Geometric Evaluation of Survivability of Disaster-affected Network with Probabilistic Failure

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Abstract—**This paper presents an algorithm for evaluating the probability that connectivity can be maintained between two given nodes in a physical network affected by a disaster. Nodes and links in a disaster area are probabilistically broken, and the disaster area is modeled using a half plane. This paper also proves that this probability of connectivity increases for a generic network topology when the perimeter length of the convex hull of a physical link route decreases, and that it becomes maximum when these physical link routes become straight line segments. In addition, this paper proposes an optimal server placement method by considering robustness against disaster and an optimal link/node replacement strategy determining which nodes or links should be replaced with those robust against disaster. Intuitive node (link) replacement strategies are also suggested based on the analysis of this paper.**

I. INTRODUCTION

A massive earthquake on March 11, 2011 and the resulting tsunami in north-eastern Japan have forced us to develop a method for designing a network that is robust against such disasters that encompass a wide area. Although there have been a large number of theoretical papers published evaluating the survivability for a given network, most were focused on a single failure (or independent failures) of a network node or a link. Some of these studies have been extended to analyze multiple or correlated failures. However, most did not take into account a physical disaster area causing a high failure rate.

A limited number of papers about network survivability have been reported that have taken into account geometrical/geographical conditions. Grubesic [1] evaluated the network survivability of the current Internet based on geographical data. Although he focused on the physical route of the network, it was a case study, so no mathematical models or methods were provided. Liew and Lu [2] proposed a framework to evaluate network survivability during a disaster and introduced a survivability function to various metrics. Although their framework can introduce correlated failures, they did not propose any method or model of correlations. Furthermore, they did not consider the physical shape of the disaster area or that of the network. Wu et al. [3] discussed the optimization of the physical route of an undersea cable by assuming a disk-shaped disaster area. Assuming a rectangular route, the length of an edge is determined by minimizing cost while maintaining a higher probability of connecting two cities than the threshold. Recently, Neumayer et al. published two papers intended to cover network survivability in a disaster [4], [5]. In their network model, there is a set of line segments of which end points are locations of network center buildings and the disaster model is a line segment or a circle [4]. They proposed to use an optimization technique to find the worst case disaster. On the other hand, Neumayer and Modiano [5] used geometric probability (integral geometry) to model the randomness of a disaster. Their network model is, again, a set of line segments of which end points are locations of network center buildings and the disaster model is a line. These papers also emphasize a polynomial order algorithm to evaluate metrics. Saito proposed geometric modeling of a network affected by a wide disaster area and analyzed the model by using integral geometry (geometric probability) under the assumption that all the network elements in the disaster area becomes unavailable [6]. This analysis provides a theoretical method of evaluating performance metrics, such as the probability of maintaining connectivity, and explicit formulas for them. Consequently, he proposes a network design rule that can make the network robust against disasters.

The minimum number of cuts disconnecting the source and sink nodes by taking into account a disaster area is discussed in the following papers. To the best of our knowledge, Bienstock [7] initiated the study of this problem. He investigated the algorithms computing the minimum number of disaster areas disconnecting the source and sink nodes when all the edges intersecting the disaster areas are removed. Sen et al. [8] proposed a region-based connectivity as a metric for fault-tolerance. They provided polynomial time algorithms for calculating region-based connectivity assuming the region is a disk-shaped disaster area. Neumayer et al. [9] discussed the geographical min-cut, defined as the minimum number of disk-shaped disaster areas to disconnect a pair of nodes, and the geographical max-flow, defined as the maximum number of paths that are not disconnected by a single disaster area, and showed that geographical min-cut is not equal to geographical max-flow. Agarwal et al. studied algorithms that find a disaster location having the highest expected impact on the network, where the impact is defined by various metrics such as the number of failed components [10]. Trajanovski et al. [11] also studied this category, and proposed a polynomial time algorithm for finding a disaster location that degrades a performance metric the most. An important contribution of that study is that the disaster area can be an ellipse, square,

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rectangle, and equilateral triangle.

Failure probabilities of network components, such as nodes and links, in a disaster area are different from those in a normal area. In our experience, in a tsunami-prone area, the failure probability of an aerial cable is much higher than that of a cable in an underground duct. The failure probability of a node in a network station is lower than that of an aerial cable in an area prone to strong winds. However, the former becomes higher in a flood area, which may be caused by a typhoon/hurricane. It is also true that failure probabilities of network components depend on the age of those components. A new network component is free from age deterioration, and it is often the case that the latest technology is more robust.

This paper proposes a method for evaluating the probability of disconnecting two given nodes under a wide disaster area when given failure probabilities of individual nodes and links during a disaster. It also derives formulas for the probability that end-to-end connectivity is maintained. This method and formulas are useful for disaster management. This paper also discusses the relationship between the probability of disconnecting two given nodes and the physical link route shape and derives an optimal physical link route. It also discusses a server placement method and node/link replacement strategies: where a server should be placed and which nodes and links should be replaced if we can replace them with new ones robust against disasters. These discussions contribute to network planning and maintenance of networks.

II. Notation and Basic Theorems

For an area C, convex sets C_1 and C_2 in 2-dimensional space $\mathbb{R}^2$, a directional line G, and a set S of points in $\mathbb{R}^2$, we use the following notations in the remainder of this paper.

- $||C||$: the size of C,
- $|C|$: the perimeter length of C,
- $\overline{C}$, $\overline{S}$: convex hull of C and convex hull of S
- $C_1 \otimes C_2$: internal cover of C_1 and C_2,
- $\sharp(S)$: the number of points in S,
- R_G, L_G: the right-half and left-half planes of G,
- $m(A)$: the measure (non-normalized probability) of the set of positions of randomly placed line G satisfying A.

When $C_1 \cap C_2 = \emptyset$, we can define the internal cover $C_1 \otimes C_2$ of C_1 and C_2 by a closed elastic string drawn around C_1 and C_2 and crossing over at a point placed between them (Fig. 1). In addition, R_G (L_G) is located at the right (left) of G when the direction of G is upward.

When $A_1 \subsetneqq A_2$, the probability of the set of positions of G satisfying A_1 among the cases satisfying A_2 is defined by the quotient of measures $m(A_1)/m(A_2)$ [12].

Equation (3.12) in [12] provides the measure of the set of positions of G that meets a fixed bounded convex set C.

Lemma 1:

$$m(G \cap C \neq \emptyset) = |C| \tag{1}$$

Lemma 1 means that the probability that G meets a fixed set C is proportional to the perimeter length of C.

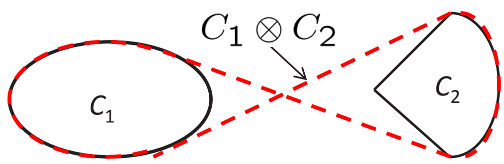


Fig. 1. Internal cover $C_1 \otimes C_2$ of C_1 and C_2

Due to the result on p.33 in [12], we obtain the following lemma.

Lemma 2: The measure of the set of positions of G that separates convex sets C_1 and C_2 is given as follows.

$$\begin{aligned} & m(G \cap C_1 = \emptyset, G \cap C_2 = \emptyset, G \cap \overline{C_1 \cup C_2} \neq \emptyset) \\ = \; & g(C_1, C_2) \end{aligned} \tag{2}$$

Here, $g(C_1, C_2) \equiv |C_1 \otimes C_2| - |C_1| - |C_2|$.

