

# EasyBid: Enabling Cellular Offloading via Small Players

Zhixue Lu<sup>†</sup>, Prasun Sinha<sup>†</sup> and R. Srikant<sup>‡</sup>

<sup>†</sup>Dept. of CSE, The Ohio State University, Columbus, OH

<sup>‡</sup>Dept. of ECE, Univ. of Illinois at Urbana-Champaign, Urbana, IL

**Abstract**—Data offloading is an increasingly popular mechanism for meeting the rising demands of cellular users. In order to enable the small players, such as businesses and individual owners to make their services available to the bigger wireless service providers (WSPs) to help offload data, a simple, practical and easy-to-use payment machinery needs to be devised. Existing auction mechanisms usually assume that bidders can precisely estimate their true valuations, and they ignore the significant overhead to sellers incurred for obtaining a precise estimation. Such assumption is unrealistic in femtocell networks. To allow imprecise valuations, we introduce the novel concept of *perceived valuation*, which is a value that can be acquired by the seller at little or no cost. We further propose two novel metrics: *partial truthfulness*, and *imprecision loss*, to measure the quality of a truthful auction that accepts perceived valuations. Based on this, we propose EasyBid, a new auction model that provides guarantees for truthfulness even when considering a system with imprecise valuations. Finally, we design a dynamic programming based algorithm which aims to maximize the WSP's utility while satisfying any given constraints on partial truthfulness and imprecision loss. Through simulations, we show that the utility achieved by EasyBid with imprecise valuations can be close to the optimal solution that assumes precise valuations.

## I. INTRODUCTION

Due to the explosive growth of cellular data traffic (world-wide 2x growth every year [1]) and the extreme scarcity of wireless spectrum, wireless service providers (WSPs) are turning to solutions that include small cells (such as femtocells and WiFi hotspots). However, deploying such small cells to provide blanket coverage is neither economically efficient nor practically feasible [2]. As a result, data offloading through third-party owned small cells becomes an increasingly popular mechanism for meeting such needs. In order to enable the small players, such as businesses and individual owners to make their services available to the bigger WSPs to help offload data, a simple, practical and easy-to-use payment machinery needs to be devised.

Auction has been regarded as one of the best known trading forms due to its perceived fairness and flexibility [3]. This paper proposes a reverse auction scheme, wherein small players bid to provide services to a WSP. Truthful auctions, which guarantee that bidding true valuation is a dominant strategy for sellers, have been recently studied in [4]–[8] for data offloading. Most of those works are derived from the well-known Vickrey-Clarke-Groves (VCG) mechanism [9].

This material is based upon work partially supported by the National Science Foundation under Grant CNS-1161490.

However, existing works usually make two unrealistic assumptions: sellers can precisely estimate their true valuations, and they do not change within the lifetime of the auction. In fact, one's true valuation might be hard to acquire, and estimating it could require a lot of personal strategizing [10]–[12] and incur *significant overhead* to sellers. For example, the true valuation of a femtocell owner depends on many factors, including the owner's degree of overload tolerance caused by admitting external users, which further depends on what types of applications (light browsing, video streaming, etc.) the owner is running; and, the electricity usage of femtocells, which might depend on the distance between the femtocells and mobile users; and, the owner's expectation on the generated utility by the femtocell. These factors are hard to precisely estimate. Moreover, the true valuation could vary within a single auction period, which makes the estimation process even more complicated. For example, depending on changes of its overload tolerances and locations of served mobile users, the femtocell owner could have a quickly varying valuation that is unpredictable at the time of submitting its bid. Surprisingly, such issues have been ignored by existing works.

Our paper focuses on solving this practical problem, and makes the following contributions:

- This paper introduces the notions of *Perceived Valuation*, *Partial Truthfulness*, and *Imprecision Loss*, which together characterize the *quality* of a truthful auction while considering imprecision in estimation of the true valuations. This is a first such notion.
- For any given values of the above parameters, with the goal of maximizing the WSP's utility, this paper develops EasyBid, a novel mechanism with heuristic algorithms which enable conducting truthful auctions, considering that the sellers only know their perceived valuations which may differ from their true valuations.
- Through simulations, we show that the utility achieved by EasyBid with imprecise valuations is close to the optimal solution that assumes precise valuations, under reasonable partial truthfulness and imprecision loss constraints.

To the best of our knowledge, this is the first paper in the literature that eliminates estimation overhead for the small players by accepting imprecise valuations in truthful auctions. The rest of the paper is organized as follows, Section II defines the problem. Section III proposes EasyBid in the context of precise valuation. Section IV applies EasyBid to the imprecise valuation scenario and solves the utility optimization problem. Section V evaluates EasyBid through simulations. Section VI

surveys the related work, and Section VII concludes this paper with a discussion on future work.

## II. PROBLEM FORMULATION

### A. Basic Settings

Consider a cellular network which consists of Macro-cell Base Stations, third-party owned Femtocells (or WiFi hotspots), and Mobile Users. This network is geographically and chronologically divided into sub-networks to conduct separate auctions. This paper focuses on one such sub-network that consists of  $M$  femtocells. The WSP and femtocell owners are the buyer and sellers of femtocell services, respectively, and the auction is transparent to mobile users.

Let  $V_f$  denote the true valuation of seller  $f$  ( $1 \leq f \leq M$ ) for each time unit of service on a single channel, which is a hidden value to seller  $f$ . We assume the true valuations are non-negative, and there is a value  $V_{max}$ , such that,  $V_f \in [0, V_{max}]$ ,  $\forall f$ .  $V'_f$  denotes the *perceived valuation* of  $f$ , which is an approximate valuation that is possible for  $f$  to acquire at little or no cost. Assume  $|V_f - V'_f| \leq \epsilon$ ,  $\forall f$  for some constant  $\epsilon$ , which denotes the estimation error of sellers. Similarly, we assume the perceived valuations are also non-negative, and bounded by  $V_{max}$ . Therefore,  $V'_f \in [\max\{0, V_f - \epsilon\}, \min\{V_{max}, V_f + \epsilon\}]$ . We introduce  $V'_f$  to address the following problems: 1)  $f$  cannot precisely estimate  $V_f$ . 2)  $V_f$  is a variable that varies within some range over the validity period of the auction, which can be defined by  $V'_f \in [\max\{0, V_f - \epsilon\}, \min\{V_{max}, V_f + \epsilon\}]$ . To participate in the auction,  $f$  submits a bid denoted by  $B_f$ . A *truthful auction* is redefined as one in which all sellers submit their perceived valuations as their bids, i.e.,  $B_f = V'_f$ ,  $\forall f$ .

Let  $G$  denote the average savings of the WSP for each unit of femtocell service, generated from the benefit of freed up cellular resources, reduced power consumption, etc. Since the WSP can arbitrarily divide the cellular network, e.g., by location, by time (weekday vs weekend, daytime vs nighttime, etc.), and conduct auctions separately, we assume  $G$  is stable and known to the WSP for a given sub-network.

We consider an online auction model in which a service request could be sent to any femtocell at any time, depending on the locations of mobile users, and the request needs to be immediately responded on arrival. Therefore, it assumes there is only one item (service) on transaction in one auction.

### B. Motivation

Let us define the seller's (buyer's) *utility* as the difference of its received payment (saving) reduced by its true valuation (its payment). We illustrate the motivation and objective of EasyBid through the following examples.

**One Femtocell, Precise Valuation:** Consider a network with one femtocell  $f$ . Let  $F_f(\cdot)$  denote the cumulative distribution function (CDF) of  $V_f$  over  $[0, V_{max}]$ , and  $U_{WSP}$  denote the utility of the WSP. Assume  $\epsilon = 0$  and a *reserve price* based optimal auction works in the following way:

- The WSP sets a reserve price  $x$ .
- $f$  submits its bid  $B_f$ .

- $x$  is a cutoff:  $f$  wins the auction and receives a payment of  $x$  if  $B_f \leq x$ .

