Delay-Throughput Tradeoff with Correlated Mobility of Ad-Hoc Networks

Shuochao Yao*, Xinbing Wang*, Xiaohua Tian*‡, Qian Zhang†
*Dept. of Electronic Engineering, Shanghai Jiao Tong University, China
†Dept. of Computer Science and Engineering, Hong Kong University of Science and Technology, China
‡State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, China
{sasukecao, xwang8, xtian}@sjtu.edu.cn, qianzh@cse.ust.hk

***Abstract*—We analyze the scaling law in wireless ad hoc networks with the correlated mobility model. The former work about correlated mobility has shown the maximum throughput and the corresponding delay of several sub-cases, but the optimal throughput performances under various delay tolerant condition (the optimal delay-throughput tradeoff) remains open. We study the properties of correlated mobility model and establish the upper bound of delay-throughput tradeoff for several sub-cases. Then we find out an achievable lower bound by studying the optimal scheduling parameters and their constrains. We exploit the node correlation to achieve the delay-throughput tradeoff and give a picture that how node correlation impacts the packet delay, asymptotic throughput, and their tradeoff.**

I. Introduction

Since the breakthrough work by Gupta and Kumar [1], people have shown great interest in the network capacity for large scale wireless ad hoc networks. Gupta and Kumar show us that the per-node throughput can only achieve $O(1/\sqrt{n \log n})$ as the number of nodes n increases in a static network. Then Franceschetti and Dousse [2] show that the $O(1/\sqrt{n})$ per-node throughput is achievable by applying percolation theory, but it is still a pessimistic conclusion for that the network capacity decay rapidly as number of node n increase.

In a seminal work [3], Grossglauser and Tse show that the per-node throughput can achieve $O(1)$ when mobility takes into account, however the cost of improvement in per-node throughput is the unbounded delay. So studies begin to focus on revealing the relationship between the per-node throughput and the packet delay (delay-throughput tradeoff). Neely *et al.* [4] study the fast mobility with *i.i.d.* model. Toumpis *et al.* [5], and Lin *et al.* [6] study the slow mobility with *i.i.d.* model. They show us the impact of *i.i.d.* mobility on the delay-throughput tradeoff.

Several works analyse the delay-throughput tradeoff for different mobility models because the details of how nodes mobile play a role in the tradeoff. From Gamal *et al.* [7] for the Brownian motion mobility model, to Bansal *et al.* [8] for the random waypoint model, Diggavi *et al.* [9] for the linear mobility model, [10], [11], [12] for the restricted mobility model, and Ciullo *et al.* [13] for the correlated mobility model. [14] for general mobility in cognitive networks.

Mobility of nodes in [7] and [8] is uniform over the network area and uncorrelated (i.e., nodes are independent from each other). Mobility of nodes in [10]- [12] introduces the nonuniform condition, but it is still uncorrelated. The real mobilities show high degree of correlation [17], [18], [19], so our work focus on the impact of correlated mobility on the delay-throughput tradeoff of ad hoc network.

Correlated mobility can be divided into three sub-cases according to the node correlation: the cluster sparse regime (nodes show strong correlation), the cluster dense regime (nodes show weak correlation), and the cluster critical regime (nodes show medium correlation). Ciullo *et al.* [13] presents a fine opening for the study of correlated mobility. They show the optimal maximum throughput with the corresponding packet delay under cluster sparse regime and the lower bound of maximum throughput with the corresponding packet delay under cluster dense regime. The optimal throughput performances under various delay tolerant condition (not only the maximum one) are important for the applications require for different packet delays [15] [16], and it is also a hard problem to establish and formulate the optimal redundance creating and messages forwarding scheme because of the various network topologies of different node correlations. So we study the following open question in this paper:

- What is the optimal delay-throughput tradeoff with correlated mobility of ad hoc network?

The goal of our work is to address the question above. We study the properties of the correlated mobility and establish an upper bound on the optimal delay-throughput tradeoff in mobile ad-hoc network with correlated mobility under some sub-cases. Further, we develop a scheduling policy that achieve the upper bound of throughput-delay tradeoff up to a logarithmic factor. During our analysis, a novel "inter-cluster duplication" is specially designed for various degree of node correlation.

According to the delay-throughput tradeoff we have deduced, we have the following observation about the correlated mobility. 1) cluster sparse regime suffers the constraints of maximum throughput and minimum delay, because extremely strong correlation among nodes causes network disconnectivity. 2) cluster critical regimes can perform better than the *i.i.d.* model. 3) Too strong node correlation cannnot help improving the network performance. The main contribution of

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our work is that we are the first to exploit the node correlation in order to give a relatively whole picture that how node correlation impacts the delay-throughput tradeoff, and show how to control the impact through system and scheduling parameters.

The rest of paper is organized as follows. In section II, we introduce our network and mobility model. In section III, we establish the upper bound of cluster sparse regime. In section IV, we show the detailed upper bound of cluster sparse regime by using the optimal scheduling parameters. In section V, we present an achievable lower bound of cluster sparse regime. In section VI, we discuss our delay-throughput tradeoff of correlated mobility. In section VII, we conclude.

II. Network and Mobility Model

A. Network Topology

We consider n nodes moving over a square with area n, and nodes are divided into $m = \Theta(n^v)$ $(0 \leq v < 1)$ groups. Each group covers a circular area with radius $R = \Theta(n^{\beta})$ $(0 \leq \beta \leq 1/2)$. In the following section, we will refer such groups as *clusters*. We assume each group contains $q = n/m$ nodes and the our result won't change if the number of nodes each cluster contains is not exactly the same but remains $\Theta(m/n)$.

We assume time is divided into time slots of unit duration. The positions of nodes are static during each time slot and nodes mobile with correlated fashion between each time slot. We describe the mobility of node i in cluster j with two steps: 1) At the begining of each time slot, the position of cluster j's center is *i.i.d.* and uniformly chosen among the whole network area at random, independently from other clusters; 2) Then the position of node i is *i.i.d.* and uniformly chosen among the circular area that cluster j covers at random, independently from other nodes in cluster j. The above two steps are called group movement and node movement respectively; the combination of them describes the correlated mobility in our work. We observe that nodes show strong correlation if we either reduce the number of clusters m (i.e., smaller value of v) or reduce the area each cluster covers (i.e., smaller value of β).

B. Transmission Protocol

To limit the interference, we adopt the protocol model proposed in [1]. Let X_i denote the position of node i $(i = 1, ..., n)$ and $|X_i - X_j|$ denote the Euclidean distance between node i and j. A transmitter i can transmit at W bit/second successfully to a destination j when

$$|X_j - X_k| \geq (1 + \Delta)|X_i - X_j|$$

for any other simultaneously active transmitters k, where Δ is a positive number.

