computed by (4.7) as the group bid. However, taking B_g^* as the group bid of group $\mathcal{S}_g$ makes it possible for a buyer to lower its payment and thus increase its utility by lying about its true valuation. To overcome these issues, we introduce a novel technique, called *cross extraction*, to compute the group bid. Instead of using its optimal group bid B_g^*, each group $\mathcal{S}_g$ is uniformly at random assigned an optimal group bid $B_{g'}^*$ of a different group $\mathcal{S}_{g'}$. The auctioneer then attempts to extract the group bid $B_{g'}^*$ from the buyers in $\mathcal{S}_g$. If successful, group $\mathcal{S}_g$ will proceed to the next step of the auction. Otherwise, it is disqualified from being a winner group. Note that the extraction is successful if and only if $B_g^* \geq B_{g'}^*$. With the cross extraction technique, we not only achieve bid-independent group bid computation, but also preserve a portion of the optimal group bids.

Step 3. Winner Determination and Pricing:

With the buyer groups formed and the group bids computed, our winner determination and pricing step is based on the double auction in [3]. Note that the payment of each buyer is the maximum between the payment to fulfill the extracted group bid and the one computed while considering the sellers.

B. Design Details

We now present our auction, named PROMISE, in details.

In Step 1, we apply bid-independent grouping algorithms, e.g., Max-IS algorithm [19], to form interference-free buyer groups. Let $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l$ be the l groups. In TRUST [28], the group bid B_g, $1 \leq g \leq l$, is computed as $B_g = \min_{S_i \in \mathcal{S}_g} b_i |\mathcal{S}_g|$. There are two drawbacks to this group bid computation.

- The auction is untruthful since the group formation in Step 1 is *public* to all agents. TRUST assumes that it is *private* to the bidders. Because the buyer with the minimum bid in the group might lower its bid and thus lower its payment, while still making the group win the auction.
- This group bid is not the maximum group bid under the single price mechanism.

In our auction, we first compute the optimal group bid for each group using (4.7). We then decide the final group bid using cross extraction technique. Before describing this technique, we introduce the Profit Extract algorithm in Algorithm 1, denoted by $ProEx$ (Algorithm 1), to make our paper self-contained. The Profit Extract algorithm was designed by Goldberg *et al.* [5], which in turn is based on the cost-sharing technique due to Moulin and Shenker [16]. The basic idea of $ProEx$ is: *Given a set of submitted values and a target profit R, it finds the largest π such that the highest π agents can equally share the target profit and the equal share is no more than any of their submitted values*. Two important properties are: 1) $ProEx$ is truthful; and 2) If $\mathcal{F}(\mathbf{h}) \geq R$, $\sum_{e_i \in \mathcal{E}^w} p_i = R$; otherwise it produces no winners and no profit.

Algorithm 1: $ProEx(\mathcal{E}, \mathbf{h}, R)$

Input: set of agents $\mathcal{E}$ and submitted values $\mathbf{h}$
Output: winning agents $\mathcal{E}^w \subseteq \mathcal{E}$ and payment p_i for each agent e_i in $\mathcal{E}$

$p_i \leftarrow 0$ for each agent $e_i \in \mathcal{E}$, $\mathcal{E}^w \leftarrow \emptyset$;
Find the largest π such that $h_{(\pi)} \geq \frac{R}{\pi}$, where $h_{(i)}$ is the i-th largest value in $\mathbf{h}$, $1 \leq i \leq |\mathbf{h}|$;
if π *exists* **then**
    $\mathcal{E}^w \leftarrow \{e_{(i)}\}_{1 \leq i \leq \pi}$, where $e_{(i)}$'s submitted value is $h_{(i)}$;
    $p_i \leftarrow \frac{R}{\pi}$ for all $e_i \in \mathcal{E}^w$;
end
return $(\mathcal{E}^w, \{p_i | e_i \in \mathcal{E}\})$

In Step 2, as shown in Algorithm 2, we first compute the optimal achievable group bid for each group based on (4.7), as in Line 2. We then uniformly at random map each group $\mathcal{S}_{(g)}$, whose optimal group bid is $B_{(g)}^*$, to an optimal group bid $B_{(r(g))}^*$ of another group. To check whether this group is qualified for Step 3, we apply $ProEx$ algorithm to the buyers in $\mathcal{S}_{(g)}$ with a target profit $B_{(r(g))}^*$. If the extraction is not successful, all the buyers in the group will be eliminated from the potential winning buyer set.