It is obvious that the auction is *truthful* and *individually rational*, i.e., submitting  $V_f$  is a dominant strategy [13] for  $f$ , and  $f$  is guaranteed to not receive a negative utility. To find an optimal reserve price, note that  $U_{WSP} = F_f(x) \times (G - x)$ , in which,  $F_f(x)$  is the probability that the auction is successful, and  $G - x$  is the utility of the WSP if it is successful.  $U_{WSP}$  can be maximized based on  $F_f(x)$ . Take the uniform distribution for example, the optimal reserve price is  $x = \min\{\frac{G}{2}, V_{max}\}$ . Let  $G = 14$  and  $V_{max} = 10$ , then  $x = \$7$  and  $U_{WSP} = \frac{7}{10} \times (14 - 7) = 4.9$ .

**One Femtocell, Imprecise Valuation:** Now, assume  $\epsilon > 0$ , and  $V'_f$  may or may not be equal to  $V_f$  (a hidden value). The previous mechanism is now revised as follows:

- The WSP sets a reserve price  $x$ .
- $f$  submits its bid  $B_f$ .
- $x - \epsilon$  is the cutoff:  $f$  wins the auction and receives  $x$  if  $B_f \leq x - \epsilon$ .

The cutoff is  $x - \epsilon$  in this auction (compare to  $x$  in the precise valuation auction), such that when  $f$  submits  $V'_f$ , its valuation  $V_f$ , upper bounded by  $V'_f + \epsilon$ , never exceeds the payment  $x$  to guarantee worst-case individual rationality (IR). Since  $U_{WSP} = Pr(V'_f \leq x - \epsilon) \times (G - x)$ , for ease of discussion, assume  $V'_f$  is also uniformly distributed over  $[0, V_{max}]$ . In this case, the optimal reserve price is  $x = \min\{\frac{G+\epsilon}{2}, V_{max}\}$ . When  $G = 14$ ,  $V_{max} = 10$  and  $\epsilon = 2$ ,  $x = \$8$ , and  $U_{WSP} = \frac{8-2}{10} \times (14 - 8) = 3.6$ . Observe that:

- 1)  $U_{WSP}$  is less in this auction, because the auction fails when  $V'_f > x - \epsilon = 6$ , even though it might be the case that  $V_f \leq 8$ , i.e., to achieve worst-case IR, some potential transactions are rejected.
- 2) Submitting  $V'_f$  truthfully is a dominant strategy (DS) for  $f$  only if  $V_f \in [0, 4]$  or  $V_f \in [8, 12]$ . Otherwise, if  $V_f$  is within  $6 \pm \epsilon$ , submitting  $V'_f$  is not necessarily the best choice: if  $V'_f > 6$ ,  $f$  loses some potential utility.
- 3) The loss is 100% if it happens: e.g., assume  $V_f = 5$  and  $V'_f = 7$ , then  $f$ 's maximum possible utility is  $\$8 - 5 = 3$  assuming  $f$  precisely knows  $V_f$ . However,  $f$  actually gets 0 utility (100%) due to the imprecision issue. We call this percentage loss of utility the Imprecision Loss (IL, formally defined later). In an imprecise valuation auction system, sellers' IL needs to be accounted for, in order to incentivize sellers to participate.

**One Femtocell, Imprecise Valuation, Multiple Reserve Prices:** EasyBid can increase the utility of the WSP and reduce the IL of sellers by placing multiple reserve prices. One possible solution with two reserve prices proposed by EasyBid works as follows:

- The WSP sets two reserve prices:  $\$8, \$10$ .
- $f$  submits its bid  $B_f$ .
- Sets a cutoff 4: if  $B_f \in [0, 4)$ , approve the transaction and pay  $f$   $\$8$ ; if  $B_f \in [4, 10]$ , approve the transaction with probability  $\frac{2}{3}$  and pay  $f$   $\$10$  only if it is approved.

This approach guarantees worst-case individual rationality ( $\$8 \geq 4 + \epsilon$ ) and has the following properties:

- 1) *Precision Compatible*: if  $f$  precisely knows  $V_f$ , bidding  $V_f$  truthfully is a dominant strategy for  $f$ . This will be clear after we present EasyBid in Section III.
- 2) *Smaller IL*: Similarly, when  $V_f$  is within  $\pm\epsilon$  of the cutoff value 4,  $f$  could lose part of its potential utility. For example, assume  $V_f = 2$  and  $V'_f = 4$ , then the maximum possible utility  $f$  can get when knowing  $V_f$  is  $(\$8 - V_f) = 6$  (simply let  $B_f = V_f$ ). Without knowing  $V_f$ ,  $f$  submits  $V'_f$ , and its utility is  $(\$10 - V_f) \times \frac{2}{3} = \frac{16}{3}$ . The IL in this case is about 11%. Actually, the worst-case IL is 25% for any pair of  $V_f$  and  $V'_f$  (details omitted).
- 3) *Higher WSP Utility*: The rationale behind decreasing IL is that if the overhead to  $f$  for acquiring a precise valuation (or strategizing on imprecise valuation) is larger than its IL in the auction,  $f$  is likely to accept the loss and submit  $V'_f$  truthfully. Assume this is the case, then the utility of WSP is given by the summation of its expected utility from two possible outcomes:  $Pr(V'_f < 4) \times 1 \times (G - 8) + Pr(4 \leq V'_f \leq 10) \times \frac{2}{3} \times (G - 10) = 4.0$ .

Table I shows a comparison of the single reserve price solution and this solution.

TABLE I. SINGE VS MULTIPLE RESERVE PRICES

Solutions	Single Reserve	Double Reserves
Worst-case IR?	Yes	Yes
WSP Utility	3.6	4.0
DS Range	$[0, 4], [8, 10]$	$[0, 2], [6, 10]$
Non-DS Range	$(4, 8)$	$[2, 6]$
Seller's IL	100%	25%
Seller's Preference	Low	High

**Multiple Competing Sellers:** Unlike the traditional auctions (e.g., VCG) that collect bids from all sellers and determine winners based on bids, EasyBid breaks the multi-seller auction into prioritized sequential one-seller auctions and pays the winner the corresponding reserve price. The priority of sellers can be determined in many ways, e.g., based on their historical service qualities, or the mobile user's received SINRs. For example, suppose sellers  $a$  and  $b$  can both serve a user who needs femtocell service, and  $a$  provides higher SINR than  $b$ . A one-seller auction is first conducted between the WSP and  $a$ . If  $a$  wins the auction, the WSP pays  $a$  a reserve price based on its bid; otherwise, the second auction is conducted between the WSP and  $b$ , and so on. Such a prioritized sequential auction model naturally suits the wireless resource auction in that: It reduces communication overhead between different parties: the WSP and the user only need to communicate with one seller at a time rather than with all sellers; and, the winner determination procedure, depending on the priority of sellers, is more flexible compared to the one that determines winners based on the outcome of the bidding process.

### C. Objective

EasyBid considers a network of  $M$  femtocells with imprecise valuations distributed over  $[0, V_{max}]$ , the CDF of which is denoted by  $F(*)$ . We define the *Partial Truthfulness Factor (PT*

*Factor*) of an auction system as the least probability (worst-case) that submitting one's perceived valuation is a dominant strategy. In the previous example, the PT can be calculated based on the DS range:  $\frac{(2-0)+(10-6)}{10} = 0.6$ . Let  $U_f(V_f)$  and  $U_f(V'_f)$  denote the utility of seller  $f$  when it bids  $V_f$  (assume it knows) and  $V'_f$ , respectively. The *Imprecision Loss (IL)* of seller  $f$ , given by  $\frac{U_f(V_f) - U_f(V'_f)}{U_f(V_f)}$ , is the worst-case fractional loss of utility when  $f$  submits  $V'_f$ . This work assumes that certain requirements over PT and IL have to be met, for the WSP to compete with its opponents and incentivize femtocell owners. Given this, EasyBid seeks to find a multi-reserve-price based solution for the WSP to maximize its utility.