Slow mobility is considered in our work, which means multihop schedule can be operated within a single time slot. Situation for fast mobility can be extended from our analysis. Moreover, we consider transmission among different clusters, because the transmission within the same cluster is a kind of *i.i.d.* model which has been discussed in many former works.

C. Traffic Model

We assume all sources communicate with their destinations at same rate λ and $\bar{D}$ denote the average delay over all messages among all source-destination pairs.

Definition of Asymptotic Throughput and Delay: Let λ_i $(i = 1, ..., n)$ denote the sustainable rate of data flow for node i and D_b $(b = 1, ..., \lambda nT)$ denote the sustainable data delay for message b. Assume that $\lambda = \min\{\lambda_1, \lambda_2, ..., \lambda_{n-1}, \lambda_n\}$ and $\bar{D} = \sum_{b=1}^{\lambda nT} D_b/\lambda nT$. Then $\lambda = \Theta\big(f(n)\big)$ is defined as the asymptotic throughput if there exist constant $c > c' > 0$, that

$$\lim_{n\to\infty} \mathbf{Pr}(\lambda = cf(n) \text{ is achievable}) < 1,$$
$$\lim_{n\to\infty} \mathbf{Pr}(\lambda = c'f(n) \text{ is achievable}) = 1.$$

And $\bar{D} = \Theta\big(g(n)\big)$ is defined as the asymptotic delay as well.

III. Upper Bound of the Cluster Sparse Regime

We divide our model into three sub-cases: the cluster sparse regime when $v + 2\beta < 1$ (*i.e.*, $mR^2 = o(n)$, strong node correlation), the cluster dense regime when $v + 2\beta > 1$ (*i.e.*, weak node correlation), and the cluster critical regime when $v + 2\beta = 1$ (*i.e.*, medium node correlation). In this paper we mainly focus on the cluster sparse regime and its extension to the cluster critical regime.

Under cluster sparse regime, m clusters only cover a negligible fraction of whole network area. In the other aspect, density in the cluster is relatively high $\big($the density is about $n/(mR^2) = \omega(1)\big)$ and overlaps between different clusters are sporadic, which indicate strong node correlation.

A. Scheduling policy

In this section, we will first design a scheduling policy, referring to some scheduling parameters. Then we propose several lemmas to exclude the parameters not affecting the asymptotic throughput and delay.

For a traffic stream $s \to d$ (where s denotes the source and d denotes the destination), we denote C_s as the cluster containing s and C_d as the cluster containing d. We assume $C_s \neq C_d$, which maximize the character of correlated mobility. Opportunistic broadcast scheme (nodes only broadcast when existing a large number of nodes around or scheme will choose another slot to broadcast) is applied here. Our original scheduling policy is shown as follow:

1) s creates R_s relays in C_s via muticast.
2) When a relay meets a cluster C_k $(k = 1, ..., {R_c}^s$, where ${R_c}^s$ is the maximum number of clusters containing relays under cluster sparse regime) not containing a relay, a relay will be created in C_k via one-hop unicast.
3) New-created relay in C_k creates R_k relays in C_k via broadcast.
4) If a relay meets C_d, a new relay will be created in C_d via one-hop unicast. If not, we come back to step 2).
5) Relays in C_d create new relays in C_d via broadcast (${R_d}^s$ denotes the overall number of relays in C_d).

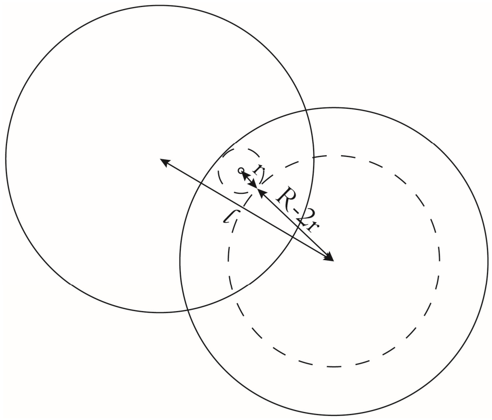

Fig. 1: Upper bound of inter-cluster transmission when only one node in source cluster contains message.

6) If one of ${R_d}^s$ relays is captured by the destination with range l^s, the message will be transmitted to destination via h^s-hop unicast. If not, we come back to step 5).

Our policy can be divided into two parts: forwarding messages form C_s to C_d and forwarding messages within C_d. The network topology of the second part is similar as *i.i.d.* model, so we focus on the first part which reflects the character of correlated mobility.

In the first part, we use term "inter-cluster duplication" to denote the cluster containing relays; we use term "intra-cluster duplication" to denote the relays in a certain cluster. In our original policy, the number of inter-cluster duplications is ${R_c}^s$, and the number of intra-cluster duplications is a set $\{R_s, R_1, ..., R_{{R_c}^s}, R_d\}$. As radio resource is needed to create relays, Lemma 3.1 will help us to simplify the schedule.

Lemma 3.1: Under cluster sparse regime, most intra-cluster duplications $\{R_s, R_1, ..., R_{{R_c}^s}\}$ will decrease the asymptotic throughput without decreasing the asymptotic delay.

Proof: The best situation is that all nodes in the source cluster contain the message. Then probability become $\mathbf{P}[Tr] = \Theta\big((2R+r)^2/n\big) = \Theta(R^2/n)$ (Tr denotes the event that message is successfully sent).

Fig. 1 shows the worst situation where only one node in the source cluster contains the message. Assume that clusters have the identical radius R, then probability become (A denotes the event that two clusters overlap a certain area)

$$\begin{aligned}\mathbf{P}[Tr] &= \int \mathbf{P}[Tr|A]\mathbf{P}[A]dA \\ &= \int_0^{2R} \frac{\arccos(l/2R)R^2 - l/2\sqrt{R^2-(l/2)^2}}{\pi R^2}\frac{2l\pi}{n}dl \\ &= \frac{\pi R^2}{2n} = \Theta\Big(\frac{R^2}{n}\Big)\end{aligned}$$

If two clusters have different radiuses, but remain $\Theta(R)$, it is easy to obtain that $\mathbf{P}[Tr] = \Theta(R^2/n)$ with our former analysis.

The two extreme cases show us the lemma directly ■

We will give detailed proof of Lemma 3.1 in Appendix A. Now we just let it as something we have already proved. So $R_k = 1$ for $k = s, 1, 2, ..., {R_c}^d$. We can particularly use "intra-cluster duplication" to denote relays in C_d. Then our scheduling policy is show below, and we illustrate the scheduling policy in Fig. 2. Opportunistic broadcast scheme (nodes only broadcast when existing a large number of nodes around or scheme will choose another slot to broadcast) is applied here.