In Step 3, we determine the winners and the payments based on the double auction in [3]. One building block of this step is the basic auction. The basic auction is originally designed for the scenario where only buyers are involved in the auction, and

Algorithm 2: $GBC(\mathbf{b}, \{\mathcal{S}_g\}_{1\le g\le l}, s)$

Input: bid vector $\mathbf{b}$, groups $\{\mathcal{S}_g\}_{1\le g\le l}$, and s
Output: group bids $\{B_{(g)}\}_{1\le g\le s}$, intra-group winners $\{\mathcal{S}^w_{(g)}\}_{1\le g\le s}$, and payments $\{\tilde{p}_i\}_{1\le i\le n}$

$\tilde{p}_i \leftarrow 0$ for $1 \le i \le n$;
$B^*_g \leftarrow \mathcal{F}(\{b_i | S_i \in \mathcal{S}_g\}), \forall 1 \le g \le l$;
for $g \leftarrow 1$ **to** s **do**
  $(\mathcal{S}^w_{(g)}, \{\tilde{p}_i | S_i \in \mathcal{S}_{(g)}\}) \leftarrow ProEx\left(\mathcal{S}_{(g)}, \{b_i | S_i \in \mathcal{S}_{(g)}\}, B^*_{(r(g))}\right)$, where $r(1), r(2), \ldots, r(s)$ is a random permutation of $1, 2, \ldots, s$ subject to the constraint that $r(g) \ne g$;
  $B_{(g)} \leftarrow \sum_{S_i \in \mathcal{S}_{(g)}} \tilde{p}_i$;
end
return $(\{B_{(g)}\}_{1\le g\le s}, \{\mathcal{S}^w_{(g)}\}_{1\le g\le s}, \{\tilde{p}_i\}_{1\le i\le n})$

thus the objective of the auctioneer is to collect as much profit from them as possible. Take, for instance, the RSEP auction proposed by Goldberg *et al.* [5]. We illustrate the pseudo code of RSEP in Algorithm 3. Note that the auction based on the computation of $\mathcal{F}(\mathbf{h})$ would not be truthful, while the truthful auction would not know the value of $\mathcal{F}(\mathbf{h})$. The essence of the basic auction is to use *random sampling profit extraction* to estimate the value of $\mathcal{F}(\mathbf{h})$. Let k be the value such that $\mathcal{F}(\mathbf{h}) = kh_{(k)}$, and let $\{e_{(i)}\}_{1\le i\le k}$ be the agents with the largest k bids. It is proved that the extracted profit is at least $\frac{1}{4}$ of $\mathcal{F}(\mathbf{h})$ [5]. Thus the competitive ratio of RSEP is $\beta = 4$.

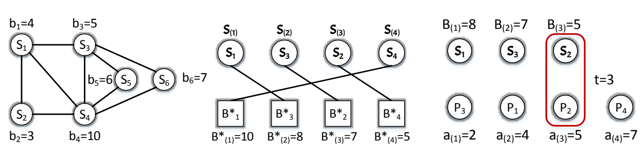
Algorithm 3: Basic Auction $\mathcal{A}(\mathcal{E}, \mathbf{h})$

Input: agent set $\mathcal{E}$ and submitted values $\mathbf{h}$
Output: winning agents $\mathcal{E}^w$ and payment p_i for each agent e_i in $\mathcal{E}$

$p_i \leftarrow 0$ for each agent $e_i \in \mathcal{E}$;
Partition $\mathbf{h}$ uniformly at random into $\dot{\mathbf{h}}$ and $\ddot{\mathbf{h}}$;
$\dot{R} \leftarrow \max_i i\dot{h}_{(i)}$ and $\ddot{R} \leftarrow \max_i i\ddot{h}_{(i)}$, where $\dot{h}_{(i)}$ and $\ddot{h}_{(i)}$ are the i-th largest value in $\dot{\mathbf{h}}$ and $\ddot{\mathbf{h}}$, respectively. Let $\dot{\mathcal{E}}$ and $\ddot{\mathcal{E}}$ denote the agents with corresponding values $\dot{\mathbf{h}}$ and $\ddot{\mathbf{h}}$, respectively;
if $\dot{R} < \ddot{R}$ **then**
  $(\mathcal{E}^w, \{p_i | e_i \in \ddot{\mathcal{E}}\}) \leftarrow ProEx(\ddot{\mathcal{E}}, \ddot{\mathbf{h}}, \dot{R})$;
else
  $(\mathcal{E}^w, \{p_i | e_i \in \dot{\mathcal{E}}\}) \leftarrow ProEx(\dot{\mathcal{E}}, \dot{\mathbf{h}}, \ddot{R})$;
end
return $(\mathcal{E}^w, \{p_i | e_i \in \mathcal{E}\})$

Finally, we present PROMISE in Algorithm 4.

C. A Walk-Through Example



(a) Conflict graph (b) Cross extraction (c) $t = 3$

Fig. 2. Walk-through example

To illustrate the basic idea of PROMISE, we use a toy example with 4 sellers and 6 buyers. The sellers' asks are $a_1 = 4, a_2 = 5, a_3 = 2$, and $a_4 = 7$. The buyers' bids are $b_1 = 4, b_2 = 3, b_3 = 5, b_4 = 10, b_5 = 6$, and $b_6 = 7$. The conflict graph of buyers is shown in Fig. 2(a).