For a solution with  $N$  reserve prices, EasyBid computes three vectors:  $\vec{S}, \vec{R}, \vec{P}$ . Vector  $\vec{S} = \{S_i, i = 1..N\}$  divides  $[0, V_{max}]$  into  $N$  segments with lengths  $S_1..S_N$ , and  $\sum_{i=1}^N S_i = V_{max}$ . This paper uses  $\sum_{i=1}^N \vec{S} = \sum_{i=1}^N S_i$  for short. If the valuation  $V_f$  of some seller  $f$  satisfies  $\sum_{i=1}^{i-1} \vec{S} \leq V_f < \sum_{i=1}^i \vec{S}$ , we say seller  $f$  is in segment  $i$  ( $i \in \{1, ..N\}$ ), denoted by  $f \in S_i$ . (With a slight abuse of notation, we also use  $S_i$  to denote segment  $i$  where it is unambiguous). Finally, let  $f \in S_N$  if  $V_f = V_{max}$ . Each  $S_i$  is also associated with an approval ratio  $R_i$  ( $0 \leq R_i \leq 1$ ) and a payment  $P_i$  ( $P_i \geq 0$ ). For sellers in  $S_i$ ,  $R_i$  denotes the probability of serving an incoming mobile user, while  $P_i$  is the amount of payment made to the sellers if the auction succeeds.  $\vec{R} = \{R_i, i = 1..N\}$ ,  $\vec{P} = \{P_i, i = 1..N\}$  are the vectors of  $R_i$  and  $P_i$  with length  $N$ , respectively. Note that  $\vec{S}$  is used to describe the "cutoffs" in the auction. The solution in the previous example can be denoted by:  $\vec{S} = \{4, 6\}$ ,  $\vec{R} = \{1, \frac{2}{3}\}$ ,  $\vec{P} = \{8, 10\}$ . Formally, for any given  $\epsilon, \alpha, \beta$ , let  $\mathbb{R}_f \triangleq [\max\{0, V_f - \epsilon\}, \min\{V_{max}, V_f + \epsilon\}]$ , then the problem can be defined as follows:

$$\begin{aligned}
 \max_{N, \vec{S}, \vec{R}, \vec{P}} \quad & \sum_{i=1}^N d_i \times R_i \times (G - P_i) \quad \text{s.t.} \quad (1) \\
 IR : \quad & U_f(V'_f) \geq 0, \forall f \in \{1, ..M\}, \forall V'_f \in \mathbb{R}_f \\
 PT : \quad & \frac{|\{f | U_f(V'_f) \geq U_f(b), \forall V'_f \in \mathbb{R}_f, \forall b \in [0, V_{max}]\}|}{M} \geq \alpha \\
 IL : \quad & \frac{U_f(V_f) - U_f(V'_f)}{U_f(V_f)} \leq \beta, \forall f \in \{1, ..M\}, \forall V'_f \in \mathbb{R}_f
 \end{aligned}$$

in which,  $d_i$  is the fraction of sellers that are within  $S_i$ , given by  $F(\sum_{i=1}^i \vec{S}) - F(\sum_{i=1}^{i-1} \vec{S})$ . For any single demand, assume that it could take place at any femtocell with equal opportunity, then  $d_i$  is the probability that it takes place at some femtocell in segment  $i$ . The objective function can be interpreted as the expected utility of WSP from a single demand. The first constraint guarantees worst-case IR for any  $f$  that submits its perceived valuation. The second constraint guarantees that submitting perceived valuations is truthful (has no IL) for at least  $\alpha$  fraction of sellers. For the remainders, the third constraint guarantees that the maximum IL is no more than  $\beta$ .

### III. THE FRAMEWORK OF EASYBID

For ease of understanding, in this section we present the EasyBid framework while assuming sellers can precisely estimate their true valuations, i.e.,  $\epsilon = 0$  and  $V_f = V'_f, \forall f$ . Given any  $N$ , we are to derive constraints over  $\vec{S}, \vec{R}, \vec{P}$  to achieve truthfulness and individual rationality. We will show how this framework is applied to address Problem (1) in Section IV.

### A. Constraints Over $\vec{S}, \vec{R}, \vec{P}$

For any femtocell  $f \in S_i$  with true valuation  $V_f$ , its expected utility from an average demand is given by,

$$U_f(V_f) = R_i \times (P_i - V_f) \quad (2)$$

in which,  $R_i$  is the probability that  $f$  serves incoming demands, and  $P_i$  is the payment it receives if it provides service. To achieve individual rationality,  $P_i$  has to be at least as large as the maximum value in segment  $S_i$ . So,

$$P_i \geq \sum_{j=1}^i \vec{S}_j, \forall i \in \{1, \dots, N\} \quad (3)$$

Note that, by manipulating its bid  $B_f$ ,  $f$  could change the segment number to which it belongs, i.e., it could claim  $f \in S_j (j \neq i)$  instead of  $f \in S_i$ , and get a different set of approval ratio and payment. To guarantee truthfulness, appropriate constraints need to be satisfied, outlined as follows:

**Lemma 3.1:**  $\vec{R}$  is non-increasing in truthful auctions.

*Proof:* It holds trivially when  $N = 1$ . For  $N \geq 2$ , assume there is some increasing subsequence in  $\vec{R}$ , and suppose it occurs at  $i$ : let  $R_i < R_{i+1}$  denote the occurrence of increasing approval ratios, where  $1 \leq i \leq N - 1$ . Let  $f_1 \in S_i$  denote a femtocell whose true valuation  $V_{f1} = \sum_{j=1}^{i-1} \vec{S}_j$  happens to be the minimum in segment  $i$ , and  $f_2 \in S_{i+1}$  denote a femtocell whose true valuation  $V_{f2} = \sum_{j=1}^i \vec{S}_j$  is the minimum in segment  $i+1$ . In truthful auction, there should be no incentive for  $f_1$  to claim  $f_1 \in S_{i+1}$ . Based on the utility function in Equation (2), the utility  $f_1$  receives when truthfully claiming  $f_1 \in S_i$  should be at least as large as when claiming  $f_1 \in S_{i+1}$ :  $R_i \times (P_i - V_{f1}) \geq R_{i+1} \times (P_{i+1} - V_{f1})$ . Given that  $V_{f1} = \sum_{j=1}^{i-1} \vec{S}_j$ , we get  $R_i \times (P_i - \sum_{j=1}^{i-1} \vec{S}_j) \geq R_{i+1} \times (P_{i+1} - \sum_{j=1}^{i-1} \vec{S}_j)$ . Similarly, there is no incentive for  $f_2$  to claim  $f_2 \in S_i$ ,  $R_{i+1} \times (P_{i+1} - V_{f2}) \geq R_i \times (P_i - V_{f2})$ . Given  $V_{f2} = \sum_{j=1}^i \vec{S}_j$ , we get  $R_{i+1} \times (P_{i+1} - \sum_{j=1}^i \vec{S}_j) \geq R_i \times (P_i - \sum_{j=1}^i \vec{S}_j)$ . By adding the previous two inequalities and simplifying, we get  $R_i \times S_i \geq R_{i+1} \times S_i$ , which means  $R_i \geq R_{i+1}$ . This contradicts with our assumption that  $R_i < R_{i+1}$ . ■

**Lemma 3.2:**  $\vec{P}$  is non-decreasing in truthful auctions.

*Proof:* Consider two segments  $S_i$  and  $S_j$ , for which  $i < j$ . Since  $R_i \geq R_j$ , then if  $P_i > P_j$ , all sellers in segment  $j$  would get higher utility if they lie and claim they are in segment  $i$ . Therefore,  $\vec{P}$  is a non-decreasing sequence. ■

Now we use lemmas 3.1 and 3.2 to derive the following constraints to achieve truthfulness. Consider the first segment, and suppose seller  $f \in S_1$ . It is clear that  $f$  will not get higher utility by submitting a bid that is within the same segment as its true valuation, as doing that will not change its approval ratio or payment. To prevent it from submitting a higher bid that belongs to the second segment,  $\forall V_f \in S_1$ , the following constraint has to be satisfied:

$$R_1 \times (P_1 - V_f) \geq R_2 \times (P_2 - V_f), \forall V_f \in S_1 \quad (4)$$

$$\Leftrightarrow \frac{R_2}{R_1} \leq \frac{P_1 - V_f}{P_2 - V_f}, \forall V_f \in S_1 \quad (5)$$

$$\Leftrightarrow \frac{R_2}{R_1} \leq \frac{P_1 - S_1}{P_2 - S_1}. \quad (6)$$

The left hand side of (4) is the utility of bidding its true valuation, while the right hand side is the utility of submitting a higher bid that is within the second segment. Note that since  $P_1/P_2 \leq 1$  and  $0 \leq V_f < S_1$ , the right hand side of (5) achieves the minimum value when  $V_f$  tends to  $S_1$ , and that is how we get (6).