1) When s and relays meet a cluster C_k ($k = 1, ..., {R_c}^s$, where ${R_c}^s$ is the maximum number of inter-cluster duplications) not containing a relay, a relay will be created in C_k via one-hop unicast.
2) If a relay meets C_d, a relay will be created in C_d via one-hop unicast. If not, we come back to step 1).
3) Relays in C_d create intra-cluster duplications in C_d via broadcast (${R_d}^s$ denotes the overall number of intra-cluster duplications).
4) If one intra-cluster duplication is captured by the destination with range l^s, the message will be transmitted to destination via h^s-hop unicast. If not, we come back to step 3)

B. Tradeoff for delay

This section will show us a fundamental tradeoff about delay, which is one of the cornerstones for deriving the upper bound of delay-throughput tradeoff. First we will give an intuitive explanation ; then we give the analysis for sophisticated strategy shown above.

Our scheduling policy can be divided into three parts. D_I^s denotes the delay of creating ${R_c}^s$ inter-cluster duplications, D_{II}^s denotes the delay of transmission from ${R_c}^s$ inter-cluster duplications to C_d, and D_{III}^s denotes the delay of transmission within C_d.

As for D_I^s, we assume that $D_I^s = \sum_{k=1}^{{R_c}^s} D_{Ik}^s$, where D_{Ik}^s stands for the delay of creating the kth inter-cluster duplication. We denote $\mathbf{P_I^s}(k)$ as the probability that inter-cluster duplications meet a cluster not containing a relay, when we have already created $k-1$ inter-cluster duplications.

$$\mathbf{P_I^s}(k) = 1 - \Big(1 - \frac{\pi k(2R+r)^2}{n}\Big)^{m-k} \quad (1)$$

Then it's easy to obtain $D_{Ik}^s = 1/\mathbf{P_I^s}(k)$, which leads to

$$\begin{aligned}D_I^s &= \sum_{k=1}^{{R_c}^s} \frac{1}{1-(1-\pi k(2R+r)^2/n)^{m-k}} \\ &\geq \sum_{k=1}^{{R_c}^s} \frac{n}{\pi k(m-k)(2R+r)^2} \\ &= \frac{n}{\pi m(2R+r)^2}\sum_{k=1}^{{R_c}^s} \frac{1}{k} + \frac{1}{m-k} \\ &\geq \Theta\Big(\frac{n}{mR^2}\big(\ln\frac{{R_c}^s m}{m-{R_c}^s} + \gamma\big)\Big)\end{aligned}$$

where γ is the Euler constant and r is the transmission range for a single hop.

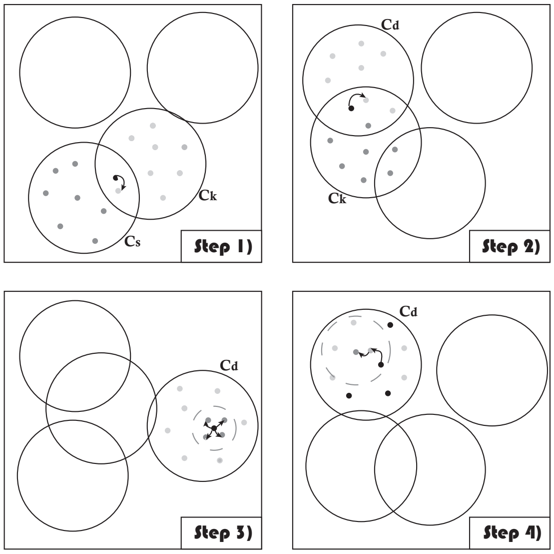

Fig. 2: Scheduling policy under cluster sparse regime.

As for D^k_{II}, the delay of forwarding a message from R_c inter-cluster duplications to C_d can be formulated by

$$\begin{aligned} D^s_{II} &= \frac{1}{1-(1-\pi(R-r)^2/n)^{R_c{}^s}} \\ &\geq \Theta\Big(\frac{n}{R_c{}^s R^2}\Big) \end{aligned}$$

Similarly,

$$D^s_{III} \geq \Theta\Big(\frac{R^2}{R_d{}^s l^2}\Big)$$

Considering these three delays D^s_I, D^s_{II}, and D^s_{III}, the total delay $D^s = \max\{D^s_I, D^s_{II}, D^s_{III}\}$, but $D^s_{I\max} \geq \Theta(n \log m/mR^2)$ and $D^s_{II\min} \geq \Theta(n/mR^2)$, which means D^s_I will not exceed D^s_{II} by a logarithmic factor when considering the bound. Omitting the logarithmic factor, we obtain

$$D^s_b = \max\{D^s_{IIb}, D^s_{IIIb}\}$$

where b stands for a particular bit.

A more sophisticated strategy (which is our schedule shown in the former section) is "opportunistic duplication scheme". A message will be transmitted to its target from the relays, if one of the relays obtains a chance to communicate with target at each time slot t. Otherwise, duplications will be created as normal. This scheme may obtain a better result for $D^s_I + D^s_{II}$ and D^s_{III}, however the following lemma shows that this scheme can only improve the delay with a $\log n$ factor.

Lemma 3.2: Under the cluster sparse regime, the delay for a particular bit b and its scheduling parameters comply the following inequality

$$c^s_1 \log n \mathbb{E}[D^s_b] \geq \max\left\{\frac{n}{R^2\mathbb{E}[R_{cb}^s]}, \frac{R^2}{\mathbb{E}[R_{db}^s]\mathbb{E}[l^s_b + \frac{mR^2}{n^2}]^2}\right\} \tag{2}$$

where c^s_1 is a positive constant and variable X^s_b denotes the variable X under cluster sparse regime for a particular bit b.

The proof of Lemma 3.2 is shown in Appendix A.

C. Tradeoff for radio resource

This section will show us another fundamental tradeoff about radio resource. Firstly, we will recall the disjoint disk. Then we will focus on some special properties under cluster sparse regime. At last, we will obtain the tradeoff between radio resource and related scheduling parameters.

We use protocol model as our communication model, so disjoint disk is a specific model describing limited radio resource, which is first proved in [1].

Consider that nodes i, j directly transmit to nodes k and l respectively, at the same time. Then, according to the interference constraint:

$$\begin{aligned} |X_j - X_k| \geq (1+\Delta)|X_i - X_k| \\ |X_i - X_l| \geq (1+\Delta)|X_j - X_l| \end{aligned}$$

Hence,

$$\begin{aligned} |X_j - X_i| &\geq |X_j - X_k| - |X_i - X_k| \\ &\geq \Delta|X_i - X_k| \end{aligned}$$

Therefore,

$$|X_j - X_i| \geq \frac{\Delta}{2}(|X_i - X_k| + |X_j - X_l|)$$

disks of radius $\Delta|X_i - X_k|/2$, where i, k is a sending-receiving pair, centering at sender are disjoint from each other.

Under the cluster sparse cluster, nodes only cover a small part of network area at each time slot. This phenomenon leads to two properties we need to notice as we deduce the tradeoff.