Algorithm 4: PROMISE

Input: primary user set $\mathcal{P}$, secondary user set $\mathcal{S}$, ask vector $\mathbf{a}$, bid vector $\mathbf{b}$, bid-independent grouping algorithm $\mathcal{G}$, and basic auction $\mathcal{A}$
Output: winning sellers $\mathcal{P}^w$, winning buyers $\mathcal{S}^w$, payment vector $\mathbf{q}$, and payment vector $\mathbf{p}$

$\mathcal{P}^w \leftarrow \emptyset$, $\mathcal{S}^w \leftarrow \emptyset$, $\mathbf{q} \leftarrow \mathbf{0}_n$, $\mathbf{p} \leftarrow \mathbf{0}_m$;
Let $P_{(i)}$, $1 \le i \le m$, denote the seller, whose ask is $a_{(i)}$;
Apply $\mathcal{G}$ to group buyers into $\{\mathcal{S}_g\}_{1\le g\le l}$, $s \leftarrow \min\{m, l\}$;
$(\{B_{(g)}\}_{1\le g\le s}, \{\mathcal{S}^w_{(g)}\}_{1\le g\le s}, \{\tilde{p}_i\}_{1\le i\le n}) \leftarrow GBC(\mathbf{b}, \{\mathcal{S}_g\}_{1\le g\le l}, s)$;
Let t be the largest integer such that $B_{(t)} \ge a_{(t)}$;
if $t = 2$ **then**
  $\mathcal{P}^w \leftarrow \{P_{(1)}\}$, $\mathcal{S}^w \leftarrow \mathcal{S}^w_{(1)}$;
  $q_{(1)} \leftarrow a_{(t)}$, $p_i \leftarrow \max\{\frac{B_{(2)}}{|\mathcal{S}^w_{(1)}|}, \tilde{p}_i\}$ for all $S_i \in \mathcal{S}^w_{(1)}$;
else if $t > 2$ **then**
  $\mathbf{a}' \leftarrow \{B_{(t)} - a_{(i)}\}_{1\le i\le t}$, $\mathbf{a}'' \leftarrow \{a'_{(i)}\}_{1\le i\le t-1}$;
  $\mathbf{B}' \leftarrow \{B_{(g)} - a_{(t)}\}_{1\le g\le t}$, $\mathbf{B}'' \leftarrow \{B'_{(g)}\}_{1\le i\le t-1}$;
  $\mathbb{S} \leftarrow \{\mathcal{S}^w_{(g)}\}_{1\le g\le t-1}$;
  /* $U(0,1)$ is a uniform distribution function on the interval $[0,1]$ */
  if $U(0,1) \le \frac{1}{2}$ **then**
    $(\mathbb{S}^w, \{\rho_{(g)} | \mathcal{S}^w_{(g)} \in \mathbb{S}\}) \leftarrow \mathcal{A}(\mathbb{S}, \mathbf{B}'')$, $\tilde{n} \leftarrow |\mathbb{S}^w|$;
    For each $\mathcal{S}^w_{(g)} \in \mathbb{S}^w$ and each $S_i \in \mathcal{S}^w_{(g)}$, $p_i \leftarrow \max\left\{\frac{\rho_{(g)} + a_{(t)}}{|\mathcal{S}^w_{(g)}|}, \tilde{p}_i\right\}$;
    $\mathcal{P}^w \leftarrow \{P_{(1)}, P_{(2)}, \ldots, P_{(\tilde{n})}\}$;
    $q_i \leftarrow a_{(\tilde{n}+1)}$ for each $P_i \in \mathcal{P}^w$;
  else
    $(\mathcal{P}^w, \{\gamma_i | P_i \in \mathcal{P}\}) \leftarrow \mathcal{A}(\mathcal{P}, \mathbf{a}'')$, $\tilde{m} \leftarrow |\mathcal{P}^w|$;
    For each $P_i \in \mathcal{P}^w$, $q_i \leftarrow \max\{a_{(t)}, B_{(t)} - \gamma_i\}$;
    $\mathcal{S}^w \leftarrow \cup_{1\le g\le \tilde{m}} \mathcal{S}^w_{(g)}$;
    For each $\mathcal{S}^w_{(g)}$, $1 \le g \le \tilde{m}$, and each $S_i \in \mathcal{S}^w_{(g)}$, $p_i \leftarrow \max\left\{\frac{B_{(\tilde{m}+1)}}{|\mathcal{S}^w_{(g)}|}, \tilde{p}_i\right\}$;
  end
end
return $(\mathcal{P}^w, \mathcal{S}^w, \mathbf{q}, \mathbf{p})$

In Step 1, assume that the buyers are divided into groups $\mathcal{S}_1 = \{S_4\}, \mathcal{S}_2 = \{S_2, S_6\}, \mathcal{S}_3 = \{S_1, S_5\}$, and $\mathcal{S}_4 = \{S_3\}$.