Similarly, to prevent  $f$  submitting a bid that is in the third or other following segments, it requires:

$$R_1 \times (P_1 - V_f) \geq R_3 \times (P_3 - V_f), \forall V_f \in S_1$$

$$\dots \dots \dots R_1 \times (P_1 - V_f) \geq R_N \times (P_N - V_f), \forall V_f \in S_1.$$

To sum up, we have:

$$\frac{R_2}{R_1} \leq \frac{P_1 - S_1}{P_2 - S_1}, \frac{R_3}{R_1} \leq \frac{P_1 - S_1}{P_3 - S_1}, \dots, \frac{R_N}{R_1} \leq \frac{P_1 - S_1}{P_N - S_1}. \quad (7)$$

Now consider sellers in other segments. Suppose  $f \in S_2$ , i.e.,  $\sum_{j=1}^1 \vec{S}_j \leq V_f < \sum_{j=1}^2 \vec{S}_j$ . To prevent  $f$  from placing a higher bid, this condition must hold:

$$\frac{R_3}{R_2} \leq \frac{P_2 - \sum_{j=1}^2 \vec{S}_j}{P_3 - \sum_{j=1}^2 \vec{S}_j}, \dots, \frac{R_N}{R_2} \leq \frac{P_2 - \sum_{j=1}^2 \vec{S}_j}{P_N - \sum_{j=1}^2 \vec{S}_j}. \quad (8)$$

Then by listing similar constraints for all other segments, eventually for segment  $S_{N-1}$ , we obtain:

$$\frac{R_N}{R_{N-1}} \leq \frac{P_{N-1} - \sum_{j=1}^{N-1} \vec{S}_j}{P_N - \sum_{j=1}^{N-1} \vec{S}_j}. \quad (9)$$

Note that, if both of the first constraint of (7) and (8) are satisfied, then by doing a multiplication, we have:

$$\frac{R_3}{R_1} \leq \frac{(P_1 - S_1)(P_2 - S_1 - S_2)}{(P_2 - S_1)(P_3 - S_1 - S_2)} \leq \frac{(P_1 - S_1)}{(P_3 - S_1 - S_2)} \leq \frac{P_1 - S_1}{P_3 - S_1},$$

which means the second constraint of (7) is redundant. Similarly, it is trivial that the  $i^{th}$  constraint of (7) is redundant if both the first constraint of (7) and the  $(i-1)^{th}$  constraint of (8) are satisfied. In the same fashion, we can find that only those first constraints in each constraint group of (7),(8), and up to the segment  $S_{N-1}$  shown in (9) are tight (due to space limitations, details are omitted). Finally, we get the following constraints from those groups after omitting the redundants:

$$\frac{R_{i+1}}{R_i} \leq \frac{P_i - \sum_{j=1}^i \vec{S}_j}{P_{i+1} - \sum_{j=1}^i \vec{S}_j}, 1 \leq i \leq N - 1. \quad (10)$$

Note that (10) lists all the constraints to prevent sellers from placing higher bids than their true valuations. To prevent sellers from receiving higher utilities by placing lower bids that fall into lower segments, we list constraints for each segment in a similar way. With the same set of techniques, we get the following results:

$$\frac{R_{i+1}}{R_i} \geq \frac{P_i - \sum_{j=1}^i \vec{S}_j}{P_{i+1} - \sum_{j=1}^i \vec{S}_j}, 1 \leq i \leq N - 1. \quad (11)$$

By putting (10) and (11) together, we conclude that:

$$\frac{R_{i+1}}{R_i} = \frac{P_i - \sum_{j=1}^i \vec{S}_j}{P_{i+1} - \sum_{j=1}^i \vec{S}_j}, 1 \leq i \leq N - 1. \quad (12)$$

Note that Equation (12) and Lemma 3.1 imply the two well-known properties of Dominant Strategy Equilibrium: *payment identity* and *monotonicity* [14].

### B. Long Term Truthfulness

A solution  $(\{\vec{S}, \vec{R}, \vec{P}\})$  based on the previous discussion guarantees truthfulness of sellers for a single arriving demand. The solution does not need to be repeatedly calculated for every femtocell at every transaction. Instead, it can be applied to all femtocells and for a long term. To guarantee their long-term truthfulness, the following requirements need to be satisfied: 1) The arrivals of demands at any seller cannot be controlled by the seller itself. This can be easily satisfied if the WSP controls the arrivals, or if lets users select femtocells (while on the go) based on their own preferences (e.g., signal strength). 2) The approval ratios are respected. Note that the approval ratio at a femtocell might not be fulfilled if a demand arrives while the resources are depleted. For this, one possible solution is to amortize the seller with a future demand. We leave this for future study, and instead assume that there are sufficient channels available at all femtocells. With increasing number of femtocells, we believe that the number of channels will typically not be limited due to increased channel reuse. We also study in the simulation section how the truthfulness gets affected when this assumption does not hold.

**Theorem 3.1:** Assume the arrivals of demands are independent of  $B_f$  for any given  $f$ , and sellers know their precise valuations. Given sufficient resources at local femtocells, a solution that follows lemma 3.1, lemma 3.2 and constraints (3) and (12) is truthful and individually rational.

*Proof:* Section III-A shows that no seller can achieve higher utility by lying for a single demand. The independence of arrivals and sufficient resources assure that the seller cannot achieve higher long-term utility by manipulating its bid. Therefore, EasyBid is truthful. ■

### C. Implement EasyBid For Data Offloading

The EasyBid based online auction model can be implemented in real systems as follows: A central server computes a solution consisting of  $\vec{S}, \vec{R}, \vec{P}$ . Local femtocells then submit their bids to the central server. For any femtocell  $f$ , the server returns one corresponding approval ratio and payment to  $f$ , based on  $B_f$  and vector  $\vec{S}$ . When a mobile user sends a demand of service to  $f$ ,  $f$  serves this user with probability  $R_i$ , and receives a payment  $P_i$  if  $f$  actually provides the service. After being admitted, the mobile user continues to receive service from  $f$  until it moves out of its range.

Consider the scenario shown in Figure 1 in which a user moves across a network of 4 femtocells. Upon reaching point A, the user sends a demand to  $a$ .  $a$  serves this user with probability  $R_1$ . If the user is approved, then it continues to receive service from  $a$ , while  $a$  receives a payment of  $P_1$ . Otherwise, it reaches B without receiving femtocell service. (this does not mean the user's cellular service is discontinued, and instead, it will receive service from the macrocell, which is offloaded by successful auctions at other users.) At point B, this user sends a new demand to seller  $c$  ( $c$ 's signal strength is higher than  $b$ ). Seller  $c$  approves with a probability of  $R_3$ . If the user is denied, then it can send a new demand to seller  $b$ . Otherwise, the user gets served until point D, at which point, a new demand is sent to Seller  $d$ . Note that the expected utility of  $a$  for a given demand is  $1 \times (4 - 1) = 3$ , while it is

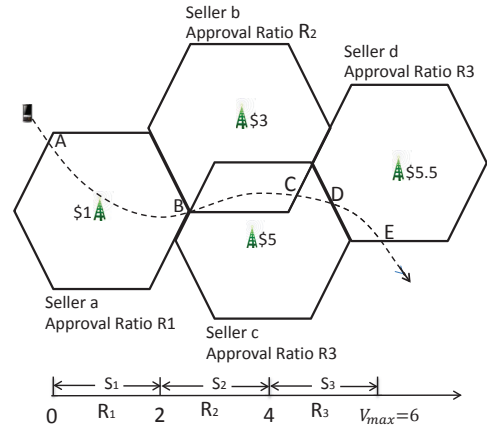


Fig. 1. A user travels across a 4-seller femtocell network. Suppose  $V_a = 1, V_b = 3, V_c = 5, V_d = 5.5$  and  $V_{max} = 6$ . For  $N = 3$ , one simple solution is  $\{S_1 = S_2 = S_3 = 2\}, \{R_1 = 1, R_2 = \frac{1}{2}, R_3 = \frac{1}{4}\}, \{P_1 = 4, P_2 = 6, P_3 = 8\}$ . Since  $a \in S_1, b \in S_2, c, d \in S_3$ , seller  $a$  uses  $R_1$  as its approval ratio, seller  $b$  uses  $R_2$ , and sellers  $c$  and  $d$  use  $R_3$ .