One is that the area of radio resource we use is only $\Theta(mR^2)$, not $\Theta(n)$ as the *i.i.d.* model. The other is that [13] have proved that a certain cluster has only a probability of mR^2/n to meet other clusters. When creating a inter-cluster duplication, $n/(mR^2)$ chances are needed to ensure a successful operation. Creating R_{cb}^s inter cluster duplications is equivalent to $nR_{cb}^s/(mR^2)$ times one-hop unicast.

Lemma 3.3: Under cluster sparse regime and concerning radio resource, the throughput for a particular bit b and its scheduling parameters comply the following inequality

$$\sum_{b=1}^{\lambda^s nT} \frac{\Delta^2}{4}\frac{\mathbb{E}[R_{db}^s]-1}{n} + \mathbb{E}\left[\sum_{b=1}^{\lambda^s nT}\sum_{h=1}^{h^s_b + \frac{nR_{cb}^s}{mR^2}} \frac{\pi\Delta^2}{4}\frac{{r^h_b}^2}{mR^2}\right] \leq c^s_2 WT \log n \tag{3}$$

where c^s_2 is a positive number, h^s_b is the number of transmission hops after message being captured by the destination, and r^h_b is the transmission range of each hop, $h = 1, ..., h^s_b$. Opportunistic broadcast scheme (nodes only broadcast when existing a large number of nodes around) is applied here.

Proof is similar to Appendix B in [6], so we omit it for simplification.

D. Tradeoff for Half Duplex and Mutihop

Since no node can transmit and receive at the same time and over same frequency, the following inequality holds, *Lemma 3.4:* The following inequality holds,

$$\sum_{b=1}^{\lambda^s nT}\sum_{h=1}^{h_b^s+\frac{nR_{cb}^s}{mR^2}} 1 \leq \frac{WT}{2}n \tag{4}$$

The following inequality holds for the nature of multihop.

Lemma 3.5: The following inequality holds,

$$\sum_{b=1}^{\lambda^s nT}\sum_{h=1}^{h_b^s} r_b^h \geq l_b^s \tag{5}$$

E. Upper bound on delay-throughput tradeoff

The upper bound under cluster sparse regime can be derived from the basic tradeoffs we have proven. In this section, we will separate our proof into two parts. One is $D_{III}^s \geq D_{II}^s$ and the other is $D_{III}^s < D_{II}^s$.

Lemma 3.6: Under cluster sparse regime, when $D_{III}^s \geq D_{II}^s$, let $\bar{D}^s$ denote the mean delay averaged over all bits and let λ^s be the throughput of each source-destination pair. The following upper bound holds,

$$(\lambda^s)^3 \leq O\Big(\frac{m\bar{D}^s}{n}\log^3 n\Big)$$

Proof: From Lemma 3.2, when $D_{III}^s \geq D_{II}^s$, we have

$$c_1^s \log n \mathbb{E}[D_b^s] \geq \frac{R^2}{\mathbb{E}[R_{db}^s](\mathbb{E}[l_b^s]+\frac{mR^2}{n^2})^2}$$

$$\begin{aligned}\sum_{b=1}^{\lambda^s nT}\mathbb{E}[R_{db}^s] &\geq \frac{1}{c_1^s\log n}\sum_{b=1}^{\lambda^s nT}\frac{R^2}{(\mathbb{E}[l_b^s]+\frac{mR^2}{n^2})^2\mathbb{E}[D_b^s]}\\ &\geq \frac{R^2}{c_1^s\log n}\frac{\sum_{b=1}^{\lambda^s nT}1}{\sum_{b=1}^{\lambda^s nT}\mathbb{E}[D_b^s]}\\ &\quad\times\frac{(\sum_{b=1}^{\lambda^s nT}1)^3}{(\sum_{b=1}^{\lambda^s nT}(\mathbb{E}[l_b^s]+\frac{mR^2}{n^2}))^2} \qquad (6)\\ &= \frac{R^2}{c_1^s\log n}\frac{(\sum_{b=1}^{\lambda^s nT}1)^3}{\bar{D}^s(\sum_{b=1}^{\lambda^s nT}(\mathbb{E}[l_b^s]+\frac{mR^2}{n^2}))^2}\end{aligned}$$

Inequality (6) is deduced by using Jensen's Inequality and Hölder's Inequality. From Lemma 3.3 and Cauchy-Schwartz inequality, we obtain

$$\frac{\pi\Delta^2}{2WTnmR^2}\Big(\sum_{b=1}^{\lambda^s nT}\mathbb{E}[\sum_{h=1}^{h_b^s+\frac{nR_{cb}^s}{mR^2}} r_b^h]\Big)^2 + \sum_{b=1}^{\lambda^s nT}\frac{\Delta^2}{4}\frac{\mathbb{E}[R_{db}^s]}{n} - \frac{\Delta^2}{4}\lambda^s T \leq c_2^s WT\log n$$

Case 1: when $h_b^s \geq \frac{nR_{cb}^s}{mR^2}$, then

$$\sum_{b=1}^{\lambda^s nT}\frac{\Delta^2}{4}\frac{\mathbb{E}[R_{db}^s]}{n} + \frac{\pi\Delta^2}{2WTnmR^2}\Big(\sum_{b=1}^{\lambda^s nT}\mathbb{E}[l_b^s]\Big)^2 \leq c_2^s WT\log n$$

$$\frac{\Delta^2 R^2}{4c_1^s n\log n}\frac{(\sum_{b=1}^{\lambda^s nT}1)^3}{\bar{D}^s(\sum_{b=1}^{\lambda^s nT}(\mathbb{E}[l_b^s]+\frac{mR^2}{n^2}))^2} + \frac{\pi\Delta^2}{2WTnmR^2}\Big(\sum_{b=1}^{\lambda^s nT}\mathbb{E}[l_b^s]\Big)^2 \leq c_2^s WT\log n$$

If $\sum_{b=1}^{\lambda^s nT}[l_b^s] < \lambda^s mR^2T/n$,

$$\frac{\Delta^2R^2}{4c_1^s n\log n}\frac{(\lambda^s nT)^3n^2}{\bar{D}^s(\lambda^s mR^2T)^2} \leq c_2^s WT\log n$$

$$\frac{\Delta^2\lambda^s n^4 T}{4c_1^s\bar{D}^s m^2R^2\log n} \leq c_2^s WT\log n$$

$$\lambda^s \leq \frac{4c_1^s c_2^s WT\bar{D}^s m^2R^2\log^2 n}{\Delta^2 n^4 T} \tag{7}$$