To simplify the notation, define $|C_1 \otimes C_2| = |C_1| + |C_2|$ when $C_1 \cap C_2 \neq \emptyset$. Then, $m(G \cap C_1 = \emptyset, G \cap C_2 = \emptyset, G \cap \overline{C_1 \cup C_2} \neq \emptyset) = 0$ in Eq. (2) when $C_1 \cap C_2 \neq \emptyset$. Thus, Eq. (2) becomes valid even when $C_1 \cap C_2 \neq \emptyset$.

Theorem 1: Let $\Phi \subset \Omega$ be a set of points in 2-dimensional space $\mathbb{R}^2$ where Ω is convex (Fig. 2). Divide Φ into two exclusive subsets: Φ_1 and Φ_2 where $\Phi = \Phi_1 \cup \Phi_2$ and $\Phi_1 \cap \Phi_2 = \emptyset$.

$$\Pr(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G | G \cap \Omega \neq \emptyset) = h(\overline{\Phi_1}, \overline{\Phi_2})/2. \tag{3}$$

$$\text{Here, } h(\overline{\Phi_1}, \overline{\Phi_2}) \equiv \begin{cases} g(\overline{\Phi_1}, \overline{\Phi_2})/|\Omega|, & \text{for } \Phi_1 \neq \emptyset, \Phi_2 \neq \emptyset, \\ (|\Omega| - |\overline{\Phi}|)/|\Omega|, & \text{for } \Phi_1 = \emptyset \text{ or } \Phi_2 = \emptyset. \end{cases}$$

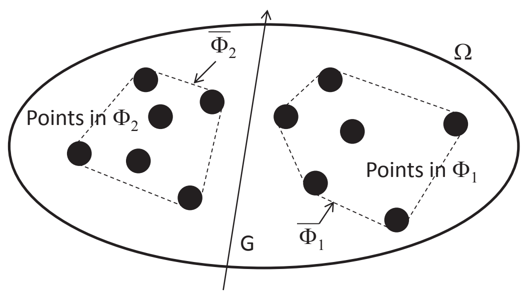


Fig. 2. Illustration of Eq. (3)

Proof: First, assume that $\Phi_1 \neq \emptyset$ and $\Phi_2 \neq \emptyset$. Consider $m(G \cap \overline{\Phi_1} = \emptyset, G \cap \overline{\Phi_2} = \emptyset, G \cap \overline{\Phi} \neq \emptyset)$. Due to symmetry of the direction of G, $m(G \cap \overline{\Phi_1} = \emptyset, G \cap \overline{\Phi_2} = \emptyset, G \cap \overline{\Phi} \neq \emptyset) = 2m(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G, G \cap \overline{\Phi} \neq \emptyset)$. According to Eq. (2) and $\overline{\overline{\Phi_1} \cup \overline{\Phi_2}} = \overline{\Phi}$, $m(G \cap \overline{\Phi_1} = \emptyset, G \cap \overline{\Phi_2} = \emptyset, G \cap \overline{\Phi} \neq \emptyset) = g(\overline{\Phi_1}, \overline{\Phi_2})$. Therefore, $m(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G, G \cap \overline{\Phi} \neq \emptyset) = g(\overline{\Phi_1}, \overline{\Phi_2})/2$. Because

$m(G \cap \Omega \neq \emptyset) = |\Omega|$ due to Lemma 1, $\Pr(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G | G \cap \Omega \neq \emptyset) = m(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G, G \cap \Omega \neq \emptyset)/|\Omega|$. Because $m(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G, G \cap \overline{\Phi} \neq \emptyset) = m(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G) = m(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G, G \cap \Omega \neq \emptyset)$, $\Pr(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G | G \cap \Omega \neq \emptyset) = h(\overline{\Phi_1}, \overline{\Phi_2})/2$.

Second, assume that $\Phi_1 = \emptyset$ and $\Phi_2 = \Phi$. Then, $\Pr(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G | G \cap \Omega \neq \emptyset) = \Pr(\overline{\Phi} \subset L_G = \emptyset | G \cap \Omega \neq \emptyset)$. According to Theorem 1 in [6], $\Pr(\overline{\Phi} \subset L_G | G \cap \Omega \neq \emptyset) = (|\Omega| - |\overline{\Phi}|)/(2|\Omega|)$.

Due to symmetry, if $\Phi_1 = \Phi$ and $\Phi_2 = \emptyset$, $\Pr(\overline{\Phi_1} \subset R_G, \overline{\Phi_2} \subset L_G | G \cap \Omega \neq \emptyset) = (|\Omega| - |\overline{\Phi}|)/(2|\Omega|)$. ■

In the later sections, this theorem provides the probability that a given set of network elements are in the disaster area.

III. Model

This paper focuses a physical network, such as an optical fiber network, in a given convex region Ω. The network consists of nodes and links. Between two nodes s and t, there is a set of routes $\{r_1(s,t), r_2(s,t), \cdots\}$.

Let $N(x)$ be the set of all the nodes in x. For example, $N(\Omega)$ is the set of all the nodes in Ω, and $N(r_i(s,t))$ is the set of nodes (including s and t) on the route $r_i(s,t)$. Divide the node set $N(x)$ into two exclusive sets $N_D(x), N_S(x)$ ($N(x) = N_D(x) \cup N_S(x)$, $N_D(x) \cap N_S(x) = \emptyset$) where $N_D(x)$ is the set of nodes in the disaster area among $N(x)$ and $N_S(x)$ is the set of nodes outside the disaster area among $N(x)$. Let $l(i,j)$ be the link between two consecutive nodes i and j. Assume that the physical route shape of a link is geometrically modeled by line segments. That is, the physical route of a link consists of several line segments. If an end point of a line segment in a link is not a node, it is called a corner (Fig. 3). Let $M(x)$ be the set of all the corners in x. For example, $M(r_i(s,t))$ is the set of corners on route $r_i(s,t)$ between s and t, and $M(l(i,j))$ is the set of corners in link $l(i,j)$. Divide the set of corners $M(x)$ into two exclusive sets $M_D(x)$ and $M_S(x)$ where $M_D(x)$ is the set of corners in the disaster area among $M(x)$, and $M_S(x)$ is the set of corners outside the disaster area among $M(x)$. To simplify the notation, $U(x) \equiv M(x) \cup N(x)$ and $U_i(x) \equiv M_i(x) \cup N_i(x)$ for $i = D, S$. That is, $U(x)$ is the set of nodes and corners in x, and $U_D(x)$ ($U_S(x)$) is the set of nodes and corners in (outside) the disaster area among $U(x)$. Sometimes, x is omitted if we can easily understand it.

We analyzed a network model affected by a large disaster. With no prior information of the disaster, we modeled the disaster area as a randomly placed area around a network in $\mathbb{R}^2$. Assume that the boundary of the large disaster can be modeled by line G. The validity of this line model was evaluated in [6]. For G, the disaster area is assumed to be R_G, the right-half plane. The set of disasters we consider is that of which the boundary intersects Ω. That is, $\Omega \cap G \neq \emptyset$. This is because the analysis becomes meaningless when the disaster does not affect Ω and because the analysis becomes trivial when the disaster area completely includes Ω. Although we focus on the positions of G such that $\Omega \cap G \neq \emptyset$, we omit the statement of this condition in the remainder of this paper for simplicity.

When node-n is in the disaster area, the node fails with probability $1 - p(n)$ and works with probability $p(n)$. If any part of $l(i,j)$ is in a disaster area R_G, the link fails with probability $1 - p(i,j)$ and works with probability $p(i,j)$. Each link or node failure in the disaster area independently occurs.