$1/2 \times (6 - 1) = 5/2$  if  $a$  lies to  $S_2$ , and  $1/4 \times (8 - 1) = 7/4$  if  $a$  lies to  $S_3$ . Therefore,  $a$  has no incentive to lie. Similar arguments also hold for other sellers.

## IV. EASYBID: DEALING WITH IMPRECISE VALUATIONS

### A. Understanding the Constraints

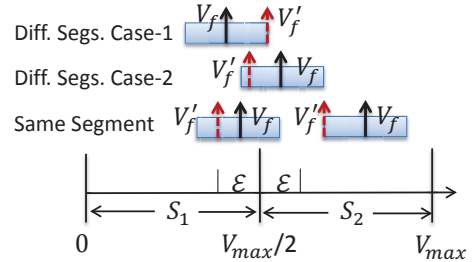


Fig. 2. Divide the range into 2 segments. If  $V_f \in \frac{V_{max}}{2} \pm \epsilon$ ,  $V_f$  and  $V'_f$  may or may not be in the same segment.

When  $\epsilon > 0$ , to understand how the constraints in Problem (1) affect the solution  $\{\vec{S}, \vec{R}, \vec{P}\}$ , consider a naive solution that pays  $V_{max}$  to all sellers with approval ratio 1, regardless of the bids. This solution satisfies all three constraints: worst-case IR,  $\alpha - PT$  (it achieves full truthfulness) and  $\beta - IL$  (0 loss if imprecise). However, it is non-optimal to the WSP due to the unique and high payment. By dividing the range into  $N = 2$  segments, say  $[0, V_{max}/2]$  and  $[V_{max}/2, V_{max}]$ , two different payments and approval ratios can be assigned based on Equation (12). The WSP could save on payment because  $P_1$  (the payment to sellers in  $S_1$ ) could be smaller than the naive solution. The consequence of this solution is that sellers in  $[\frac{V_{max}}{2} - \epsilon, \frac{V_{max}}{2} + \epsilon]$  are not guaranteed of truthfulness (see Figure 2). If either  $\{V_f \in S_1, V'_f \in S_2\}$ , or  $\{V_f \in S_2, V'_f \in S_1\}$  happens, an IL occurs. The IL of those two conditions are  $\frac{R_1 \times (P_1 - V_f) - R_2 \times (P_2 - V_f)}{R_1 \times (P_1 - V_f)}$  and  $\frac{R_2 \times (P_2 - V_f) - R_1 \times (P_1 - V_f)}{R_2 \times (P_2 - V_f)}$ . Based on the constraints, this solution should satisfy: 1)  $P_1 \geq S_1 + \epsilon, P_2 \geq V_{max}$ . 2)  $F(\frac{V_{max}}{2} + \epsilon) - F(\frac{V_{max}}{2} - \epsilon) \leq 1 - \alpha$ . 3) IL (given above)  $\leq \beta, \forall f$ . To conclude, if seller  $f$ 's true valuation  $V_f$  is at least  $\epsilon$  far from the boundary of any segment (except 0 and  $V_{max}$ ), then  $f$  is guaranteed of PT. Otherwise, it is not. Intuitively, for the WSP, a smaller  $N$  results in higher average

payment and higher  $\alpha$  value, while a larger  $N$  can decrease the average payment as well as the  $\alpha$  value.

### B. Algorithms

#### Algorithm 1: Solve Utility Maximization Problem

```

1 input:  $\alpha, \beta, \epsilon$ , range  $[0, 1]$ ,  $F(*)$ 
2 output:  $\{\vec{S}, \vec{R}, \vec{P}, U\}$  //  $U$ : utility
3 for ( $x \leftarrow 1; x \geq 0; x \leftarrow x - 1/n$ ) do
4   for ( $w \leftarrow 0; w \leq 1 - \alpha; w \leftarrow w + 1/m$ ) do
5     for ( $y \leftarrow x + 1/n; y \leq 1; y \leftarrow y + 1/n$ ) do
6       if ( $NPT([x, y]) \leq w$ ) [use alg.2] then
7         if ( $y = 1$ ) then
8            $e' \leftarrow \{1 - x, 1, 1, (1 - F(x)) \times (G - 1)\}$ 
9           if ( $\max[x][w] = \phi \mid e' > \max[x][w]$ ) then
10             $\max[x][w] \leftarrow e'$ 
11           end
12           continue
13         end
14          $e \leftarrow \max[y][w - NPT([x, y])]$ 
15         if ( $e = \phi$ ) then continue
16          $S_y \leftarrow e.\vec{S}[0]$  // 1st seg in  $e$ 
17          $P_y \leftarrow e.\vec{P}[0]$  // 1st pmt in  $e$ 
18          $S_x \leftarrow y - x$  // length of seg  $[x, y]$ 
19         for ( $P_x \leftarrow y + \epsilon; P_x \leq P_y; P_x \leftarrow P_x + 1/n$ ) do
20            $e' \leftarrow \{[S_x, e.\vec{S}], [1, \frac{P_x - y}{P_y - y} * e.\vec{R}], [P_x, e.\vec{P}]\}$ 
21           if  $!check\beta IL([x, 1], e', \epsilon)$  [use alg.3] then
22             continue
23           end
24            $e'.U \leftarrow (F(y) - F(x)) \times (G - P_x) + \frac{P_x - y}{P_y - y} \times e.U$ 
25           if ( $\max[x][w] = \phi \mid e' > \max[x][w]$ ) then
26              $\max[x][w] \leftarrow e'$ 
27           end
28         end
29       end
30     end
31   end
32 end
33 return
    ( $\max[0][1 - \alpha].U > 0$ )?  $\max[0][1 - \alpha] : \{V_{max}, 0, 0, 0\}$ 

```

Without loss of generality, let  $[0, 1]$  denote the normalized range of  $[0, V_{max}]$ , and assume other values are normalized accordingly. Our solution considers a discrete version of problem (1), in which, the boundaries of segments can only be placed on a finite number of candidate locations. Selecting a subset of boundaries from  $n$  given candidate boundaries with constraints of  $\alpha$  for PT and  $\beta$  for IL is a combinatorial problem: each boundary introduces different percent of non-PT sellers and gives different increases in the utility, depending on the locations of other chosen boundaries. While holding the belief that this problem is NP-hard, we leave it for future work.

We design a dynamic programming based heuristic algorithm (Algorithm 1). The algorithm discretizes the original problem from two perspectives: the range  $[0, 1]$  with an integer  $n$  and the non-PT budget  $(1 - \alpha)$  with an integer  $m$ , wherein  $n$  and  $m$  are two integers that represent the number of discrete values considered. For example, if  $n = 10$ , then only multiples of  $1/10 = 0.1$  are considered as candidate locations of boundaries. If  $1 - \alpha = 0.2$  and  $m = 2$ , then only multiples of  $0.2/2 = 0.1$  are considered as legal budgets. The algorithm takes  $n$  steps to finish (step size  $\frac{1}{n}$ ). At each step, it finds and saves a set of  $m$  solutions for range  $[x, 1]$ ,

with  $x$  started from 1 and decreased to 0 finally. Those  $m$  solutions denote the best solutions for  $[x, 1]$  with  $m$  possible non-PT budget values. At the  $i^{th}$  step (so,  $x = 1 - \frac{i}{n}$ ), the algorithm finds the best solution for  $[x, 1]$  with non-PT budget  $w$  ( $w = \{1..m\} \times \frac{1}{m}$ ) by exhausting all possible lengths of its first segment ( $\frac{1}{n}, \frac{2}{n} \dots \frac{i}{n}$ ). To construct the solution, take  $S_1 = \frac{2}{n}$  for example, now that the first segment is  $[x, x + \frac{2}{n}]$ , the fraction of non-PT users in the first segment can be found based on  $F(x)$ . Let  $NPT[x, y]$  denote this value. Since we are computing a solution with no more than  $w$  budget, the budget left for  $[x + \frac{2}{n}, 1]$  is  $w - NPT[x, y]$ . Then it constructs the solution under consideration by expanding the previously saved solution of range  $[x + \frac{2}{n}, 1]$ , budget  $w - NPT[x, y]$ .