In Step 2, according to Algorithm 2, we first compute the optimal group bid for each group, as shown in Fig. 2(b). We have $B^*_1 = 10$, $B^*_2 = 7$, $B^*_3 = 8$, and $B^*_4 = 5$. Hence $B^*_{(1)} = B^*_1$, $\mathcal{S}_{(1)} = \mathcal{S}_1$; $B^*_{(2)} = B^*_3$, $\mathcal{S}_{(2)} = \mathcal{S}_3$; $B^*_{(3)} = B^*_2$, $\mathcal{S}_{(3)} = \mathcal{S}_2$; and $B^*_{(4)} = B^*_4$, $\mathcal{S}_{(4)} = \mathcal{S}_4$. We then use the cross extraction technique to decide the group bid for each group. Assume that $r(1) = 2$, $r(2) = 3$, $r(3) = 4$, and $r(4) = 1$, as shown in Fig. 2(b). Therefore we have $B_{(1)} = B^*_{(2)} = 8$, $\mathcal{S}^w_{(1)} = \{S_4\}$, $\tilde{p}_4 = 8$; $B_{(2)} = B^*_{(3)} = 7$, $\mathcal{S}^w_{(2)} = \{S_1, S_5\}$, $\tilde{p}_1 = \tilde{p}_5 = \frac{7}{2}$; $B_{(3)} = B^*_{(4)} = 5$, $\mathcal{S}^w_{(3)} = \{S_2, S_6\}$, $\tilde{p}_2 = \tilde{p}_6 = \frac{5}{2}$; and $B_{(4)} = 0$ as the buyer(s) in group $\mathcal{S}_4$ cannot provide profit of

10, $\mathcal{S}^w_{(4)} = \emptyset$. Since $\mathcal{S}_{(4)} = \{P_3\}$, $\tilde{p}_3 = 0$.

In Step 3, we first find the largest t such that $B_{(t)} \geq a_{(t)}$ as shown in Fig. 2(c). Since $t = 3$, we compute $\mathbf{a}'' = \{B_{(3)} - a_{(1)}, B_{(3)} - a_{(2)}\} = \{3, 1\}$ and $\mathbf{B}'' = \{B_{(1)} - a_{(3)}, B_{(2)} - a_{(3)}\} = \{3, 2\}$. Assume that the auction proceeds with the branch in Lines 13–13. In the basic auction on the buyer groups, we may have two group bid vectors $\{3\}$ and $\{2\}$. Thus $\dot{R} = 3$ and $\ddot{R} = 2$. Since $\dot{R} > \ddot{R}$, the only buyer S_4 in group $\mathcal{S}^w_{(1)}$ is the winner and $p_4 = \max\{\frac{\ddot{R}+a_{(3)}}{|\mathcal{S}^w_{(1)}|}, \tilde{p}_4\} = \max\{\frac{2+5}{1}, 8\} = 8$. Since there is only one winning buyer group, only seller $P_{(1)} = P_3$ is the winning seller and receives payment of $q_3 = a_{(2)} = 4$. The profit of the auction is $p_4 - q_3 = 4$.

6. Analysis of PROMISE

In this section, we prove important properties of PROMISE. Specifically, we prove that PROMISE is computationally efficient (Lemma 6.1), individual-rational (Lemma 6.2), truthful (Lemma 6.3), and competitive (Lemma 6.4).

Lemma 6.1: PROMISE is computationally efficient. □

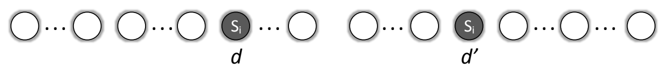
Proof: We first spend $O(n \log n)$ time sorting the buyers according to their bids in non-decreasing order to make the computation of $\mathcal{F}$ linear in the rest of the algorithm. Grouping the buyers into l groups using algorithm $\mathcal{G}$ takes $T_{\mathcal{G}}$ time. GBC takes $O(\min\{n^2, m^2\})$ time. In Line 5, it takes $O(m \log m)$ time to sort the sellers. It takes $O(n \log n)$ to sort the group bids. It is obvious that Lines 9–24 dominate the running time of the rest of the algorithm. The time complexity of this part is dominated by algorithm $\mathcal{A}$, i.e., $T_{\mathcal{A}}$. Thus the time complexity of PROMISE is $O(\min\{n^2, m^2\} + m \log m + T_{\mathcal{G}} + T_{\mathcal{A}})$. ■

Lemma 6.2: PROMISE is individual rational. □

Proof: Assume that each seller P_i and buyer S_i asks and bids truthfully, i.e., $a_i = c_i$ and $b_i = \nu_i$. We first prove individual rationality for the sellers. If P_i is a winner, the payment in Line 8 is $q_i = q_{(1)} = a_{(t)} \geq a_{(1)} = a_i = c_i$. The payment in Line 17 is $q_i = a_{(\tilde{n}+1)} \geq a_i = c_i$. The payment in Line 20 is guaranteed to be no less than c_i due to the individual rationality of $\mathcal{A}$. In summary, we have $q_i \geq c_i$ for any $P_i \in \mathcal{P}^w$. Therefore we have $u_i \geq 0$ for any $P_i \in \mathcal{P}^w$, and $u_i = 0$ otherwise. We next prove individual rationality for the buyers. For any S_i, $\tilde{p}_i > 0$ implies that there exists π such that $\nu_i \geq \frac{B^*_{(r(g))}}{\pi} = \tilde{p}_i$. When $t = 2$, $\frac{B_{(2)}}{|\mathcal{S}^w_{(1)}|} \leq \frac{B_{(1)}}{|\mathcal{S}^w_{(1)}|} \leq \nu_i$ for all $S_i \in \mathcal{S}^w_{(1)}$ (Line 8). In addition, $\frac{\rho_{(g)}+a_{(t)}}{|\mathcal{S}^w_{(g)}|} \leq \frac{B^*_{(r(g))}}{|\mathcal{S}^w_{(g)}|} \leq \nu_i$ (Line 15), where the first inequality follows from the individual rationality of $\mathcal{A}$, and the second inequality follows from the condition in $ProEx$. Finally, $\frac{B_{(\tilde{m}+1)}}{|\mathcal{S}^w_{(g)}|} \leq \frac{B_{(t)}}{|\mathcal{S}^w_{(g)}|} \leq \frac{B^*_{(r(g))}}{|\mathcal{S}^w_{(g)}|} \leq \nu_i$ (Line 22). We conclude that $p_i \leq \nu_i$ for any $S_i \in \mathcal{S}^w$. Hence $\mu_i \geq 0$, $\forall S_i \in \mathcal{S}^w$, and $\mu_i = 0$ otherwise. ■

