A Credit-Token-Based Spectrum Etiquette Framework for Coexistence of Heterogeneous Cognitive Radio Networks

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***Abstract*—The coexistence of cognitive radio (CR) networks in the same swath of spectrum has become an increasingly important problem, which is especially challenging when coexisting networks are heterogeneous (i.e., use different air interface standards), such as the case in TV white spaces. In this paper, we propose a credit-token-based spectrum etiquette framework that enables spectrum sharing among distributed heterogeneous CR networks with equal priority. Specifically, we propose a game-auction coexistence framework. Each network acts as either an offerer or a requester, and coexists with other networks via a non-cooperative game and a truthful multi-winner auction. The framework addresses the trade-offs among social welfare and offerer's revenue in the auction and requester's utility in the game. We prove that the framework guarantees system stability. Our simulation results show that the proposed coexistence framework always converges to a near-optimal distributed solution and improves coexistence fairness and spectrum utilization.**

I. Introduction

The increasing level of congestion in wireless spectrum has spurred a growing interest in cognitive radio (CR) technology [1][2]. Secondary CR networks can coexist with primary incumbent networks by opportunistically accessing fallow white spaces to improve overall spectrum utilization. One major challenge that needs to be addressed in opportunistic spectrum access is the potential interference from secondary users to primary users. For the most part, the problem of primary-secondary spectrum sharing has been addressed by the use of incumbent geolocation databases [3][4] augmented with spectrum sensing techniques. In contrast, however, the coexistence of secondary CR networks with equal priority, especially in the context of heterogeneous coexistence [5], is still a new area. We predict that the problem of secondary-secondary network coexistence will garner great attention in the near future, as the proliferation of secondary white space networks becomes closer to reality, especially in dense urban environments. The IEEE 802 standards committee has recently approved the P802.19.1 project to specify methods for effective coexistence of dissimilar networks in TV white spaces [6].

In this paper, we propose a *credit-token*-based spectrum etiquette framework for the coexistence of distributed heterogeneous CR networks. The framework was inspired by the credit-token-based coexistence protocol (CT-CXP) in IEEE 802.16h [7]. In specific, a two-stage *game-auction* coexistence framework is proposed. At the first stage, coexisting requesting networks (requesters) compete with each other as players in a non-cooperative game to obtain extra access to under-utilized white-space spectrum from collocated offering networks (offerers). At the second stage, each offerer rents out surplus spectrum to requesters through a truthful auction and allows spectrum sharing among multiple auction winners.

In this framework, we address the following challenges. First, coexistence fairness should be ensured. Unlike classical monetary spectrum trading that leads to prioritized spectrum access, our framework aims to ensure fair spectrum sharing, since all offerers and requesters are equal secondary networks. In auction theory, social welfare characterizes the efficiency of resource reallocation, and is an indication of coexistence fairness. Maximizing social welfare avoids the unfair case where under-utilized spectrum is reallocated to a network who has less interest in it. To achieve this, our auction design has to thwart untruthful reporting of spectrum valuations from bidding requesters. Second, spectrum utilization should be maximized dynamically. When the availability of auctioned spectrum is very limited, our auction design has to consider spatial reuse of the spectrum. In a case where bidding requesters can dynamically arrive and depart, online updating of spectrum sharing results has to be supported. In addition, spectrum utilization needs to be maximized to improve each offerer's revenue in a multi-winner auction. Third, the convergence of our framework to a near-optimal equilibrium solution should be guaranteed in a distributed manner. Without a centralized coexistence infrastructure, such as the IEEE 802.19.1 system [6], each requester in a non-cooperative game is likely to act greedily. In the presence of conflicts of interest, our game and auction designs have to always guarantee system stability.

In this paper, we make the following contributions:

- We propose a game-auction coexistence framework, which incorporates a game formulation for supporting distributed spectrum sharing and an auction formulation for ensuring system stability, coexistence fairness, and spectrum reuse. To the best of our knowledge, this framework is the first one addressing all the above aspects for the coexistence of wireless networks with equal priority.
- We address the trade-offs among social welfare, offerer's revenue, and requester's utility. The offerer's strategy tries

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to maximize social welfare as well as offerer's revenue in the auction, while the requester's strategy aims to maximize requester's utility in the game. The proposed strategies guarantee truthful, fair, and efficient spectrum reallocation.
- We show that the proposed coexistence framework is stable, and the resulting distributed equilibrium solution is fairly close to the centralized optimal solution.