Because R_G is the disaster area, $U_D \subset R_G, U_S \subset L_G$. That is, giving M_D and N_D determines which links and nodes may fail.

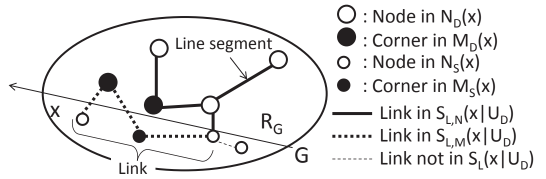


Fig. 3. Model and definition of $S_L(x|U_D)$

IV. Analysis

In the remainder of this paper, $S_L(x|U_D)$ is the set of links at which failure can happen among links in x for a given U_D. We should note that a link can fail when part of it is in R_G, which means that at least a node or a corner is in R_G. Therefore, $S_L(x|U_D)$ can be divided into two exclusive sets: $S_{L,N}(x|U_D)$ and $S_{L,M}(x|U_D)$ (Fig. 3). Among the links in x, $S_{L,N}(x|U_D)$ is the set of links of which at least one of the end nodes is in the disaster area, and $S_{L,M}(x|U_D)$ is the set of links of which both end nodes are outside the disaster area and at least one of the corners is in the disaster area. Therefore, for a given U_D, $S_{L,N}(x|U_D) = \{l(i,j) | i \oplus j \in N_D(x)\}$ where $\{i \oplus j \in N_D(x)\} \equiv \{i < j, i,j \in N_D(x)\} \cup \{i \in N_D(x), j \in N_S(x)\} \cup \{i \in N_S(x), j \in N_D(x)\}$. Similarly, $S_{L,M}(x|U_D) = \{l(i,j) | i,j \in N_S(x), M(l(i,j)) \cap M_D(x) \neq \emptyset\}$.

Let $\alpha(x|U_D)$ be the probability that all the links in $S_{L,N}(x|U_D)$ are working, $\beta(x)$ be the probability that all the links in $S_{L,M}(x|U_D)$ are working, and $P_L(x|U_D)$ be the probability that all the links in $S_L(x|U_D)$ are working. Because link failure in a given $S_{L,N}(x|U_D)$ independently occurs, $\alpha(x|U_D)$ is given as follows:

$$\alpha(x|U_D) = \Pi_{l(i,j) \in S_{L,N}(x|U_D)} p(i,j). \tag{4}$$

(If $\{l(i,j) \in S_{L,N}(x|U_D)\} = \emptyset$, define $\alpha(x|U_D) = 1$.) Similarly, $\beta(x|U_D)$ is given as follows:

$$\beta(x|U_D) = \Pi_{l(i,j) \in S_{L,M}(x|U_D)} p(i,j). \tag{5}$$

(If $\{l(i,j) \in S_{L,M}(x|U_D)\} = \emptyset$, define $\beta(x|U_D) = 1$.)

The occurrence of the event in which all links in $S_L(x|U_D)$ are working is identical to the occurrence of two independent events: in which all the links in $S_{L,N}(x|U_D)$ and all those in $S_{L,M}(x|U_D)$ are working. Therefore,

$$P_L(x|U_D) = \alpha(x|U_D)\beta(x|U_D) = \Pi_{l(i,j) \in S_L(x|U_D)} p(i,j). \tag{6}$$

(If $\{l(i,j) \in S_L(x|U_D)\} = \emptyset$, define $P_L(x|U_D) = 1$.) We should note that $P_L(x|U_D)$ is equal to the probability that all the links in x are working for a given U_D because all the links outside the disaster area are working.

Similarly, define $P_N(x|N_D)$ as the probability that all the nodes in $N_D(x)$ are working. Because node failure in a given $N_D(x)$ independently occurs,

$$P_N(x|N_D) = \Pi_{i \in N_D(x)} p(i). \quad (7)$$

Note that $P_N(x|N_D)$ is equal to the probability that all the nodes in x are working for a given N_D.

A. Tree network

In this subsection, the network we consider is a tree. Thus, there is a single route $r_1(s,t)$ between s and t.

1) Evaluation of $\Pr(s \not\leftrightarrow t)$*:* We evaluate the probability $\Pr(s \not\leftrightarrow t)$ that s and t are disconnected. Because a given U_D determines which links and nodes may fail, we can divide the event $s \not\leftrightarrow t$ into exclusive events by changing U_D.

For a given U_D, the probability that all the links and nodes in $r_1(s,t)$ are working is $P_L(r_1(s,t)|U_D)P_N(r_1(s,t)|N_D)$. Thus,

$$\begin{aligned}\Pr(s \not\leftrightarrow t) &= \sum_{U_D(r_1(s,t)) \in 2^{U(r_1(s,t))}} \\ &\quad \Pr(U_D \subset R_G, U_S \subset L_G) \\ &\quad (1 - P_L(r_1(s,t)|U_D)P_N(r_1(s,t)|N_D)).\end{aligned}$$

Apply Eq. (3) to this equation.

Result 1:

$$\begin{aligned}&\Pr(s \not\leftrightarrow t) \\ = &\sum_{U_D(r_1(s,t)) \in 2^{U(r_1(s,t))}, \overline{U_D(r_1(s,t))} \cap \overline{U_S(r_1(s,t))} = \emptyset} \\ &(1 - P_L(r_1(s,t)|U_D)P_N(r_1(s,t)|N_D)) \\ &h(\overline{U_D(r_1(s,t))}, \overline{U_S(r_1(s,t))})/2. \quad (8)\end{aligned}$$

The number of $U_D(r_1(s,t)) \in 2^{U(r_1(s,t))}$ satisfying $\overline{U_D(r_1(s,t))} \cap \overline{U_S(r_1(s,t))} = \emptyset$ is a polynomial function of $\sharp(U(r_1(s,t)))$ if no three elements in $U(r_1(s,t))$ are on a single line. We can find such $U_D(r_1(s,t))$ by drawing all the lines passing through each pair of elements in $U(r_1(s,t))$. It is clear that the computation time for drawing these lines is polynomial regarding the number of elements in $U(r_1(s,t))$. Because a polynomial time algorithm regarding the number of nodes is known to obtain a convex hull for a set of nodes [13], the computation time for Eq. (8) is also polynomial regarding the number of elements in $U(r_1(s,t))$.

As a reference of $\Pr(s \not\leftrightarrow t)$, we consider its approximations: the independence, straight-line independence, and line approximations.