Lemma 6.3: PROMISE is truthful. □

Proof: The truthfulness for the sellers can be proved similarly as in [3, Theorem 4]. We next prove the truthfulness for the buyers. Since the group formation is independent of the bids, we focus on the group bid computation algorithm and final winners determination. Algorithm 2 is truthful for buyers in groups $\{\mathcal{S}_{(g)}\}_{1 \leq g \leq s}$ because the extracted profit $B^*_{(r(g))}$ for $\mathcal{S}_{(g)}$ is independent of the bids in $\mathcal{S}_{(g)}$, and $ProEx$ is truthful for any target profit [5]. If $s < l$, for those buyers in $\{\mathcal{S}_{(g)}\}_{s+1 \leq g \leq l}$ while bidding truthfully, i.e., $b_i = \nu_i$, if any buyer S_i wants to make its group qualified for winner determination, it has to increase its bid, i.e., $b_i > \nu_i$. Let $b_{(i)}$ and $b'_{(i)}$ denote the ith-largest bid in S_i's group when S_i bids truthfully and when S_i bids $b_i > \nu_i$, respectively. As shown in Fig. 3, its position changes from d to d', where $d' \leq d$. Then in $ProEx$, we must have $\pi \leq d$ if π exists. Thus $p_i \geq \tilde{p}_i = \frac{B^*_{(r(g))}}{\pi} \geq \frac{B^*_{(s)}}{\pi} \geq \frac{b_{(\pi)}\pi}{\pi} = b_{(\pi)} \geq \nu_i$, where the 3rd inequality follows from the fact that S_i's group was not among the top s groups when S_i bids truthfully. In winner determination step, the truthfulness for all buyers can be proved similarly as in [3, Theorem 4]. ■



(a) Buyer S_i bids truthfully (b) Buyer S_i increases it bid

Fig. 3. Bids sorted in a nondecreasing order

Lemma 6.4: PROMISE is α-competitive, where

$$\alpha = 4\beta(s-1), \tag{6.1}$$

β is the competitive ratio of $\mathcal{A}$, and $s = \min\{n, m\}$. □

Proof: Let $k \in [2, s]$ be such that $\mathcal{M}^{[2]}(\mathbf{a}, \mathbf{b}) = k(B^*_{(k)} - a_{(k)})$. Similar to (4.8), we define $\mathcal{F}^{[2]}(\mathbf{b}) = \max_{2 \leq i \leq |\mathbf{b}|} ib_{(i)}$, and $\mathcal{F}^{[2]}(\mathbf{a}, \mathbf{b}) = \max_{i \geq 2} i(b_{(i)} - a_{(i)})$. Hence $\mathcal{M}^{[2]}(\mathbf{a}, \mathbf{b}) = \mathcal{F}^{[2]}(\mathbf{a}, \mathbf{B}^*)$ by (4.6), where $\mathbf{B}^* = (B^*_1, B^*_2, \ldots, B^*_l)$.

Then we have $\mathcal{F}^{[2]}(\mathbf{B}') \geq k(B_{(k)} - a_{(t)})$, and $\mathcal{F}^{[2]}(\mathbf{a}') \geq k(B_{(t)} - a_{(k)})$. Let k' be such that $\mathcal{F}^{[2]}(\mathbf{B}') = k'B'_{(k')}$. If $k' = t$, $\mathcal{F}^{[2]}(\mathbf{B}') - B'_{(t)} = (t-1)B'_{(t)} \leq (t-1)B'_{(t-1)} \leq \mathcal{F}^{[2]}(\mathbf{B}'')$. If $k' < t$, $\mathcal{F}^{[2]}(\mathbf{B}') = \mathcal{F}^{[2]}(\mathbf{B}'')$. Hence $\mathcal{F}^{[2]}(\mathbf{B}') \leq \mathcal{F}^{[2]}(\mathbf{B}'') + (B_{(t)} - a_{(t)})$. Similarly, $\mathcal{F}^{[2]}(\mathbf{a}') \leq \mathcal{F}^{[2]}(\mathbf{a}'') + (B_{(t)} - a_{(t)})$. Therefore,