The remainder of this paper is organized as follows. Related work is discussed in section II. The game-auction coexistence framework is introduced in section III. The problems of auction and game are studied in sections IV and V, respectively. System stability is proved in section VI. Implementation issues are discussed in section VII. Simulation results are presented in section VIII, and conclusion is drawn in section IX.

II. Related Work

In IEEE 802.16h [7], CT-CXP enables dynamic subframe sharing among collocated coexisting systems. This protocol is based on the idea of auction-based spectrum leasing to reallocate under-utilized spectrum from offering networks to requesting networks. Each network follows certain coexistence etiquette. However, CT-CXP only defines a protocol for the exchange of control messages between offerers and requesters, and it does not provide any details regarding actual algorithm designs. We study auction-based spectrum sharing in the context of a non-cooperative game, and consider many important coexistence issues, such as the trade-offs among objectives of offerers and requesters, spectrum reuse, online auction winning, bidding truthfulness, and system stability.

In CR networks, auction models are usually used to study centralized spectrum trading between primary spectrum sellers (or spectrum brokers) and secondary spectrum buyers [8]-[15]. The maximization of seller's revenue is the primary objective in such monetary auction problems. In our spectrum etiquette framework, however, each offerer maximizes social welfare as well as its revenue to achieve equal spectrum sharing. The need for bidding truthfulness is important to prevent market manipulation [8]-[11]. A truthful double auction is studied in [8], which requires a centralized entity. In contrast, we study a distributed coexistence problem. In [9], bidding truthfulness is achieved by utilizing the distribution of buyers' valuations, which is hard to get in practice. We do not assume such a priori knowledge. The truthful second-price auction is applied to a multi-winner case in [10], where a set of bidders is grouped as a "virtual bidder". However, each individual bidder can still untruthfully get extra benefits and harm other bidders in the same winning group. On the contrary, we ensure that each individual bidder stays truthful to its spectrum valuation. The idea of pricing strategy in [11] is to charge each winner by a critical value, but this method cannot be applied to a multi-winner auction, which is necessary to enable spatial reuse of auctioned spectrum. Bidding truthfulness is not addressed in [12]-[15], which assume that each bidder always reports its true spectrum valuation. However, untruthful bidding can greatly harm the maximization of social welfare.

As for distributed spectrum trading between multiple sellers and multiple buyers, game models are used to characterize the competition among sellers who want to attract buyers so as to maximize their revenue [16]-[19]. In the secondary spectrum market [19], for example, each secondary service provider buys spectrum from primary users and sells the spectrum to secondary users to make a profit. The spectrum resource for sale comes from primary users and is thus relatively abundant. In contrast, we study the competition among requesters in a over-demand (instead of over-supply) case, where the spectrum resource from each secondary offerer is temporary.

III. System Model

In this section, we discuss the system model that underpins the proposed game-auction coexistence framework.

The credit-token-based spectrum etiquette concept in IEEE 802.16h CT-CXP [7] serves as the basis of our auction problem. In this reservation-based coexistence protocol, each collocated network can be either an *offerer* or a *requester* to "sell" or "buy" the rights to access shared spectrum, respectively, in certain under-utilized time subframes for a finite amount of time. An auction is needed if an offerer's claimed subframes cannot accommodate all the requesters. In this case, the selected auction winners need to "pay" *credit tokens* for the granted spectrum access. A credit token is defined as a pseudo monetary unit. For each network, the amount of credit tokens it owns is proportional to its likelihood of getting fallow spectrum from other networks. The coexisting networks can exchange control messages locally by means of a common radio access technology and a common control channel. A mechanism like CT-CXP based on frame reservation has to rely on tight synchronization among coexisting networks, which is challenging to achieve when the coexisting heterogeneous networks use different time frame formats. Hence, we consider a coexistence framework based on channel reservation. Moreover, we study auction-based spectrum sharing in the context of a non-cooperative game, and consider many important coexistence issues, such as optimization trade-offs, spectrum reuse, online auction winning, bidding truthfulness, and system stability. As shown in Fig. 1, we model credit-token-based spectrum sharing problem as a *game-auction* coexistence framework.