With the independence approximation, it is assumed that failure at each node or link on the route in the disaster area independently occurs without giving U_D. Let nodes $i_1, \cdots, i_{k-1}$ be intermediate nodes between $s(= i_0)$ and $t(= i_k)$ on route $r_1(s,t)$. Because the probability that a node i_j is in R_G is 1/2, the probability that the node is working is $(1 + p(i_j))/2$. In addition, the probability $\Pr(l(i_j, i_{j+1}) \cap R_G = \emptyset)$ that a link $l(i_j, i_{j+1})$ is outside the disaster area is given by $(|\Omega| - |\overline{l(i_j, i_{j+1})}|)/(2|\Omega|)$ due to Theorem 1 in [6], and the probability that the link is working is $\Pr(l(i_j, i_{j+1}) \cap R_G = \emptyset) + \Pr(l(i_j, i_{j+1}) \cap R_G \neq \emptyset)p(i_j, i_{j+1})$. Therefore, $\Pr(s \not\leftrightarrow t)$ under the independence approximation is

$$\begin{aligned}&\Pr(s \not\leftrightarrow t) \\ \approx\ &1 - \Pi_{j=0}^{k}\{(1 + p(i_j))/2\} \\ &\Pi_{j=0}^{k-1}\{\frac{|\Omega| - |\overline{l(i_j, i_{j+1})}|}{2|\Omega|} + \frac{|\Omega| + |\overline{l(i_j, i_{j+1})}|}{2|\Omega|}p(i_j, i_{j+1})\}.\end{aligned}$$

In addition to this assumption of independence, $l(i_j, i_{j+1})$ is assumed to be a straight line segment between nodes-i_j and i_{j+1} with the straight-line independence approximation. Then, $|\overline{l(i_j, i_{j+1})}|$ is equal to double the distance $d(i_j, i_{j+1})$ between nodes i_j and i_{j+1}. Therefore, $\Pr(s \not\leftrightarrow t)$ under the straight-line independence approximation is

$$\begin{aligned}&\Pr(s \not\leftrightarrow t) \\ \approx\ &1 - \Pi_{j=0}^{k}\{(1 + p(i_j))/2\} \\ &\Pi_{j=0}^{k-1}\{\frac{|\Omega| - 2d(i_j, i_{j+1})}{2|\Omega|} + \frac{|\Omega| + 2d(i_j, i_{j+1})}{2|\Omega|}p(i_j, i_{j+1})\}.\end{aligned}$$

It is assumed with the line approximation that the physical route of a link is given by a straight line segment (a flybird connection [14], [15], so to speak). Under the line approximation, $\Pr(s \not\leftrightarrow t)$ is given by Eq. (8) by replacing $M(r_1(s,t))$ with $\emptyset$.

2) Evaluation of p_0*:* Let p_0 be the probability that connectivity is maintained between any pair of nodes in Ω. When the network we are discussing is a tree, p_0 is equal to the probability that all nodes and all links are working.

Result 2: Because the probability that all the links are working is $P_L(\Omega|U_D)$ and the probability that all the nodes are working is $P_N(\Omega|U_D)$ for a given U_D,

$$\begin{aligned}p_0 = &\sum_{U_D(\Omega) \in 2^{U(\Omega)}, \overline{U_D(\Omega)} \cap \overline{U_S(\Omega)} = \emptyset} P_L(\Omega|U_D)P_N(\Omega|N_D) \\ &h(\overline{U_D(\Omega)}, \overline{U_S(\Omega)})/2 \quad (9)\end{aligned}$$

B. Ring network

In this subsection, the network we consider is a ring. In the ring network, there are two routes $r_1(s,t), r_2(s,t)$ between s and t. If and only if neither routes are available, s and t are disconnected.

1) Evaluation of $\Pr(s \not\leftrightarrow t)$*:* In the ring network, $s \not\leftrightarrow t$ occurs if (a) s is not working, (b) s is working but t is not working, or (c) s and t are working but neither routes $r_1(s,t), r_2(s,t)$ are available because of failure at an intermediate node or a link. Note that events (a)-(c) are exclusive. Define $r(s,t) \equiv r_1(s,t) \cup r_2(s,t)$.

Event (a) occurs with probability $1 - p(s)$ when $s \in N_D$ and does not occur otherwise. Event (b) occurs with probability $1 - p(t)$ when $s \in N_S, t \in N_D$, with probability $p(s)(1 - p(t))$ when $s, t \in N_D$, and does not occur when $t \in N_S$.

Event (c) is a little bit complicated. First, assume that $s, t \notin N_D$. Then, for a given U_D, the probability that all the links and nodes in $r_i(s,t)$ are working is

$P_L(r_i(s,t)|U_D)P_N(r_i(s,t)|N_D)$. Because the disconnection of $r_1(s,t)$ and that of $r_2(s,t)$ are independent under a given U_D, the probability $\xi(r(s,t)|U_D)$ that neither routes are available because of failure at an intermediate node or a link for a given U_D is given as follows.

$$\xi(r(s,t)|U_D) = (1-P_L(r_1(s,t)|U_D)P_N(r_1(s,t)|N_D)) (1-P_L(r_2(s,t)|U_D)P_N(r_2(s,t)|N_D))$$

Second, assume that $s \in N_D(r_i(s,t)), t \in N_S(r_i(s,t))$. Then, for a given U_D, the probability that all the links and intermediate nodes in $r_i(s,t)$ are working is $P_L(r_i(s,t)|U_D)P_N(r_i(s,t)|N_D - s)$. Similarly,

$$\begin{aligned} & \xi(r(s,t)|U_D) \\ = \; & (1-P_L(r_1(s,t)|U_D)P_N(r_1(s,t)|N_D - s)) \\ & (1-P_L(r_2(s,t)|U_D)P_N(r_2(s,t)|N_D - s)) \end{aligned}$$

As a whole,

$$\begin{aligned} & \xi(r(s,t)|U_D) \\ = \; & (1-P_L(r_1(s,t)|U_D)P_N(r_1(s,t)|N_D - x)) \\ & (1-P_L(r_2(s,t)|U_D)P_N(r_2(s,t)|N_D - x)), \end{aligned} \quad (10)$$

where

$$\begin{cases} x = \{s,t\}, & \text{if } s,t \in N_D(r_i(s,t)), \\ x = \{s\}, & \text{if } s \in N_D(r_i(s,t)), t \in N_S(r_i(s,t)), \\ x = \{t\}, & \text{if } s \in N_S(r_i(s,t)), t \in N_D(r_i(s,t)), \\ x = \emptyset, & \text{if } s,t \in N_S(r_i(s,t)). \end{cases}$$

(When $\{N_D(r_j(s,t)) - x\} = \emptyset$, set $P_N(r_j(s,t)|N_D - x) = 1$ for $j = 1, 2$ and $x = \emptyset, s, t, \{s,t\}$.)

Therefore, event (c) occurs with probability $p(s)p(t)\xi(U_D(r(s,t)))$ when $s,t \in N_D(r(s,t))$, with probability $p(s)\xi(U_D(r(s,t)))$ when $s \in N_D(r(s,t)), t \in N_S(r(s,t))$, with probability $p(t)\xi(U_D(r(s,t)))$ when $s \in N_S(r(s,t)), t \in N_D(r(s,t))$, and with probability $\xi(U_D(r(s,t)))$ when $s,t \in N_S(r(s,t))$. Hence, the probability $p_{ring}(U_D(r(s,t)))$ that s and t are disconnected for a given U_D is given as follows.

$$\begin{aligned} & p_{ring}(U_D(r(s,t))) \\ = \; & \begin{cases} \zeta_{11}, & \text{if } s,t \in N_D(r(s,t)), \\ \zeta_{12}, & \text{if } s \in N_D(r(s,t)), t \in N_S(r(s,t)), \\ \zeta_{21}, & \text{if } s \in N_S(r(s,t)), t \in N_D(r(s,t)), \\ \zeta_{22}, & \text{if } s,t \in N_S(r(s,t)), \end{cases} \end{aligned} \quad (11)$$

where

$$\begin{cases} \zeta_{11} \equiv 1 - p(s)p(t) + p(s)p(t)\xi(r(s,t)|U_D), \\ \zeta_{12} \equiv 1 - p(s) + p(s)\xi(r(s,t)|U_D), \\ \zeta_{21} \equiv 1 - p(t) + p(t)\xi(r(s,t)|U_D), \\ \zeta_{22} \equiv \xi(r(s,t)|U_D). \end{cases}$$

Therefore, we obtain $\Pr(s \not\leftrightarrow t)$ for the ring network.