In Algorithm 1,  $\max[y][w]$  is used to save the optimal solution of range  $[y, 1]$  with non-PT budget  $w$  ( $0 \leq w \leq 1 - \alpha$ ). Lines 3-4 iterate for  $x$  and  $w$ , respectively. Line 5 tries to locate the first segment in  $[x, 1]$  by locating its upper boundary  $y$ . Once the first segment  $[x, y]$  is given, the portion of non-PT sellers in this segment is calculated (Line 6) using Algorithm 2. and it has to be less than the current budget  $w$ . If  $y = 1$  (Line 7), which means there is only one segment within  $[x, 1]$ , we construct the solution  $e'$  (Line 8), and save it if it is a better solution (Lines 9-11). Otherwise, it finds the saved solution for segment  $[y, 1]$  with the remaining budget  $[w - NPT(x, y)]$  in Line 14. (The  $\lfloor \cdot \rfloor$  operation rounds the value down to multiples of  $1/m$ .) Line 15 skips current solution if it is not feasible. Line 19 iterates the payment  $P_x$  for the segment  $[x, y]$ , and Line 20 builds the new solution of  $[x, 1]$  using the solution of  $[y, 1]$ . Lines 21-23 check if this solution satisfies  $\beta - IL$  constraint using Algorithm 3. Lines 24-27 calculate the utility of current solution based on the utility of  $[y, 1]$ , and substitute the best solution if it is better than the saved solution in  $\max[x][w]$ . Finally, the best solution for  $[0, 1]$  is returned (Line 33). Note that the three constraints in Problem (1) are addressed in Lines 19, 4, 21, respectively.

Specifically, In Lines 20 and 24, a factor of  $\frac{P_x - y}{P_y - y}$  is multiplied to  $e.\vec{R}$  and  $e.U$ . This is because the utility of  $[y, 1]$  was previously calculated based on the assumption that the approval ratio of its first segment is 1. When another segment  $[x, y]$  is added ahead of it, its new approval ratio is now given by  $\frac{P_x - y}{P_y - y}$  (see Equation 12), in which  $y$  equals to the summed length of all segments between  $[0, y]$ . This factor needs to be multiplied to all other approval ratios in  $e.\vec{R}$ , which depend on the first one. Due to this, a better solution ( $>$ ) in Lines 9 and 25 is defined in the way that takes this factor into account: given two solutions of range  $[x, 1]$ ,  $e1$  and  $e2$ , let  $e1.\vec{P}[0]$  and  $e2.\vec{P}[0]$  denote the first payment element in their payment vectors,  $e1$  is better than  $e2$  when,

$$\begin{cases} \frac{e1.U}{e1.\vec{P}[0] - x} > \frac{e2.U}{e2.\vec{P}[0] - x} & \text{if } x > 0 \\ e1.U > e2.U & \text{if } x = 0 \end{cases}$$

Algorithm 2 calculates the portion of non-PT sellers for a given segment  $[x, y]$ . The idea is that if  $V_f$  is at least  $\epsilon$  far from the boundary (when the boundary is not 0 or 1), then  $f$  is counted as PT, otherwise, it is non-PT. Lines 4-5 require the minimum true valuation of a PT seller has to be at least  $\epsilon$  larger than its lower boundary (except 0), and at least  $\epsilon$  smaller

**Algorithm 2:** Calculate Non-PT In  $[x, y]$ 


---

```

1 input:  $[x, y], \epsilon, F(\cdot)$ 
2 output: Fraction of Non-PT Sellers
3  $\alpha' \leftarrow 0$  // the PT fraction
4  $dsStart \leftarrow (x = 0)?x : x + \epsilon$ 
5  $dsEnd \leftarrow (y = 1)?y : y - \epsilon$ 
6 if  $(dsEnd \geq dsStart)$  then
7    $\alpha' \leftarrow F(dsEnd) - F(dsStart)$ 
8 end
9 Return  $F(y) - F(x) - \alpha'$ 

```

---

than its upper boundary (except 1). The fraction of sellers that satisfy this constraint is calculated in Line 7. Line 9 returns the portion of non-PT as the complement of PT.

Algorithm 3 checks if a given solution for range  $[x, 1]$  meets  $\beta - IL$  requirement. For seller  $f$ , it can be shown that the maximum loss happens when the imprecision is maximized, i.e.,  $|V_f - V'_f| = \epsilon$ . Due to space limitations, we omit the details here. In Algorithm 3, for each segment  $S_i$ , it finds the minimum  $V_f$  that could have a corresponding  $V'_f$  in  $S_i$  (Line 6), and the maximum  $V_f$  that could have its  $V'_f$  in  $S_i$  (Line 14). The losses of those two cases are calculated (Line 9 and 17), and both of them have to be less than  $\beta$  (Lines 10-12, Lines 18-20). Based on Lines 5 and 6, the complexity of Algorithm 3 is  $O(n \log n)$  (binary search on Line 6). For this, the complexity of Algorithm 1 is  $O(mn^4 \log n)$ .

**Algorithm 3:** Check For  $\beta - IL$  Requirement

---

```

1 input:  $[x, 1], \{\vec{S}, \vec{R}, \vec{P}\}, \epsilon$ 
2 output: True or False
3  $\vec{L} \leftarrow$  find the sequence of lower boundaries based on  $x$  and  $\vec{S}$ 
4  $\vec{U} \leftarrow$  find the sequence of upper boundaries based on  $x$  and  $\vec{S}$ 
5 for  $i \leftarrow 1$  to  $\vec{S}.length$  do
  // Assume the index of  $\vec{L}, \vec{U}$  starts at 1.
6    $V_f \leftarrow \vec{L}[i] - \epsilon, j \leftarrow$  the segment number of  $V_f$ 
7    $j \leftarrow (j < 1)?1 : j$ 
8   if  $(j < i)$  then
9      $loss = \frac{\vec{R}[j] * (\vec{P}[j] - V_f) - \vec{R}[i] * (\vec{P}[i] - V_f)}{\vec{R}[j] * (\vec{P}[j] - V_f)}$ 
10    if  $(loss > \beta)$  then
11      return False
12    end
13  end
14   $V_f \leftarrow \vec{U}[i] + \epsilon, k \leftarrow$  the segment number of  $V_f$ 
15   $k \leftarrow (k > \vec{S}.length)?\vec{S}.length : k$ 
16  if  $(k > i)$  then
17     $loss = \frac{\vec{R}[k] * (\vec{P}[k] - V_f) - \vec{R}[i] * (\vec{P}[i] - V_f)}{\vec{R}[k] * (\vec{P}[k] - V_f)}$ 
18    if  $(loss > \beta)$  then
19      return False
20    end
21  end
22 end
23 return True

```

---

## V. SIMULATION

## A. Simulation Settings

Our simulation considers a region of  $1000m \times 1000m$ , consisting of  $M$  femtocells (40 by default, varied from 10 – 100). Each femtocell is placed at the center of a  $20m \times 20m$  building, the position of which is randomly selected. Femtocells are

by default assigned sufficient subchannels, while the case of limited subchannels will also be evaluated. We use the LTE module in ns-3 [15] to simulate the wireless communication. The transmission and interference radii are  $100m$  and  $250m$ . For simplicity and ease of understanding, the simulation assumes the true valuations (in \$/second/subchannel) of femtocells are uniformly distributed within  $[0, 1]$  (other distributions are also evaluated), and the perceived valuations are randomly generated within  $\pm\epsilon$  of the true valuations (also  $\in [0, 1]$ ).

Mobile users arrive at this network with certain arrival rate, and perform a random walk after that. Their speeds are randomly generated within  $0 - 2m/s$ . The arrival rate is by default  $5usrs/min$ , and varied from  $1 - 10usrs/min$  (based on the speed profile, the average number of users within the region at any time is about 20 – 200, correspondingly). Users send requests to femtocells if they come across new femtocells and are not receiving services from other femtocells. The requested data rate of each user is randomly chosen from 3 categories:  $\{5Mbps, 500Kbps, 50Kbps\}$ , which represent heavy, intermediate and light network usages, respectively.