$$\begin{aligned}
& \mathcal{F}^{[2]}(\mathbf{a}, \mathbf{B}) && (6.2)\\
= \; & k(B_{(k)} - a_{(k)}) && (6.3)\\
= \; & k(B_{(k)} - a_{(t)}) + k(B_{(t)} - a_{(k)}) - k(B_{(t)} - a_{(t)}) && (6.4)\\
\leq \; & \mathcal{F}^{[2]}(\mathbf{a}') + \mathcal{F}^{[2]}(\mathbf{B}') - k(B_{(t)} - a_{(t)}) && (6.5)\\
\leq \; & \mathcal{F}^{[2]}(\mathbf{a}'') + \mathcal{F}^{[2]}(\mathbf{B}''). && (6.6)
\end{aligned}$$

Since $\mathcal{A}$ is β-competitive against $\mathcal{F}^{[2]}$, we have

$$E[\mathcal{M}_{\mathcal{A}}(\mathbf{a}, \mathbf{b})] \geq \frac{\mathcal{F}^{[2]}(\mathbf{a}'')}{2\beta} + \frac{\mathcal{F}^{[2]}(\mathbf{B}'')}{2\beta} \geq \frac{\mathcal{F}^{[2]}(\mathbf{a}, \mathbf{B})}{2\beta}. \tag{6.7}$$

Let $i \leq k$ be the number of bids in $\{B_{(g)}\}_{1 \leq g \leq s}$ that are no less than $B^*_{(k)}$, we have

$$\mathcal{F}^{[2]}(\mathbf{a}, \mathbf{B}^*) = k(B^*_{(k)} - a_{(k)}) \leq k(B_{(i)} - a_{(i)}) \leq \frac{k}{i}\mathcal{F}^{[2]}(\mathbf{a}, \mathbf{B}). \tag{6.8}$$

Since we use random cross checking, i is a random variable. Let $Pr[i = j]$ denote the probability that $i = j$ for $j \in [1, k]$. Combining (6.7) and (6.8), we have

$$\frac{E[\mathcal{M}_{\mathcal{A}}(\mathbf{a}, \mathbf{b})]}{\mathcal{F}^{[2]}(\mathbf{a}, \mathbf{B}^*)} \geq \frac{\sum_{j=1}^{k} Pr[i = j] \cdot j}{2k\beta}. \tag{6.9}$$

However, we do not know the value of k *a priori*, except that $2 \leq k \leq s$. Considering the worse case, we have

$$\frac{E[\mathcal{M}_{\mathcal{A}}(\mathbf{a}, \mathbf{b})]}{\mathcal{F}^{[2]}(\mathbf{a}, \mathbf{B}^*)} \geq \min_{2 \leq k \leq s} \frac{\sum_{j=1}^{k} Pr[i = j] \cdot j}{2k\beta}. \tag{6.10}$$

By Lemma A.2 in Appendix, $\frac{\sum_{j=1}^{k} Pr[i=j]\cdot j}{2k\beta} = \frac{k-1}{4\beta(s-1)}$. Hence,

$$E[\mathcal{M}_{\mathcal{A}}(\mathbf{a},\mathbf{b})] \geq \frac{1}{\alpha}\mathcal{F}^{[2]}(\mathbf{a},\mathbf{B}^*) = \frac{1}{\alpha}\mathcal{M}^{[2]}(\mathbf{a},\mathbf{b}), \qquad (6.11)$$

where α is defined as in (6.1). ■

Our theoretical derivation of α is very conservative, as demonstrated by extensive numerical results in Fig. 6.

By Lemmas 6.1 to 6.4, we have the following main theorem.

Theorem 4: PROMISE is computationally efficient, individual-rational, truthful, and competitive. □

7. Performance Evaluation

We evaluate the performance of PROMISE and compare it with TRUST, as the model in TRUST is closest to ours.

A. Evaluation Setup

In all the simulations, we randomly generated secondary users in a 1000×1000 square. Two secondary users interfere with each other if the Euclidean distance between them is no more than 300. We varied n from 100 to 200 with an increment of 20, and m from 10 to 20 with an increment of 2. As in [28], each bid b_i is uniformly distributed over $(0, 1]$, while each ask a_i is uniformly distributed over $(0, 2]$. The secondary user distribution and grouping algorithms are as follows.

Secondary User Distribution: We consider two different distributions of the secondary users:

- **Uniform (UNF):** Secondary users are randomly and uniformly distributed in the whole area.
- **Cluster (CLTR):** Several cluster heads are randomly generated, and then each secondary user is randomly generated within range 100 from a cluster head.

Grouping Algorithms: As in [28], we apply four different grouping algorithms:

- **Random:** Each group is formed by choosing conflict-free nodes randomly.
- **Max-IS [19]:** Each group is formed by iteratively adding the node with the minimum maximum independent set in its neighborhood, and then removing it and its neighbors. To remove the node degree constraint and guarantee efficiency, we use maximal independent set instead.
- **Greedy-U [17]:** Each group is formed by recursively adding a node with the minimum degree in the conflict graph and updating all the other nodes' degrees.
- **Greedy [17]:** It is the same as Greedy-U, except that nodes's degrees are not updated.