If $\sum_{b=1}^{\lambda^s nT}[l_b^s] \geq \lambda^s mR^2T/n$,

$$\frac{\Delta^2R^2}{4c_1^s n\log n}\frac{(\sum_{b=1}^{\lambda^s nT}1)^3}{\bar{D}^s(\sum_{b=1}^{\lambda^s nT}\mathbb{E}[l_b^s])^2} + \frac{\pi\Delta^2}{2WTnmR^2}\Big(\sum_{b=1}^{\lambda^s nT}\mathbb{E}[l_b^s]\Big)^2 \leq c_2^s WT\log n$$

$$\sqrt{\frac{\pi\Delta^2T^2}{8c_1^s W\log n}\frac{(\lambda^s)^3 n}{m\bar{D}^s}} \leq c_2^s WT\log n \tag{8}$$

$$(\lambda^s)^3 \leq \frac{8c_1^s(c_2^s)^2W^3m\bar{D}^s\log^3 n}{\pi\Delta^2 n} \tag{9}$$

Case 2: when $h_b^s \leq \frac{nR_{cb}^s}{mR^2}$, then

Decreasing number of hops h_b^s for each bit will not consume the radio resource asymptotically, however it may decrease the capture range and increase the delay. So we assume $h_b^s = \Theta(nR_{cb}^s/(mR^2))$; all h_b^s and $nR_{cb}^s/(mR^2)$ above are interchangeable when we consider asymptotic throughput and delay.

Finally we compare the two Inequalities (7) and (9). Inequality (9) is the upper bound of delay-throughput tradeoff when $D_{III}^s \geq D_{II}^s$.

$$(\lambda^s)^3 \leq O\Big(\frac{m\bar{D}^s}{n}\log^3 n\Big)$$

■

Lemma 3.7: Under cluster sparse regime, when $D_{III}^s < D_{II}^s$, let $\bar{D}^s$ denote the mean delay averaged over all bits and let λ^s be the throughput of each source-destination pair. The following upper bound holds,

$$\lambda^s \leq O\Big(\frac{mR^4\bar{D}^s}{n^2}\log^3 n\Big)$$

Proof: From Lemma 3.2, when $D_{III}^s < D_{II}^s$, we have

$$c_1^s\log n\mathbb{E}[D_b^s] \geq \frac{n}{R^2\mathbb{E}[R_{cb}^s]}$$

$$\begin{aligned}\sum_{b=1}^{\lambda^s nT}\mathbb{E}[R_{cb}^s] &\geq \frac{1}{c_1^s\log n}\sum_{b=1}^{\lambda^s nT}\frac{n}{R^2\mathbb{E}[D_b^s]}\\ &\geq \frac{n}{c_1^s\log nR^2}\frac{(\sum_{b=1}^{\lambda^s nT}1)^2}{\sum_{b=1}^{\lambda^s nT}\mathbb{E}[D_b^s]} \qquad (10)\\ &= \frac{n(\sum_{b=1}^{\lambda^s nT}1)}{c_1^s\log nR^2\bar{D}^s} \qquad (11)\end{aligned}$$

Inequality (10) is deduced using Jensen's Inequality. From Lemma 3.3 and assume $h_b^s = n^\gamma nR_{cb}^s/(mR^2)$, $(1 \leq h_b^s \leq n/m)$ we obtain

TABLE I: The order of optimal values of the scheduling parameters under cluster sparse regime when $D^s_{III} \geq D^s_{II}$.

$R_{d_b^s}$: # of Intra-cluster duplications	$\Theta(n^{\frac{1-\tilde{d}-v}{3}})$
$R_{c_b^s}$: # of Inter-cluster duplications	$\Theta(n^{1-\tilde{d}-2\beta}/\log n)$
l_b^s: Capture Range	$\Theta(n^{\frac{v+6\beta-2\tilde{d}-1}{6}}/\log^{\frac{1}{2}} n)$
h_b^s: # of Hops	$\Theta(n^{\frac{1-v-\tilde{d}}{3}}/\log n)$
r_b^h: Transmission range of Each Hop	$\Theta(n^{\frac{v-1+2\beta}{2}}\log^{\frac{1}{2}} n)$

TABLE II: The order of optimal values of the scheduling parameters under cluster sparse regime when $D^s_{III} < D^s_{II}$.

$R_{d_b^s}$: # of Intra-cluster duplications	$\Theta(n^{2-v-4\beta-\tilde{d}}/\log^3 n)$
$R_{c_b^s}$: # of Inter-cluster duplications	$\Theta(n^{1-\tilde{d}-2\beta}/\log n)$
l_b^s: Capture Range	$\Theta(\min\{R, n^{\frac{3-v-6\beta-2\tilde{d}}{2}}/\log n\})$
h_b^s: # of Hops	$\Theta(\min\{n^{\frac{1-v}{2}}, n^{2-v-4\beta-\tilde{d}}/\log n\})$
r_b^h: Trans. range of Each Hop	$\Theta(n^{\frac{v-1+2\beta}{2}})$

$$\frac{\pi\Delta^2}{4mnR^2}\sum_{b=1}^{\lambda^s nT}\mathbb{E}\Big[\sum_{h=1}^{\frac{(1+n^\gamma)nR_{c_b^s}}{mR^2}} n{r_b^h}^2\Big] + \sum_{b=1}^{\lambda^s nT}\frac{\Delta^2}{4}\frac{\mathbb{E}[R_{d_b^s}]}{n} \leq c_2^s WT\log n$$

$$\frac{\pi\Delta^2 n}{4m^2R^4}\sum_{b=1}^{\lambda^s nT}\frac{(1+n^\gamma)\mathbb{E}[R_{c_b^s}]\mathbb{E}[{r_b^h}^2]}{\log n} + \sum_{b=1}^{\lambda^s nT}\frac{\Delta^2}{4}\frac{\mathbb{E}[R_{d_b^s}]}{n} \leq 2c_2^s WT\log n \quad (12)$$

$$\frac{\pi\Delta^2 n}{4m^2R^4}\sum_{b=1}^{\lambda^s nT}\frac{(1+n^\gamma)\mathbb{E}[R_{c_b^s}]\mathbb{E}[r_b^h]^2}{\log n} + \sum_{b=1}^{\lambda^s nT}\frac{\Delta^2}{4}\frac{\mathbb{E}[R_{d_b^s}]}{n} \leq 2c_2^s WT\log n \quad (13)$$

Inequality (12) is deduced by Chernoff bound. If the first term in Inequality (13) domains, we use Inequality (11).

$$\frac{\pi\Delta^2 n(1+n^\gamma)}{4m^2R^4\log^2 n}\frac{n\lambda^s nT}{c_1^s R^2\bar{D}^s}\mathbb{E}[r_b^h]^2 \leq 2c_2^s WT\log n$$

$$\lambda^s \leq \frac{8c_1^s c_2^s WT}{\pi\Delta^2}\frac{m^2R^6\bar{D}^s}{n^3} \times\frac{\log^3 n}{(1+n^\gamma)\mathbb{E}[r_b^h]^2} \quad (14)$$