Result 3:

$$\begin{aligned} & \Pr(s \not\leftrightarrow t) \\ = \; & \sum_{U_D(r(s,t)) \in 2^{U(r(s,t))}, \overline{U_D(r(s,t))} \cap \overline{U_S(r(s,t))} = \emptyset} \\ & p_{ring}(U_D(r(s,t)))h(\overline{U_D(r(s,t))}, \overline{U_S(r(s,t))})/2. \end{aligned} \quad (12)$$

Similar to the tree network, the computation time for Eq. (12) is also polynomial.

Again, we consider three approximations of $\Pr(s \not\leftrightarrow t)$: the independence, straight-line independence, and line approximations. Because the network is a ring, the independence approximation is given by the product of $\Pr(s \not\leftrightarrow t)$ of the independence approximation for $r_1(s,t)$ and that for $r_2(s,t)$. The straight-line independence approximation is also given by the product of $\Pr(s \not\leftrightarrow t)$ of the straight-line independence approximation for $r_1(s,t)$ and that for $r_2(s,t)$. Similar to the tree network, $\Pr(s \not\leftrightarrow t)$ under the line approximation is given by Eq. (12) by replacing $M(r(s,t))$ with $\emptyset$.

2) Evaluation of p_0: For the ring network, no failure other than a single link failure is acceptable to maintain the connectivity between every pair of nodes. For a given $U_D(\Omega)$, the probability that all the nodes and links are working is $P_L(\Omega|U_D)P_N(\Omega|N_D)$. The probability $P_{L,1}(\Omega|U_D)$ that a single link in $S_L(\Omega|U_D)$ is disconnected and all the other links are working for a given U_D is

$$P_{L,1}(\Omega|U_D) = P_L(\Omega|U_D) \sum_{l(i,j) \in S_L(\Omega|U_D)} (1-p(i,j))/p(i,j).$$

This is because $P_L(\Omega|U_D)(1-p(i,j))/p(i,j) = (1-p(i,j))\Pi_{(i,j)\neq(i',j'), l(i',j') \in S_L(\Omega|U_D)} p(i',j')$, which is the probability that $l(i,j)$ is disconnected and all the other links are working. Because $P_L(\Omega|U_D)$ includes the term $p(i,j)$, this equation can be defined even when $p(i,j) = 0$. If $\{l(i,j) \in S_L(\Omega|U_D)\} = \emptyset$, define $P_{L,1}(\Omega|U_D) = 0$.

Result 4:

$$\begin{aligned} p_0 = & \sum_{U_D(\Omega) \in 2^{U(\Omega)}, \overline{U_D(\Omega)} \cap \overline{U_S(\Omega)} = \emptyset} (P_L(\Omega|U_D) + P_{L,1}(\Omega|U_D)) \\ & P_N(\Omega|N_D)h(\overline{U_D(\Omega)}, \overline{U_S(\Omega)})/2. \end{aligned} \quad (13)$$

C. Generic network

The analysis applicable to tree and ring networks can be extended to a generic network. This subsection briefly describes this.

Roughly speaking, the method consists of four steps. (1) By using G, divide U into U_D and U_S; (2) determine whether U_D includes a cutset disconnecting s and t; (3) if U_D includes a cutset, evaluate $\Pr(s \not\leftrightarrow t, U_D) = \Pr(U_D)\Pr(s \not\leftrightarrow t|U_D)$ for the given U_D, where $\Pr(U_D)$ is the probability that this U_D will occur and the $\Pr(s \not\leftrightarrow t|U_D)$ is the probability of disconnecting s and t for the given U_D; (4) sum up $\Pr(s \not\leftrightarrow t, U_D)$ for $U_D \in \{2^{U(r(s,t))}, \overline{U_D(r(s,t))} \cap \overline{U_S(r(s,t))} = \emptyset\}$ to obtain $\Pr(s \not\leftrightarrow t)$. In this method, a cutset is defined as a set of nodes and links that disconnect s and t if and only if none in the cutset are working. The determination in step (2) is easy because we can apply an ordinary polynomial time algorithm for determining the connectivity of a graph, which is made from the graph of the original network by removing all the nodes and corners in U_D. In step (3), $\Pr(U_D)$ is given by $h(\overline{U_D}, \overline{U_S})/2$. Therefore, in step (4),

$$\Pr(s \not\leftrightarrow t) = \sum_{U_D \in 2^{U(r(s,t))}, \overline{U_D(r(s,t))} \cap \overline{U_S(r(s,t))} = \emptyset} \Pr(s \not\leftrightarrow t | U_D) h(\overline{U_D}, \overline{U_S})/2. \quad (14)$$

This is consistent with Eqs. (8) and (12).

Derivation of $\Pr(s \not\leftrightarrow t | U_D) = 1 - p_{work}(Cut(U_D))$ depends on the network. Here, $p_{work}(Cut(U_D))$ is the probability that none of the cutsets in $Cut(U_D)$ actually disconnect s and t, and $Cut(U_D)$ is the set of cutsets for a given U_D. Roughly speaking, the evaluation method for $p_{work}(Cut(U_D))$ is as follows. (Details are in [16].) (1) Classify cutsets into exclusive cutset groups $\{G_i(Cut(U_D))\}_i$. Then, $p_{work}(Cut(U_D)) = \Pi_i \, p_{work}(G_i(Cut(U_D)))$ where $p_{work}(G_i(Cut(U_D)))$ is the probability that none of the cutsets in $G_i(Cut(U_D))$ will disconnect between s and t. (2) Introduce $p_{disc}(Cut(i_1, \cdots, i_k | U_D))$. This is the probability that cutsets $i_1, \cdots, i_k$ in $Cut(U_D)$ actually disconnect s and t, and is given by

$$\begin{aligned} & p_{disc}(Cut(i_1, \cdots, i_k | U_D)) \\ = \ & \Pi_{i \in N_{cut}(i_1) \cup \cdots \cup N_{cut}(i_k)} (1 - p(i)) \\ & \Pi_{i,j \in L_{cut}(i_1) \cup \cdots \cup L_{cut}(i_k)} (1 - p(i,j)). \quad (15) \end{aligned}$$

Here, $N_{cut}(i)$ and $L_{cut}(i)$ are the set of nodes and the set of links in cutset i. (3) By noting that the cutsets in a cutset group are non-exclusive for a given U_D,

$$\begin{aligned} & p_{work}(G_i(Cut(U_D))) \\ = \ & 1 - \{ \sum_{i \in G_i(Cut(U_D))} p_{disc}(Cut(i|U_D)) \\ & - \sum_{i_1 < i_2 \in G_i(Cut(U_D))} p_{disc}(Cut(i_1, i_2 | U_D)) + \cdots \}. \end{aligned}$$

V. Optimality

A. Optimal physical route

Here, we discuss the change in $\Pr(s \not\leftrightarrow t)$ when the physical route of a link changes. In particular, we show that $\Pr(s \not\leftrightarrow t)$ becomes smaller as the convex hull of the physical route of a link becomes smaller, where the locations of nodes are fixed, and that it becomes minimum when the physical route of each link becomes a straight line segment. The key observation is that, when the physical route of a link changes with the fixed locations of nodes, the change in $\Pr(s \not\leftrightarrow t)$ can occur only with the probability that a part of the link is in the disaster area. This result is an extension of a result in [6].