Performance Metrics: Our performance metrics are the profit, the number of winning sellers and winning buyers.

For each case, we randomly generated 1000 instances and averaged the results. As PROMISE is a randomized auction, we ran $10n$ times and averaged the results for each instance.

B. Evaluation Results and Analysis

We first evaluate the effects of different distributions and grouping algorithms on the profit of PROMISE compared to TRUST, as shown in Fig. 4. One observation is that the profit of PROMISE is 4.32(CLTR-Greedy) to 6.47(UNF-Greedy) times the profit of TRUST. In addition, except RAND, PROMISE

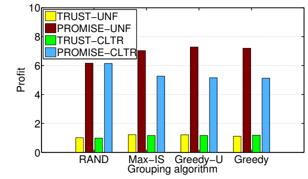
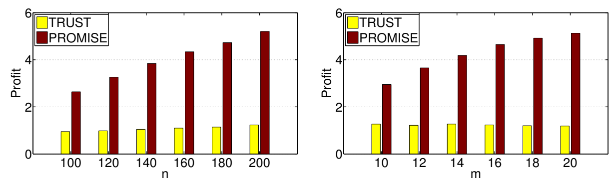


Fig. 4. Comparison among grouping algorithms with $n = 200$ and $m = 20$

performs better in CLTR than in UNF, because RAND is independent of the topology of the conflict graph.

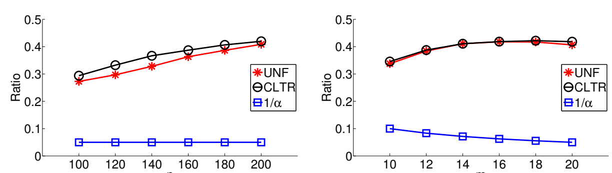
Since CLTR-Greedy combination has the lowest advantage over TRUST, our evaluations on the profit based on this setting. Fig. 5 shows the profit of PROMISE and TRUST as functions of n and m. When there are more secondary users or primary users, we see that the profit of PROMISE increases fairly rapidly while that of TRUST increases slowly or even decreases, resulting in an increasing gap between them. The reason of TRUST's decline in profit is that the difference between $B_{(t)}$ and $a_{(t)}$, where t is the largest integer such that $B_{(t)} \geq a_{(t)}$, becomes smaller and eventually reaches 0.



(a) $m = 20$ (b) $n = 200$

Fig. 5. Profit

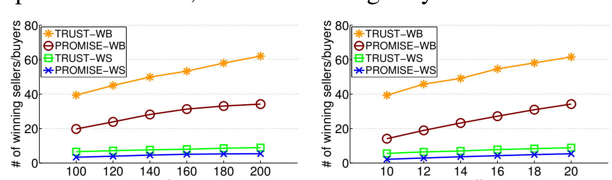
We also compare the ratios of the profit of PROMISE over that of $\mathcal{M}^{[2]}$ with our theoretical ratios α. Fig. 6 illustrates the results using the Greedy algorithm, as other algorithms have the similar pattern. We can see the rise of ratios in both figures, meaning that PROMISE performs better and better when there are more users participating in the auction. Note that the user's distribution does not impact on the performance.



(a) $m = 20$ (b) $n = 200$

Fig. 6. Comparison with $\mathcal{M}^{[2]}$ using Greedy

We now compare TRUST and PROMISE in terms of the number of winning buyers and sellers, which is not the objective of PROMISE though. Fig. 7 shows the results using Greedy-UNF setting. We observe that the number of winning buyers or winning sellers in PROMISE is approximately half of that in TRUST. Hence PROMISE dramatically increases the profit compared to TRUST, while sacrificing only half of the winners.



(a) $m = 20$ (b) $n = 200$

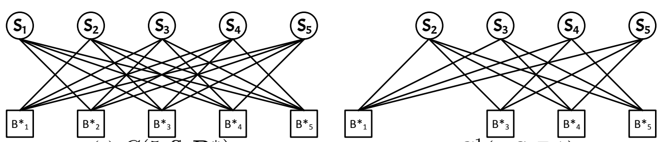
Fig. 7. Number of winning buyers (WB) and sellers (WS)

8. Conclusion

In this paper, we proposed PROMISE, a general framework for truthful and profit maximizing spectrum double auctions. It jointly considers the spatial reusability of the spectrum, truthfulness and profit maximization. We designed a novel technique, called *cross extraction*, to compute the bid representing a group of secondary users, who can share a common channel. Finally, we proved that PROMISE is computationally efficient, individual-rational, truthful, and competitive.