The less $\mathbb{E}[r_b^s]$ and γ are, the better tradeoff will be. Unfortunately, $\mathbb{E}[r_b^s]$ exist a minimum value $\Theta(\sqrt{m/n}R)$, because small $\mathbb{E}[r_b^s]$ may cause connectivity problem [1]. The Inequality (14) become

$$\lambda^s \leq \frac{8c_1^s c_2^s WT}{\pi\Delta^2}\frac{mR^4\bar{D}^s\log^3 n}{n^2}$$
$$\lambda^s \leq O\Big(\frac{mR^4\bar{D}^s}{n^2}\log^3 n\Big) \quad (15)$$

If the second term in Inequality (13) domains,we find $\lambda^s \leq o\big(\frac{mR^4\bar{D}^s}{n^2}\log^3 n\big)$. Then we obtain the result with Inequality (15) ■

Theorem 3.1: Under cluster sparse regime, let $\bar{D}^s$ denote the mean delay averaged over all bits and let λ^s be the throughput of each source-destination pair. The following upper bound holds,

$$\begin{cases} (\lambda^s)^3 \leq O(\frac{m\bar{D}^s}{n}\log^3 n) & D^s_{III} \geq D^s_{II} \\ \lambda^s \leq O(\frac{mR^4\bar{D}^s}{n^2}\log^3 n) & D^s_{III} < D^s_{II} \end{cases}$$

where $\lambda^s \leq mR^2/n$ and $\bar{D}^s \geq n/(mR^2)$

Proof: Using Lemma 3.6, Lemma 3.7, the minimum value of D^s_{II}, and the maximum per-node throughput derived in [13], we can obtain the Theorem directly. ■

IV. Detailed Upper Bound of the Cluster Sparse Regime

In this section, we will obtain the optimal values of scheduling parameters and a delay-throughput tradeoff with detailed separation.

A. Optimal values of scheduling parameters

We assume that the mean delay is $\Theta(n^{\tilde{d}})$. In order to obtain the tight upper bound of tradeoff, Equalities in inequalities (2), (4), (5) and (8) should hold, and some constrains such as l_b^s ($l_b^s \leq R$) and r_b^h ($r_b^h \geq \sqrt{m/n}R$) should be considered. By solving these equations with constraints, we can obtain the optimal values of scheduling parameters. As these process are trivial, we omit it for simplification and show Table I and Table II directly.

B. Detailed tradeoff with optimal values

In this section, we will obtain a detailed picture about the delay-throughput tradeoff with optimal values of scheduling parameters. As a blurry separation ($D^s_{III} \geq D^s_{II}$ and $D^s_{III} < D^s_{II}$) is used in Theorem 3.1, which isn't a direct expression, we can use the optimal values of scheduling parameters to decide the precise separation of our upper bound.

The scheduling parameters suffer some common constraints, $R_{c_b^s} \leq m$, $R_{d_b^s} \leq q$, $l_b^s \leq R$ and $h_b^s \geq 1$. Other constraints are different for two situations $D^s_{III} \geq D^s_{II}$ and $D^s_{III} < D^s_{II}$, so we will discuss them separately.

Case 1: When $D^s_{III} \geq D^s_{II}$

We solve the common constraints with the optimal values of scheduling parameters in Table I omitting the logarithmic factor. Discarding the meaningless results, we obtain $\tilde{d} \geq 1 - v - 2\beta$ and $\tilde{d} \leq 1 - v$, which are equivalent to $\bar{D}^s \geq n/(mR^2)$ and $\bar{D}^s \leq n/m$. These two inequalities are the nature lower bound [13] and upper bound of $\bar{D}^s$ for our cluster spare regime.

There exist two other constraints. One is $D^s_{III} \geq D^s_{II}$ and the other is $h^s_b \geq nR_{cb}^s/(mR^2)$. The first one also comes into the nature bound $\bar{D}^s \geq n/(mR^2)$ and the second one comes into $\tilde{d} \geq 5/2 - v - 6\beta$, which is one of our targets. So our tradeoff can be partly written as

$$(\lambda^s)^3 \leq O\Big(\frac{m\bar{D}^s}{n}\log^3 n\Big) \quad \tilde{d} \geq \frac{5}{2} - v - 6\beta \tag{16}$$

Case 2: when $D^s_{III} < D^s_{II}$

We omit the solution of common constraints, which becomes the nature bound of our network. Another constraint is $D^s_{III} < D^s_{II}$, which comes into $\tilde{d} < 5/2 - v - 6\beta$. Hence

$$\lambda^s \leq O\Big(\frac{mR^2\bar{D}^s}{n^2}\log^3 n\Big) \quad \tilde{d} < \frac{5}{2} - v - 6\beta \tag{17}$$

Theorem 4.1: Under cluster sparse regime, let $\bar{D}^s$ denote the mean delay averaged over all bits and let λ^s be the throughput of each source-destination pair. The following upper bound holds,

$$\begin{cases} (\lambda^s)^3 \leq O(\frac{m\bar{D}^s}{n}\log^3 n) & \tilde{d} \geq \frac{5}{2} - v - 6\beta \\ \lambda^s \leq O(\frac{mR^4\bar{D}^s}{n^2}\log^3 n) & \tilde{d} < \frac{5}{2} - v - 6\beta \end{cases}$$

where $\lambda^s \leq mR^2/n$ and $\bar{D}^s \geq n/(mR^2)$

Proof: Using Inequality (16), Inequality (17), the minimum value of D^s_{II}, and the maximum per-node throughput derived in [13], we can obtain this theorem directly ∎

V. Lower Bound of the Cluster Sparse Regime

We have gotten the upper bound as well as the optimal values of scheduling parameters, so we will construct an achievable lower bound in this section.

We divide our normal time slot into three subslots. The operations of each slot are shown below:

1) The nodes (source node and relays) create inter-cluster duplications and the destination cluster C_d receives messages from inter-cluster duplications via one hop unicast with transmission range r^h_b.
2) R_{db}^s intra-cluster duplications are created via broadcast.
3) Intra-cluster duplication is captured by range l^s_b and transmitted to the destination via h^s_b-hop unicast with single-hop transmission range r^h_b.

The scheduling parameters in our scheme use the optimal values in Table I and Table II. The operations in each slot are similar to the scheduling policy in our upper bound.

In each subslot, we tessellate the network into several cells and employ a cellular time-division multi-access (TDMA) transmission scheme so that each cell is scheduled to be active regularly. When a cell is activated, nodes within it are allowed to transmit to nodes inside the same cell or neighbouring cells. The TDMA transmission scheme allow each cell to have a $1/c^s_3$ fraction of subslot to transmit, where c^s_3 is a constant being independent of the tessellation information. We describe our tradeoff achieving scheme and the network tessellation then.