For the analysis, we provide the following lemma. This lemma is clear because of the definition of the convex hull.

Lemma 3: For sets $X_1, X_2 \subset \mathbb{R}^2$ where $\overline{X_1} \subseteq \overline{X_2}$, if $X_1 \cap R_G \neq \emptyset$, then $X_2 \cap R_G \neq \emptyset$ for any G.

In the remainder of this section, let $\mathcal{L}_{original}$ denote the original physical route of the links between s and t (the series of solid line segments in each figure in Fig. 4), and $\mathcal{L}_{line}$ denote the straight line segment physical route of the links between s and t (the series of dotted line segments in each figure in Fig. 4). In addition, let $\mathcal{L}_{small}$ be a physical route of the links between s and t such that $\overline{\mathcal{L}_{small}} \subseteq \overline{\mathcal{L}_{original}}$, and $\mathcal{L}_{any}$ be any physical route of the links between s and t.

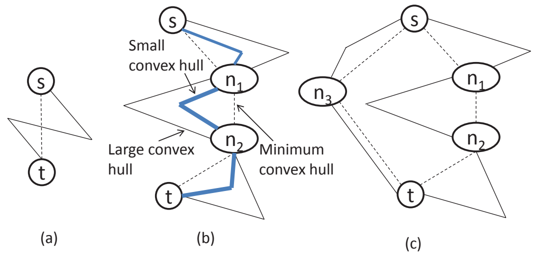


Fig. 4. Model of optimal physical route

1) Tree network: We analyze $\Pr(s \not\leftrightarrow t)$ when there is a single route between s and t.

First, consider the case in which there is no intermediate node between s and t (Fig. 4-(a)). Because of Lemma 3 and the facts that $\overline{\mathcal{L}_{line}} \subseteq \overline{\mathcal{L}_{any}}$ and that $\overline{\mathcal{L}_{small}} \subseteq \overline{\mathcal{L}_{original}}$, (i) if $l(s,t) \cap R_G \neq \emptyset$ for $\mathcal{L}_{small}$, $l(s,t) \cap R_G \neq \emptyset$ for $\mathcal{L}_{original}$ and (ii) if $l(s,t) \cap R_G \neq \emptyset$ for $\mathcal{L}_{line}$, $l(s,t) \cap R_G \neq \emptyset$ for $\mathcal{L}_{any}$. Therefore, we obtain the following lemma.

Lemma 4: The probability that $l(s,t) \cap R_G \neq \emptyset$ for $\mathcal{L}_{small}$ is equal to or smaller than that for $\mathcal{L}_{original}$ and the probability that $l(s,t) \cap R_G \neq \emptyset$ becomes minimum for $\mathcal{L}_{line}$.

As a result, $\Pr(s \not\leftrightarrow t)$ for $\mathcal{L}_{small}$ is equal to or smaller than that for $\mathcal{L}_{original}$, and $\Pr(s \not\leftrightarrow t)$ becomes minimum for $\mathcal{L}_{line}$ when there is no intermediate node.

Next, consider the case in which there are intermediate nodes between s and t (Fig. 4-(b)). Note that, for a link $l(i_j, i_{j+1})$, Lemma 4 is valid. That is, the probability that $l(i_j, i_{j+1}) \cap R_G \neq \emptyset$ for $\mathcal{L}_{small}$ is equal to or smaller than that for $\mathcal{L}_{original}$ and the probability that $l(i_j, i_{j+1}) \cap R_G \neq \emptyset$ becomes minimum for $\mathcal{L}_{line}$ for G. Redefine the physical routes $\mathcal{L}_{line}$ and $\mathcal{L}_{small}$: In $\mathcal{L}_{line}$, all the physical routes of all the links are straight line segments (the series of dotted line segments in Figure 4-(b)). The convex hull of the physical route of each link between s and t in $\mathcal{L}_{small}$ is included in or equal to that of each link in $\mathcal{L}_{original}$ (the series of thick solid line segments in Fig. 4-(b)). Then, $\Pr(s \not\leftrightarrow t)$ for $\mathcal{L}_{small}$ is equal to or smaller than that for $\mathcal{L}_{original}$, and $\Pr(s \not\leftrightarrow t)$ becomes minimum for $\mathcal{L}_{line}$.

2) Generic network: We can easily extend the analysis for a tree network to a generic network (Fig. 4-(c)).

Result 5: $\Pr(s \not\leftrightarrow t)$ for $\mathcal{L}_{small}$ is equal to or smaller than that for $\mathcal{L}_{original}$, and $\Pr(s \not\leftrightarrow t)$ becomes minimum for $\mathcal{L}_{line}$.

This result shows that the line approximation gives the minimum $\Pr(s \not\leftrightarrow t)$ when the physical route of links changes with fixed locations of nodes.

B. Optimal server placement

It is often the case that some equipment, such as a server, is co-located at a physical network node. Because network services are possible with such equipment, a network operator can determine the equipments location by considering its robustness against disaster.

Consider a metric in which node i (or subscribers accommodating node i) cannot access a server co-located at node j. This metric is $\Pr(i \not\leftrightarrow j)$. One of the optimal placement methods of this server is minimizing the worst (or the sum) of $\Pr(i \not\leftrightarrow j)$. That is, to place the server at i^* where $i^* = \mathrm{argmin}_i \max_j$ (or $\sum_j$) $\Pr(i \not\leftrightarrow j)$. The solution of these optimizations can be easily obtained because $\Pr(i \not\leftrightarrow j)$ is given from the results in this paper.

C. Optimal replacement

This subsection analyzes $\Pr(s \not\leftrightarrow t)$ when $p(i)$ or $p(i,j)$ changes. This analysis corresponds to the situation in which a new network component robust against disaster replaces an existing one, and suggests which link or node should be replaced with a new one. In the remainder of this subsection, the locations of nodes and corners are fixed.

First, node replacement on a single route between s and t is analyzed. If we increase $p(j)$, we should choose which j to decrease $\Pr(s \not\leftrightarrow t)$. Due to Eq. (8) and the fact that $\sum_{U_D(r_1(s,t)) \in 2^{U(r_1(s,t))}} h(\overline{U_D(r_1(s,t))}, \overline{U_S(r_1(s,t))})$ is a constant independent of $p(j)$,

$$\Pr(s \not\leftrightarrow t) - \mathrm{const} = -p(j) w_n(j), \tag{16}$$

where $w_n(j) \equiv \sum_{U_D(r_1(s,t)) \in 2^{U(r_1(s,t))}, j \in N_D} P_L(r_1(s,t)|U_D)$ $\Pi_{i \in N_D(r_1(s,t)), i \neq j} p(i) h(\overline{U_D(r_1(s,t))}, \overline{U_S(r_1(s,t))})/2 \geq 0$. Therefore,

Result 6: $\Pr(s \not\leftrightarrow t)$ becomes smaller when we increase $p(j)$ of node-j, which has a larger $w_n(j)$ than when we increase $p(j)$ of node-j, which has a smaller $w_n(j)$.