Appendix

Let $\mathbb{S} = (\mathcal{S}_1, \ldots, \mathcal{S}_s)$ be a vector of s groups, and $\mathbf{B}^* = (B_1^*, \ldots, B_s^*)$ be a vector of s bids. We construct a bipartite graph $G(s, \mathbb{S}, \mathbf{B}^*)$ with $2s$ vertices and $s^2 - s$ edges. The vertices are $\mathcal{S}_1, \ldots, \mathcal{S}_s$ (denoted by circles) and $B_1^*, \ldots, B_s^*$ (denoted by squares). The edges are $(\mathcal{S}_i, B_j^*)$ for $i \neq j$. Fig. 8(a) shows $G(5, \mathbb{S}, \mathbf{B}^*)$. For any $i \neq j$, we use $G_j^i(s, \mathbb{S}, \mathbf{B}^*)$ to denote the subgraph of $G(s, \mathbb{S}, \mathbf{B}^*)$ with nodes $\mathcal{S}_i$ and B_j^* deleted. Fig. 8(b) shows $G_2^1(5, \mathbb{S}, \mathbf{B}^*)$. We use $f(s)$ to denote the number of perfect matchings in $G(s, \mathbb{S}, \mathbf{B}^*)$, and $g_j^i(s)$ the number of perfect matchings in $G_j^i(s+1, \mathbb{S}, \mathbf{B}^*)$ [10]. Since $g_j^i(s) = g_2^1(s)\ \forall i \neq j$, we use $g(s)$ to denote $g_j^i(s), \forall i \neq j$.



(a) $G(5, \mathbb{S}, \mathbf{B}^*)$ (b) $G_2^1(5, \mathbb{S}, \mathbf{B}^*)$

Fig. 8. Illustration of $G(5, \mathbb{S}, \mathbf{B}^*)$ and $G_2^1(5, \mathbb{S}, \mathbf{B}^*)$.

Lemma A.1: $f(2) = 1$, $g(2) = 1$. For $s \geq 2$, we have

$$f(s+1) = s \times g(s), \quad g(s+1) = f(s) + s \times g(s). \qquad (A.1)$$

Proof: $G(2, \mathbb{S}, \mathbf{B}^*)$ has two edges: $(\mathcal{S}_1, B_2^*)$ and $(\mathcal{S}_2, B_1^*)$. $G_2^1(3, \mathbb{S}, \mathbf{B}^*)$ has three edges: $(\mathcal{S}_2, B_1^*)$, $(\mathcal{S}_2, B_3^*)$, and $(\mathcal{S}_3, B_1^*)$. Therefore $f(2) = g(2) = 1$. Now we prove (A.1).

For each $j = 2, \ldots, s+1$, there are $g(s)$ perfect matchings in $G(s+1, \mathbb{S}, \mathbf{B}^*)$ that contain edge $(\mathcal{S}_1, B_j^*)$. Therefore $f(s+1) = s \times g(s)$. $G_j^i(s+1, \mathbb{S}, \mathbf{B}^*)$ has $f(s)$ perfect matchings that contain edge $(\mathcal{S}_j, B_i^*)$. For each $l \in \{1, \ldots, s+1\} \setminus \{i, j\}$, $G_j^i(s+1, \mathbb{S}, \mathbf{B}^*)$ has $g(s)$ perfect matchings that contain edge $(\mathcal{S}_j, B_l^*)$. Therefore $g(s+1) = f(s) + s \times g(s)$. ■

For each $j = 1, \ldots, s$, let $\psi(s, j)$ denote the number of perfect matchings in $G(s, \mathbb{S}, \mathbf{B}^*)$ that contain an edge of the form $(\mathcal{S}_i, B_j^*)$ where $i < j$. Then $\psi(s, j) = (j-1) \times g(s-1)$, since for each $i < j$, the number of perfect matchings of $G(s, \mathbb{S}, \mathbf{B}^*)$ containing the edge $(\mathcal{S}_i, B_j^*)$ is $g(s-1)$.

For each $k = 2, 3, \ldots, s$ and each $r = 1, 2, \ldots, k$, we use $\phi(s, k, r)$ to denote the number of perfect matchings in $G(s, \mathbb{S}, \mathbf{B}^*)$ that contain exactly r edges of the form $(\mathcal{S}_i, B_j^*)$ where $i < j \leq k$. We have the following important result.

Lemma A.2: For any $k = 2, 3, \ldots, s$, we have

$$\sum_{r=1}^{k} \frac{\phi(s,k,r)}{f(s)} \times \frac{r}{k} = \frac{1}{2} \times \frac{k-1}{s-1}. \qquad (A.2)$$

Proof: To prove (A.2), it suffices to prove

$$\sum_{r=1}^{k} \phi(s,k,r) \times r = \frac{f(s)}{s-1} \times \frac{(k-1) \times k}{2}. \qquad (A.3)$$

The left-hand-side of (A.3) is the number of perfect matchings of $G(s, \mathbb{S}, \mathbf{B}^*)$ that contain the edge $(\mathcal{S}_i, B_j^*)$ for $1 \leq i < j \leq k$. Therefore we have

$$\sum_{r=1}^{k} \phi(s,k,r) \times r = \sum_{j=1}^{k} \psi(s,j) = \frac{f(s)}{s-1} \times \frac{(k-1) \times k}{2}. \quad (A.4)$$

This proves (A.3). Hence (A.2) is true. ■

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