1) In the 1st subslot, we divide each cluster $\Theta(R^2)$ into $\mathbb{T}^s_1 = q = n^{1-v}$ equal-area cells. Assume that each message has a length of $\lambda^s/\log^2 n \leq mR^2/(nR^s_{cb})$, and all transmissions are employed by one-hop unicast. So each node can transmit at least $nR_{cb}^s/(mR^2)$ messages when it is scheduled to be active. Each cluster has at least a chance of $\Theta\big(mR^2/(n\log n)\big)$ per time slot to communicate with other clusters, which indicates at least $R^s_{cb}/\log n$ messages can be sent per slot and network can sustain $\lambda^s/\log^2 n$ per slot throughput. If each time the network cannot sustain $\Theta\big(mR^2/(n\log n)\big)$ per-node throughput of inter-cluster communication, we denote it as *Error*^s_I. If the network falls to forward a message to its C_d during $\Theta(D^s_{II}\log^2 n)$ time slots, we denote it as *Error*$^s_{II}$.

2) & 3) In the 2rd and 3th subslot, all messages are transmitted in their C_d. Nodes in a certain cluster follow the uniform distribution.The achievable lower bound under uniform condition have been studied widely that the network can achieve $\Theta(\lambda^s/\log n)$ throughput with $\Theta(\bar{D}^s)$ delay. But there exists a problem. If different clusters overlap at a certain area, they will take turns to transmit (under all three subslots). *Error*$^s_{III}$ denote that more that c^s_4 overlap at a certain area, where c^s_4 is a positive number. So each cluster will take at least $1/c^s_4$ length of subslot to transmit.

Theorem 5.1: Error$^s_I \to 0$, *Error*$^s_{II} \to 0$, and *Error*$^s_{III} \to 0$ as $n \to \infty$, So our lower bound under cluster sparse regime can achieve the per-node throughput of $\Theta(\lambda^s/\log^2 n)$ with $\Theta(\bar{D}^s \log^2 n)$ delay.

Proof: We start to prove that *Error*$^s_I \to 0$, *Error*$^s_{II} \to 0$, and *Error*$^s_{III} \to 0$ as $n \to \infty$. All the values of scheduling parameters used in the following proof are chosen from Table I and II.

1) *Error*^s_I:
Let Λ_i $(i = 1, 2, ..., n^2/(mR^2))$ be the amount of data can be transmit under a cell with area of mR^2/n in the network. And $\Lambda = \sum_{i=1}^{n^2/(mR^2)} \Lambda_i$. The probability that at least two nodes from different clusters staying in the same cell area, which is first proposed in [13]:

$$E[\Lambda_i] \leq \big(1 - (1 - \frac{(R - r/\sqrt{2})^2}{n})^m\big)\big(1 - (1 - \frac{r^2}{R^2})^q\big)^2 \times\big(1 - (1 - \frac{(R-r)^2}{n})^{m-1}\big) = \Theta\Big(\frac{m^2R^4}{n^2}\Big)$$

By Chernoff bound, we can obtain that:

$$\mathbf{P}[\Lambda < \Theta(\frac{mR^2}{\log n})] \leq \frac{1}{e^{mR^2/4}} < O\Big(\frac{1}{n}\Big)$$

With $n \to \infty$, $\mathbf{P}[\Lambda < \Theta(mR^2/\log n)] \to 0$, which means that our network can at least sustain a per-node throughput of $\Theta(mR^2/(n\log n))$ of inter-cluster communication. So $\mathbf{P}[$*Error*$^s_I] \to 0$, as $n \to \infty$

2) *Error*$^s_{II}$:
Equivalently, we can calculate the experiment that we throw $n\bar{D}^s\log^2 n$ balls to $(n/R^s_{cb})(n/R^2)$ urns. If no ball falls in a certain urn, *Error*$^s_{II}$ happens (all value of the scheduling parameters are chosen from Table I and

II). We denote the number of ball in each certain urn as X_d^s.

$$\mathbb{E}[X_d^s] = \frac{n\bar{D}^s \log^2}{\frac{n\bar{D}^s R^2 \log n}{n} \frac{n}{R^2}} = \log n$$

Using multiplicative form of Chernoff bound,

$$\mathbf{P}[X_d^s = 0] < \mathbf{P}[X_d^s < \frac{\log n}{2}] < \left(\sqrt{\frac{2}{e}}\right)^{\log n} = O\left(\frac{1}{n}\right)$$

With $n \to \infty$, $\mathbf{P}[X_d^s = 0] \to 0$, which indicates that $\mathbf{P}[Error_{II}^s] \to 0$, as $n \to \infty$.

3) $Error_{III}^s$:
We know that if two cluster have a overlap part, their cluster center must stay in an circle with radius R. Using Chernoff's bound, let $X^o = \sum_{i=1}^{m} X_i^o$ be a random variable, with parameter m and R^2/n (the probability of success of each X_i^o).

$$\mathbf{P}[X^o > c_4^s] < \left(\frac{mR^2 e}{n}\right)^{\frac{n}{mR^2}} < O\left(\frac{1}{n}\right)$$

With $n \to \infty$, $\mathbf{P}[X^o > c_4^s] \to 0$. The overlap in cluster sparse regime only affects the tradeoff with a constant factor, which indicates that $\mathbf{P}[Error_{III}^s] \to 0$, as $n \to \infty$.

$\mathbf{P}[Error_I^s]$, $\mathbf{P}[Error_{II}^s]$, and $\mathbf{P}[Error_{III}^s]$ all come to 0, as $n \to \infty$. ■

VI. Discussion

A. Cluster Critical Regime

We have studied the delay-throughput tradeoff of correlated mobility for the cluster sparse regime ($v+2\beta < 1$). The upper and lower bound of the cluster critical regime ($v + 2\beta = 1$) can be derived from the similar analysis, but the network can perform better, which will be illustrated below.

B. Discussion for correlated mobility

Under the cluster sparse regime, the nodes mobile with strong correlation (i.e., nodes form a few number of clusters or nodes in each cluster mobile within a small range). Then clusters in the network suffer a certain degree of disconnectivity, which restricts the maximum per-node throughput (mR^2/n) and minimum packet delay $\left(n/(mR^2)\right)$. These two constrains greatly degrade the performance of the network; we can neither obtain a high per-node throughput nor a low packet delay, so the application will be limited.

Fig. 3 is the delay-throughput tradeoff of the cluster sparse regime when $v = 5/12$, $\beta = 1/4$. We can see that there still exist a certain range for us to tradeoff. When we design the network to perform with high throughput, the tradeoff will be better than the optimal tradeoff of *i.i.d.* slow mobility model [6]. But our tradeoff become bad if the network is design to perform with low delay.