Next, link replacement on a single route between s and t is analyzed. If we increase $p(i_1, i_2)$, we should choose which i_1, i_2 to decrease $\Pr(s \not\leftrightarrow t)$. Because $P_L(x|U_D) = \Pi_{l(i,j) \in S_L(x|U_D)} p(i,j)$ due to Eq (6), similarly to node replacement,

$$\Pr(s \not\leftrightarrow t) - \mathrm{const} = -p(i_1, i_2) w_l(i_1, i_2), \tag{17}$$

where $w_l(i_1, i_2) \equiv \sum_{U_D(r_1(s,t)) \in 2^{U(r_1(s,t))}, l(i_1,i_2) \in S_L(r_1(s,t)|U_D)}$ $\Pi_{l(i,j) \in S_L(r_1(s,t)|U_D), l(i,j) \neq (i_1,i_2)} p(i,j) \Pi_{i \in N_D(r_1(s,t))} p(i)$ $h(\overline{U_D(r_1(s,t))}, \overline{U_S(r_1(s,t))})/2 \geq 0$. Therefore,

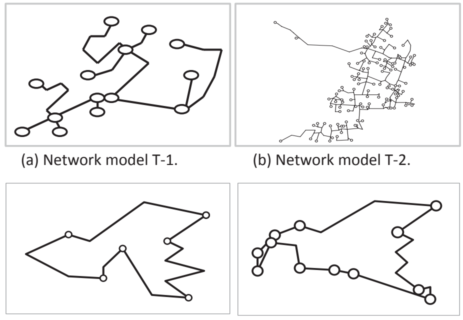
Result 7: $\Pr(s \not\leftrightarrow t)$ becomes smaller when we increase $p(i_1, i_2)$ of $l(i_1, i_2)$, which has a larger $w_l(i_1, i_2)$ than when we increase $p(i_1, i_2)$ of $l(i_1, i_2)$, which has a smaller $w_l(i_1, i_2)$.

Roughly speaking, $w_n(j)$ ($w_l(i_1, i_2)$) is the probability that node-j ($l(i_1, i_2)$) is in the disaster area and that other nodes and links in the disaster are working. Because it is likely that nodes and links near node-j ($l(i_1, i_2)$) are in the disaster when this node (link) is in the disaster, the following node (link) replacement strategy is suggested: If there is a robust part of the network, we should make a node (link) near that part robust.

VI. Numerical examples

A. Validity of results

The proposed formulas were applied to the two tree networks and two ring networks in Fig. 5 to evaluate the validity of the proposed formulas.



(a) Network model T-1. (b) Network model T-2.

(c) Network model R-1. (d) Network model R-2.

Fig. 5. Network models

In the following figures, $\Pr(s \not\leftrightarrow t)$ are plotted when $p(i) = 0.8$ and $p(i,j) = 0.3$ for any i, j. In these figures, $\Pr(s \not\leftrightarrow t)$ is sorted by the exact $\Pr(s \not\leftrightarrow t)$ derived from Eq. (8) for tree networks and that derived from Eq. (12) for ring networks.

For the tree networks, as expected, the independent approximation overestimated $\Pr(s \not\leftrightarrow t)$ and the line approximation underestimated $\Pr(s \not\leftrightarrow t)$ (Fig. 6-(a)). Simultaneously, the independent line approximation, which is a combination of the independent and line approximations, was nearly equal to the independent approximation in these examples. The simulation had good agreement with the exact $\Pr(s \not\leftrightarrow t)$ in Fig. 6-(a).

For the ring networks, the independent approximation may not always overestimate $\Pr(s \not\leftrightarrow t)$. This is because the independent approximation underestimates (actually ignores) the positive correlation of multiple failures due to the simultaneous occurrence of these failure points in the disaster area. As a result, the independent approximation overestimates $\Pr(s \not\leftrightarrow t)$ for a single route due to the independent assumption, but the independent assumption underestimates the occurrence of simultaneous disconnection of two routes. Nevertheless, Fig. 6-(b) shows that the independent approximation overestimates $\Pr(s \not\leftrightarrow t)$ and provides a nearly equal result to the independent line approximation. It also shows that the line approximation underestimates $\Pr(s \not\leftrightarrow t)$, and that the difference between the line approximation and the exact one is small. The simulation had good agreement with the exact $\Pr(s \not\leftrightarrow t)$. (The results for network models T-2 and R-2 are similar to those for network models T-1 and R-1, respectively; thus, they are omitted.)

B. Optimal server placement

For the network model R-2 in Fig. 5, find i^* where $i^* = \mathrm{argmin}_i \max_j \Pr(i \not\leftrightarrow j)$. In this example, node 1 is the node at the upper right corner of R-2 in Fig. 5 and the node

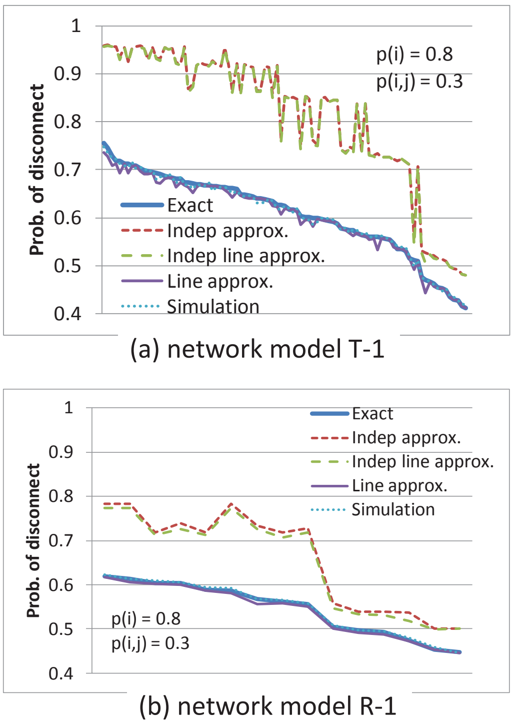


Fig. 6. $\Pr(s \nleftrightarrow t)$ for network models T-1 and R-1

number of each node is placed along the counter-clockwise route. Assume that $p(i) = p(i, i+1) = 0.8$ for all i except for a single weak link. When $l(i', i'+1)$ is the weak link, $p(i', i'+1) = 0.1$. In this example, i' is a variable.

When there is no weak link, the optimal server location i^* is node 2, which is near the center of the network. (Because nodes are densely located in the left half of the network, node 2, which is near the center and slightly in the left half, seems to be optimal.) When there is a single weak link, $i^* = 8$ for $i' = 1, 2, 3, 10, 11$; $i^* = 7$ for $i' = 4$; $i^* = 2$ for other i's. This result means that i^* is often far away from the weak link.

C. Optimal replacement

1) Single route: For network model T-1, each pair of nodes are chosen as node s and node t, the following replacement strategies are tried, and $\Pr(s \nleftrightarrow t)$ is compared under these strategies. Denote the i-th intermediate node as node-i, and the number of intermediate nodes between s and t as n_{int}. As node replacement strategies, strategy N1 assigns large $p(i)$ to node s, node-1 to node-$(n_{int}/2)$, and small $p(i)$ to other intermediate nodes and node t. The other node replacement strategy, strategy N2, assigns large $p(i)$ and small $p(i)$ alternatively to the nodes on the route. That is, it assigns large $p(i)$ to node-s, node-2, node-4, ... and small $p(i)$ to node-1, node-3, ..., and node t.

Similarly, link replacement strategies, strategies L1 and L2, are tried, which are similar to strategies N1 and N2. Strategy L1 assigns large $p(i,j)$ for the first half of the links and small $p(i,j)$ for other links. Strategy L2 assigns large $p(i,j)$ and small $p(i,j)$ alternatively. Strategies N1 and L1 were suggested for node replacement and for link replacement in Section V-C, and strategies N2 and L2 are reference strategies.