The default value of  $G$  is 1 by default (varied in some setups). The parameter  $n$  and  $m$  in Algorithm 1 are 500 and 100, respectively. The auction was conducted for one week in each setup. To evaluate the performance of EasyBid, we first compare EasyBid with VCG and the optimal auction assuming precise valuations, and then assume imprecise valuations and evaluate EasyBid alone by varying different factors ( $\alpha, \beta, \epsilon$ ). VCG is not evaluated for imprecise valuations since it loses truthfulness and worst-case individual rationality: Consider two sellers:  $a$  and  $b$ .  $V_a = \$5$  and  $V'_a = \$3$ , while  $V_b = \$2$  and  $V'_b = \$4$ . If they both submit their perceived valuations truthfully,  $a$  wins since  $3 < 4$ . The payment to  $a$  is the secondary bid  $\$4$ . As a result,  $a$  actually receives  $4 - 5 = -1$  utility, and  $a$  could increase its utility to 0 by lying: submitting any value larger than 4.

## B. Simulation Results

**Precise Valuations: Utilities of the WSP with Variable Femtocell Density, User Density, and Average Saving:** We first assume the valuations are precise, and compare EasyBid with two auction schemes: 1) VCG1: the VCG auction with a reserve price 1. 2) Optimal: the VCG auction with reserve price 0.5, which is the optimal auction in our setup [16].

Figures 3(a) and 3(b) show that the WSP benefits from the increased density of femtocells and users in all three models. This is because higher density of femtocells results in larger coverage area and higher competition among femtocells, which can reduce the average payment. (Note that EasyBid can also take advantage of the competition by polishing the approval ratios and payments.) Meanwhile, higher user density increases the chances of data offloading, thus it also benefits the WSP. The value of  $G$  unsurprisingly affects WSP as shown in Figure 3(c). Overall, EasyBid performs close to the optimal when assuming precise valuations, in spite of the small gap between them, which might be caused by the sub-optimality of the algorithm.

**Imprecise Valuations: Utilities of the WSP with Variable  $\alpha, \beta, \epsilon$ :** When  $\epsilon > 0$ , we study how the values of  $\alpha, \beta$  and  $\epsilon$



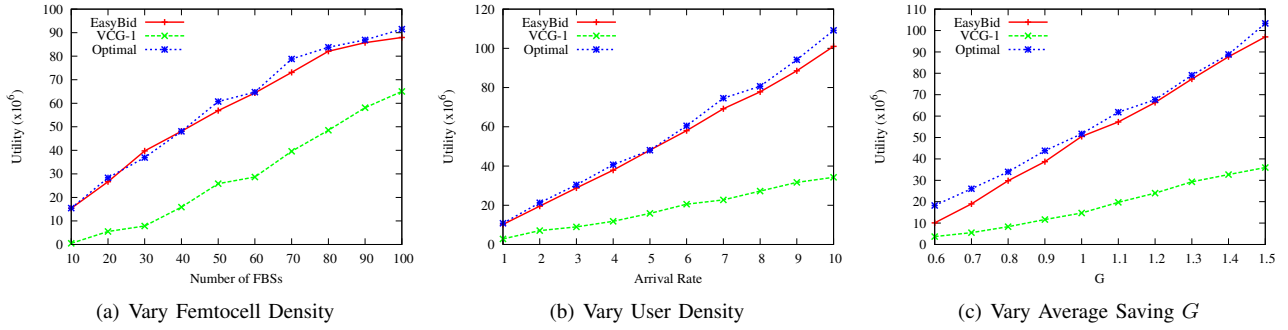


Fig. 3. The utility of the WSP under different femtocell density, user density and average cost, assuming precise valuations. EasyBid performs close to the optimal solution when assuming precise valuations. (a) Vary the number of femtocells, the arrival rate is 5,  $G = 1$ . (b) Vary the arrival rate,  $M = 40$  femtocells and  $G = 1$ . (c) Vary the average saving  $G$ ,  $M = 40$  and arrival rate is 5.

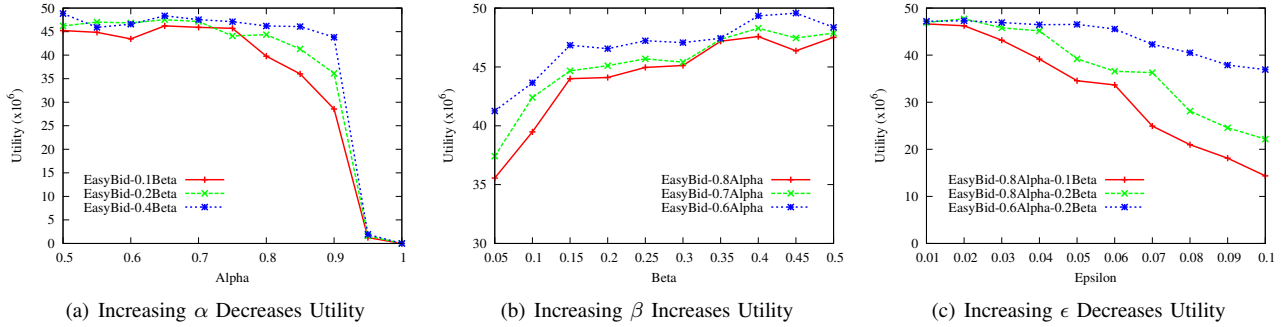


Fig. 4. The utility of the WSP under variables  $\alpha, \beta, \epsilon$  using  $M = 40$  femtocells, 5 arrival rate and  $G = 1$ : assume imprecise valuations. (a) Vary  $\alpha$ , with  $\beta = \{0.1, 0.2, 0.4\}$  and  $\epsilon = 0.04$ . (b) Vary  $\beta$ , with  $\alpha = \{0.6, 0.7, 0.8\}$  and  $\epsilon = 0.04$ . (c) Vary  $\epsilon$  with different  $\alpha, \beta$ .

affect the utility of the WSP, with the default setting of  $M = 40$ , arrival rate = 5 and  $G = 1$ .

Figure 4(a) shows that increasing the value of  $\alpha$  can decrease the utility of the WSP, which means higher partial truthfulness factor is achieved at the cost of compromised utility of the WSP. However, given the same value of  $\alpha$ , increasing the value of  $\beta$  can increase the utility of the WSP, i.e., if sellers are to afford higher imprecision loss, the WSP will receive higher utility. This can be explained by Equation (1): a larger  $\alpha$  means a tighter constraint, while a larger  $\beta$  corresponds to a looser constraint. Note that the utilities in all three conditions are substantially decreased at  $\alpha = 0.95$ . This is because the simulation uses  $\epsilon = 0.04$ , and the algorithm can hardly find a multi-segment solution that guarantees more than 95% of sellers being PT. Note that under the same setup ( $M = 40$ , arrival rate = 5,  $G = 1$ ), the utility was 48.5 in the optimal solution assuming precise valuations. Therefore, we conclude that EasyBid can handle imprecise valuations (given reasonable  $\alpha, \beta, \epsilon$ ) without significantly compromising the utility of the WSP. The result of Figure 4(b) is consistent with above discussion. Figure 4(c) shows that the performance of EasyBid is relatively sensitive to the value of  $\epsilon$ , but its affect could be mitigated by relaxing  $\alpha$  and  $\beta$ .

**Imprecise Valuations: Limited Subchannels:** This simulation evaluates how limiting subchannels affects the truthfulness of the system. It uses 50 femtocells and  $5 \text{ users/min}$  arrival rate. Assuming  $\alpha = 0.8, \beta = 0.1, \epsilon = 0.04$ , we assign different number of subchannels to the femtocells. The  $\{25, \dots, 125\}$  subchannels in Figure 5 correspond to  $5 \text{ MHz} - 25 \text{ MHz}$  in LTE network. For each setup of subchannels, same simulations are

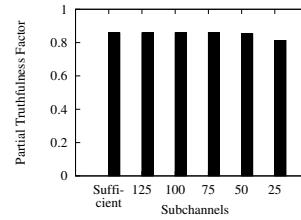


Fig. 5. Limited Subchannels

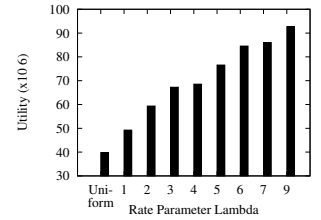


Fig. 6. Non-uniform distribution

repeated such that, for each run, one out of 50 sellers lies to one of the  $N$  segment while others being truthful, i.e.,  $50 * N$  simulations are repeated for one channel profile. The percent of sellers that received their maximum utility when being truthful is calculated and shown in Figure 5.