Under the cluster critical regime, the mobility of nodes show medium correlation, which helps cluster critical regime being the better-performance regime. From Fig. 3, we can see that the delay-throughput tradeoff of the cluster critical regime perform better than the tradeoff of the cluster sparse regime

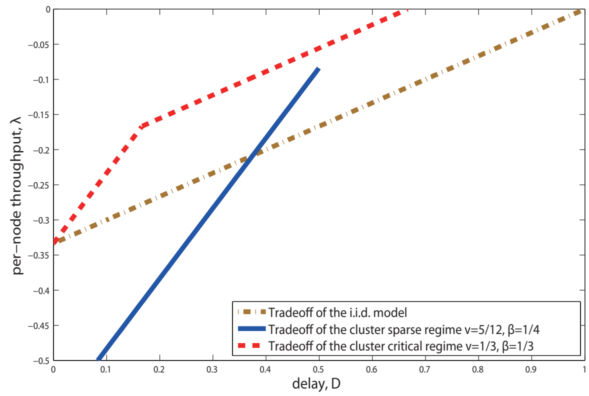

Fig. 3: Tradeoff of the cluster critical regime when $v = 1/3$, $\beta = 1/3$; Tradeoff of the cluster sparse regime when $v = 5/12$, $\beta = 1/4$; Tradeoff of the *i.i.d.* model (The marks on the axes represent the orders asymptotically in n).

and i.i.d. mobility model. A network with specific tradeoff can be designed by controlling the system parameters $m = n^v$ and $R = n^\beta$ using our upper bounds.

VII. Conclusion

The correlated mobility has a huge impact on the delay-throughput tradeoff of mobile ad hoc network. In this paper, we give an relatively whole picture of this mobility model and exploit the node correlation to achieve the delay-throughput tradeoff. We show that medium node correlation can greatly help improving the tradeoff performance. Our study reveals how the system parameters of node correlation improve the tradeoff performance and how to control the improvement through system and scheduling parameters. In the further study about the delay-throughput tradeoff of ad hoc mobile network, node correlation can be introduced to improve the network performance, and we predict that the cluster dense model can also improve the delay-throughput tradeoff.

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APPENDIX A
PROOF OF LEMMA 3.2

To simplify our proof process, some notation are needed. Under the cluster sparse regime, let $X_i^s(t)$ denote the position of node i at time slot t. Let b denote a bit message in our network. Let ${t_0}_b^s$ denote time when bit b is generated. Let $l_b^s\hat{(t)}$ denote the minimum distance form the edge of cluster containing duplication nodes (inter cluster duplication) to the edge of C_d at time slot t, and $l_b^s\hat{(t)}$ can be negative if inter cluster duplication and C_d overlap. Let $L_b^s\hat{(t)} = \max\{0, l_b^s\hat{(t)}\}$. Let ${R_c}_b^s(t)$ denote the number of inter cluster duplications at slot t. Let ${t_s}_b^s$ denote the time when bit b is captured by C_d.

We focus on the transmission of sending bit b from source to its C_d and $\mathbb{I}_A$ be the indicator function on set A

$$\mathbb{E}\left[\frac{n}{(R+L_b^s\hat{(t)})^2}\right] = \mathbb{E}\left[\frac{n}{R^2}\mathbb{I}_{L_b^s(t)\le 0}\right] + \mathbb{E}\left[\frac{n}{(R+l_b^s\hat{(t)})^2}\mathbb{I}_{L_b^s(t)>0}\right]$$

Since the definition of expectation,

$$\begin{aligned}
&\mathbb{E}\left[\frac{n}{(R+l_b^s\hat{(t)})^2}\mathbb{I}_{L_b^s(t)>0}\right] \\
&= \int_0^{\sqrt{n}} \frac{n}{(R+u)^2} d\mathbf{P}[l_b^s\hat{(t)}) \le u] \\
&= 1 - \frac{n}{R^2}\mathbf{P}[l_b^s\hat{(t)} \le 0] + \int_0^{\sqrt{n}} \frac{2n}{(R+u)^3}\mathbf{P}[l_b^s\hat{(t)} \le u]du
\end{aligned}$$

Therefore,

$$\begin{aligned}
\mathbb{E}\left[\frac{n}{(R+L_b^s\hat{(t)})^2}\right] &= 1 + \int_R^{\sqrt{n}} \frac{2n}{u'^3}\mathbf{P}[R + l_b^s\hat{(t)} \le u]du' \\
&= 1 + \int_R^{\sqrt{n}} 2\pi {R_c}_b^s(t)\frac{(R+u')^2}{u'^3}du' \\
&\le 6\pi {R_c}_b^s(t)\log n
\end{aligned}$$

We let

$$W(t) = 6\pi\log n[t - {t_0}_b^s] - \sum_{{t_0}_b^s+1}^{t} \mathbb{E}\left[\frac{n}{(R+L_b^s\hat{(t)})^2 {R_c}_b^s(t)}\mathbb{I}_{t={t_s}_b^s}\right]$$

Then

$$\begin{aligned}
&\mathbb{E}[W(t) - W(t-1)] \\
&= 6\pi\log n - \mathbb{E}\left[\frac{n}{(R+L_b^s\hat{(t)})^2 {R_c}_b^s(t)}\mathbb{I}_{t={t_s}_b^s}\right] \\
&\ge 6\pi\log n - \mathbb{E}\left[\frac{n}{(R+L_b^s\hat{(t)})^2 {R_c}_b^s(t)}\right] \ge 0
\end{aligned}$$

which means that $W(t)$ is a sub-martingale. By the Optional Stopping Theorem. We obtain

$$6\pi\log n\mathbb{E}[{D_{II}}_b^s] \ge \mathbb{E}\left[\frac{n}{(R+L_b^s\hat{(t)})^2 {R_c}_b^s}\right]$$

By Hölder's Inequality

$$\begin{aligned}
6\pi\log n\mathbb{E}[{D_{II}}_b^s] &\ge \frac{n}{\mathbb{E}^2[R+L_b^s\hat{(t)}]\mathbb{E}[{R_c}_b^s]} \\
&\ge \frac{n}{(2R+r_b^s)^2\mathbb{E}[{R_c}_b^s]}
\end{aligned}$$

Therefore,

$$54\pi\log n\mathbb{E}[{D_{II}}_b^s] \ge \frac{n}{R^2\mathbb{E}[{R_c}_b^s]} \quad (18)$$

The part for ${D_{III}}_b^s$ is similar, so we directly give the result:

$$8\pi\log n\mathbb{E}[{D_{III}}_b^s] \ge \frac{R^2}{\mathbb{E}[{R_d}_b^s]\mathbb{E}[l_b^s + \frac{mR^2}{n^2}]^2} \quad (19)$$

Inequality (18) and (19) lead to the Lemma 3.2 directly