Figure 7 shows the performance results of the node and link replacement strategies. The relative difference in $\Pr(s \nleftrightarrow t)$ under two node replacement strategies is defined as $(\Pr(s \nleftrightarrow t|N2) - \Pr(s \nleftrightarrow |N1))/\Pr(s \nleftrightarrow |N1)$, where $\Pr(s \nleftrightarrow t|X)$ is $\Pr(s \nleftrightarrow t)$ using strategy X. For two link replacement strategies, N1 and N2 should be replaced with L1 and L2.

In Fig. 7-(a), large $p(i)$ means $p(i) = 1$ and small $p(i)$ means $p(i) = 0$, while $p(i,j) = 1$ for all (i,j). Similarly, in Fig. 7-(b), large $p(i,j)$ means $p(i,j) = 1$ and small $p(i,j)$ means $p(i,j) = 0$, while $p(i) = 1$ for all i.

Figure 7 shows that the suggested strategies, strategies N1 and L1, can reduce $\Pr(s \nleftrightarrow t)$ for almost all pairs of s and t. Because the suggested strategies do not always assign a large $p(i)$ for node-i of a large $w_n(i)$ or a large $p(i,j)$ for $l(i,j)$ of a large $w_l(i_1, i_2)$, they sometimes fail to reduce $\Pr(s \nleftrightarrow t)$. However, it is impressive that these simple strategies can reduce $\Pr(s \nleftrightarrow t)$ without calculating $w_n(i)$ or $w_l(i_1, i_2)$. In these examples, strategy N1 is slightly more effective than strategy L1.

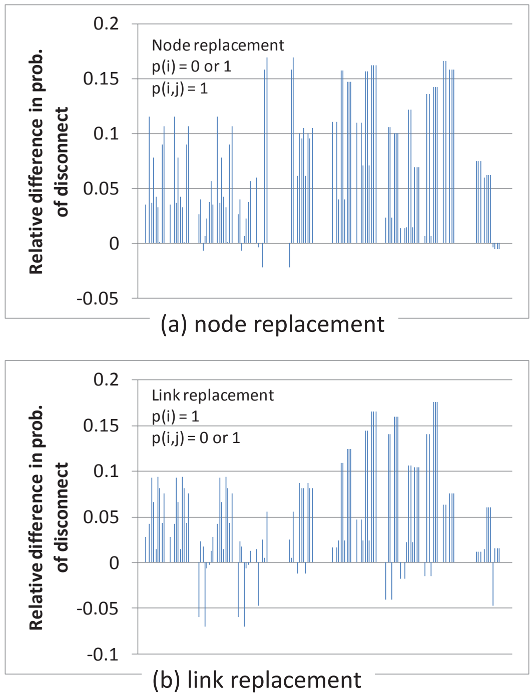


Fig. 7. Relative difference in $\Pr(s \nleftrightarrow t)$ under two node/link replacement strategies for single route

2) Ring network: For a ring network, there are no explicit results such as Results 6 and 7. However, the suggested strategies may work as a rule of thumb to replace nodes or links robust against disasters even for a ring network. Thus, similar to the single route case, the following strategies for network model R-1 are tried, which has six nodes and six links. Strategy N1-(k) for node replacement assigns large $p(i)$ for three consecutive nodes from node-k and small $p(i)$ for other nodes while $p(i,j) = 1$ for all (i,j). In addition, strategy N2-(k) alternatively assigns large $p(i)$ and small $p(i)$ from node-k while $p(i,j) = 1$ for all (i,j). (Strategy N2-(1) is equal to strategy N2-(3), and so on. Therefore, strategy N2-(k) with $k > 2$ can be omitted.) For link replacement, strategy L1-(k) assigns large $p(i,j)$ for three consecutive links from $l(k, k+1)$ and small $p(i,j)$ for other links while $p(i) = 1$ for all i. In addition, strategy L2-(k) alternatively assigns large $p(i,j)$ and small $p(i,j)$ from $l(k, k+1)$ while $p(i) = 1$ for all i. Define $\Pr(s \not\leftrightarrow t|ave)$ as the average of $\Pr(s \not\leftrightarrow t)$ under eight node (link) replacement strategies, that is, Strategies N1-(1) (L1-(1)), ..., N1-(6) (L1-(6)), N2-(1) (L2-(1)), and N2-(2) (L2-(2)), and the normalized probability of disconnect for strategy X as $(\Pr(s \not\leftrightarrow t|X) - \Pr(s \not\leftrightarrow t|ave))/\Pr(s \not\leftrightarrow t|ave)$.

Figure 8 shows the normalized probability of disconnect for each pair of nodes under various node (link) replacement strategies. A negative value means that $\Pr(s \not\leftrightarrow t)$ under a strategy is smaller than the average $\Pr(s \not\leftrightarrow t)$. Figure 8-(a) shows that suggested strategy N1-(k) works, although the effectiveness is limited when we compare it with Fig. 7-(a). On the other hand, Fig. 8-(b) shows that suggested strategy L1-(k) demonstrates almost similar performance with reference strategies L2-(1) and L2-(2).

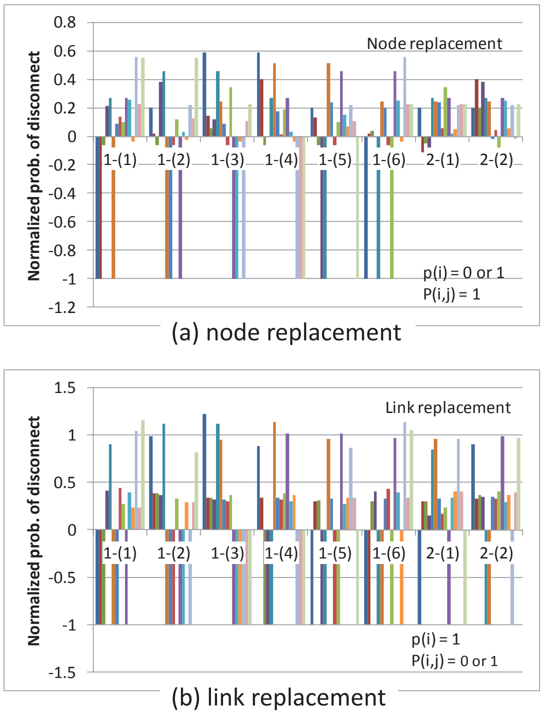


Fig. 8. Normalized $\Pr(s \not\leftrightarrow t)$ under various node/link replacement strategies for ring network

VII. Conclusion

This paper proposed a method for evaluating the probability of end-to-end connectivity with probabilistic failure of links and nodes in a disaster area. Its computing time is the polynomial time of nodes and links for practical cases. The probability that every source-destination pair maintains connectivity was also derived.

In addition, an optimal physical link route, optimal server placement, and optimal node/link replacement were investigated and the following results were derived. (1) By reducing the length of the convex hull of the physical link route, the probability of end-to-end connectivity increases. It reaches maximum when the physical link route of a link between a pair of nodes is on a line. (2) If there is a weak link, the optimal server location is often far away from the weak link. (3) By making a node with a larger w_n or a link with a larger w_l robust, we can make the probability of end-to-end connectivity larger than that by making a node with a smaller w_n or a link with a smaller w_l robust. As a rule of thumb, the following strategy is suggested: if there is a robust part of the network, we should make a node (link) near that part robust.

It was assumed in the analysis of this paper that the boundary of a disaster area can be modeled by a line. Analysis under the assumption that the disaster area is modeled by a bounded region will be shown soon.

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