Note that the PT constraint  $\alpha = 0.8$  is a worst case bound. That is why the percentage of sellers being PT in the figure is actually larger than 0.8 when assuming sufficient subchannels. If the number of subchannels is limited, the percent of truthfulness is not affected until a very small number (50, 25 subchannels), for which the overall truthfulness is still above 0.8. The result shows that the  $\alpha$  constraint is not violated even with a limited number of subchannels.

**Imprecise Valuations: Non-uniform Distribution:** This simulation uses the exponential distribution function within the interval  $[0, 1]$  to generate the true valuations of 40 sellers, such that the distribution of true valuations are biased (to 0) instead of being uniform. Figure 6 shows that the utility of the WSP in general is much higher in this biased distribution, as there are more low-valuation sellers that can accept lower payments.



## VI. RELATED WORK

*Efficient Auctions* and *Optimal Auctions* are the main two types of auctions that aim to maximize the social welfare and the buyer's utility (in reverse auctions), respectively. VCG auctions [9], [17]–[19] are among the most well-studied truthful efficient auctions. Unlike VCG auctions, [16] focuses on designing optimal auctions by a set of tools including posing a reserve price or charging an entry fee. One well-known result is that the optimal auction is simply the VCG auction with an optimal reserve price in simple environments (regular and i.i.d. distribution). Those works assume precise valuations. The topic of imprecise valuation has been covered in some economic works, including [10]–[12], [20]. In [20], the author brings out an intriguing phenomenon called Winner's Curse, which says the winner will tend to overpay (i.e., receive negative utility) in common value forward auctions when bidders cannot precisely estimate the item. [10]–[12] focus on the procedure of valuation discovery, and strategy analysis of bidders. In contrast, this work focuses on handling imprecise valuations through mechanism design.

Existing auction works in the literature of wireless network can be roughly categorized as follows: wireless spectrum trade [21]–[25], cooperative communication [26], [27], and data offloading [4]–[8]. Wireless spectrum trade between primary and secondary owners has been studied with double auctions [21], [22], efficient VCG-based single auction [23], [24] and optimal single auction [25]. The problems studied in those works are different with this paper. [26], [27] study the topic of auction design in cooperative communication. The objective is to maximize bandwidth by maximizing the efficiency of auction, which is different from ours. Finally, efficient [5]–[8] and optimal [4] auction based data offloading has recently gained a lot of interests. [7] focuses on the access control problem in femtocell networks. The authors propose a VCG-based reverse auction framework for fair and efficient access control. [5] proposes to use WiFi for cellular data offloading, and aims to maximize the system efficiency. [8] considers data offloading between multiple network operators and multiple femtocells by proposing a double auction framework aiming to maximize the social welfare. [6] proposes a VCG-based auction framework that aims to maximize efficiency. [4] is the closest work to ours. It proposes an auction framework that allows the WSP to leverage resources from third-party resource owners on demand. The problem of resource allocation between the WSP and third parties is formulated as a linear program, which aims to minimize the cost of the WSP. However, all those existing works simply assume precise valuations and ignore the problem of imprecise valuations, which, in contrast, is considered in this work.

## VII. CONCLUSION AND DISCUSSION

This paper proposes EasyBid, a multiple reserve price based auction mechanism that considers imprecise valuations. Heuristic algorithms that aim to maximize the utility of the WSP under given constraints were presented. Note that although the algorithms assume a priori knowledge of the CDF of true valuations, they can also be applied to the case when the CDF of perceived valuations is known.

Our future works include: 1) Investigate the amortized

arrival method to relax the sufficient-resource assumption. 2) Solve the optimal constraint parameters  $\alpha$  and  $\beta$  by integrating them into the objective function.

## REFERENCES

- [1] Ericsson, "Cellular Data Traffic Keeps Doubling Every Year," <http://arstechnica.com/business/2013/02/cellular-data-traffic-keeps-doubling-every-year>.
- [2] S. C. Forum, "Small Cells, Femtocells and the LTE Business Model," <http://www.smallcellforum.org/resources-presentations>.
- [3] KRISHNA, V., "Auction Theory," *Academic Press*, 2002.
- [4] W. Dong, S. Rallapalli, R. Jana, L. Qiu, K. K. Ramakrishnan, L. V. Razoumov, Y. Zhang, and T. W. Cho, "iDEAL: Incentivized Dynamic Cellular Offloading via Auctions," *In Proc. of IEEE INFOCOM*, 2013.
- [5] S. Paris, F. MARTIGNON, I. Filippini, and L. Chen, "A Bandwidth Trading Marketplace for Mobile Data Offloading," *In Proc. of IEEE INFOCOM (Mini-conference)*, 2013.
- [6] S. Hua, X. Zhuo, and S. S. Panwar, "A Truthful Auction based Incentive Framework for Femtocell Access," *in Proc. of WCNC*, 2013.
- [7] Y. Chen, J. Zhang, Q. Zhang, and J. Jia, "A Reverse Auction Framework for Access Permission Transaction to Promote Hybrid Access in Femtocell Network," *IEEE INFOCOM mini-symposium*, 2012.
- [8] G. Iosifidis, L. Gao, J. Huang, and L. Tassiulas, "An Iterative Double Auction Mechanism for Mobile Data Offloading," *IEEE WiOpt*, 2013.
- [9] J. MacKie-Mason and H. Varian, "Generalized Vickrey Auctions," *working paper*, 1994.
- [10] O. Compte and P. Jehiel, "The Wait-and-See Option in Ascending Price Auctions," *Jnl. of Euro. Economic Association*, vol. 2, no. 2-3, 2004.
- [11] T. Hossain, "Learning by Bidding," *working paper, Deptt of Economics, Hong Kong University of Science and Technology, Hong Kong*, 2004.
- [12] E. Rasmusen, "Strategic Implications of Uncertainty over One's Own Private Value in Auctions," *Advances in Theoretical Economics*, vol. 6, no. 1, 2006.
- [13] Shor, Mikhael, "Dictionary of Game Theory Terms," <http://www.gametheory.net/dictionary/DominantStrategy.html>.
- [14] J. Hartline, "Lectures on Approximation and Mechanism Design," [users.eecs.northwestern.edu/~hartline/amd.pdf](http://users.eecs.northwestern.edu/~hartline/amd.pdf).
- [15] The NS-3 Network Simulator, <http://www.nsnam.org> (June 3, 2013).
- [16] R.B. Myerson, "Optimal Auction Design," *MATH. OPER. RES.*, vol. 6, no. 1, pp. 58–73, 1981.
- [17] W. Vickrey, "Counter-speculation, Auctions, and Competitive Sealed Tenders," *The Journal of Finance*, vol. 16, no. 1, pp. 8–37, 1961.
- [18] E. H. Clarke, "Multipart Pricing of Public Goods," *Public Choice*, vol. 11, pp. 17–33, 1971.
- [19] T. Groves, "Incentives in Teams," *Econometrica*, 1973.
- [20] R. Thaler, "Anomalies: The Winner's Curse," *Journal of Economic Perspectives*, vol. 2, no. 1, pp. 191–202, 1988.
- [21] X. Zhou and H. Zheng, "TRUST: A General Framework for Truthful Double Spectrum Auctions," *In the Proceedings of IEEE INFOCOM*, April 2009.
- [22] S. Wang, P. Xu, X. Xu, S. Tang, and X. Li, "TODA: Truthful Online Double Auction for Spectrum Allocation in Wireless Networks," *DySPAN*, 2010.
- [23] X. Zhou, S. Gandi, S. Suri, and H. Zheng, "eBay in the Sky: Strategy-Proof Wireless Spectrum Auctions," *In proc. of MobiCom*, 2008.
- [24] M. Dong, G. Sun, X. Wang, and Q. Zhang, "Combinatorial Auction with Time-Frequency Flexibility in Cognitive Radio Networks," *In Proc. of IEEE INFOCOM*, 2012.
- [25] J. Jia, Q. Zhang, Q. Zhang, and M. Liu, "Revenue Generation for Truthful Spectrum Auction in Dynamic Spectrum Access," *MobiHoc*, 2009.
- [26] D. Yang, X. Fang, and G. Xue, "Truthful Auction for Cooperative Communications," *MobiHoc*, 2011.
- [27] Y. Zhu, B. Li, and Z. Li, "Truthful Spectrum Auction Design for Secondary Networks," *In the Proceedings of IEEE INFOCOM*, 2012.