Suppose that there are N collocated CR networks. In each network n for $n \in \mathcal{N} \triangleq \{1, \cdots, N\}$, the base station on behalf of the entire network claims a set of master channels, denoted by $\mathcal{M}_n$, via local channel reservation. The initial channel reservation can be on a first-come-first-serve basis, which is assumed in CT-CXP. The channels in $\mathcal{M}_n$ are fallow white spaces locally available to network n. Each network queries a geolocation database or performs spectrum sensing to identify the set of white-space channels and claims $\mathcal{M}_n$ from it. The number of master channels and the MAC protocol (with or without channel aggregation [20]) used to access those channels may be different for each network, since coexisting networks may use heterogeneous air interface standards. Other neighboring networks can still access $\mathcal{M}_n$ as slave channels under the control of network n. When a network o has surplus spectrum, it can act as an offerer and rent out a subset of $\mathcal{M}_o$, say $\mathcal{G}_o \subseteq \mathcal{M}_o$. The set of offerers is denoted by $\mathcal{O} \subseteq \mathcal{N}$. When a network r needs extra spectrum, it can act as a requester and

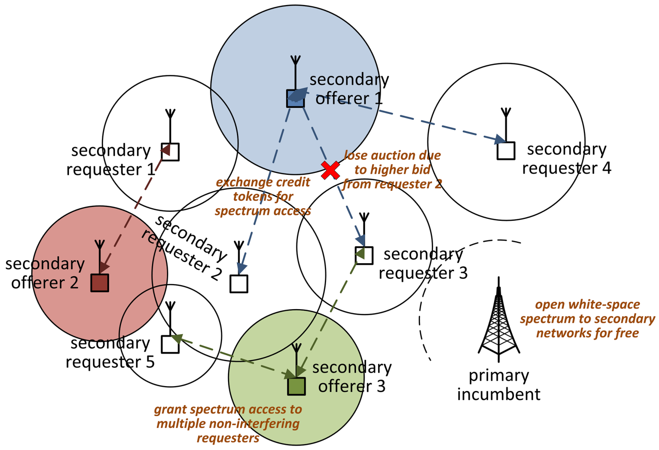

Fig. 1. A credit-token-based spectrum sharing scenario.

bid for the access to $\mathcal{G}_o$ for $o \in \mathcal{O}$ using credit tokens. The set of requesters is denoted by $\mathcal{R} \subseteq \mathcal{N}$. We assume that collocated coexisting networks exchange control messages over a common control channel [21]. The initial channel reservation and fallow spectrum reallocation are limited to a local region. More details will be given in section VII. Unlike the fixed sellers and buyers in most existing work, the sets $\mathcal{O}$ and $\mathcal{R}$ dynamically change over time in our coexistence framework.

Upon receiving the announcement of $\mathcal{G}_o$ from offerer o, each requester $r \in \mathcal{R}$ first makes sure that $\mathcal{G}_o$ is locally available, and then comes up with a *valuation* of $\mathcal{G}_o$, denoted by v_{or}. Different networks may have different valuations for the same spectrum according to their special needs. Among the set of available offerers, denoted by $\mathcal{O}_r \subseteq \mathcal{O}$, who have announced fallow spectrum locally available to requester r, a set of preferred offerers, denoted by $\mathcal{D}_r \subseteq \mathcal{O}_r$, are selected for bidding. In section V, we will discuss the requester's dynamic offerer selection strategy that maximizes *requester's utility*, denoted by U_r. For each preferred offerer, say o, requester r submits a *bid*, denoted by b_{or}, showing interest for the channels in $\mathcal{G}_o$. Note that the value of b_{or} is not necessarily equal to v_{or}. We assume that the amount of credit tokens owned by each network is limited to avoid arbitrarily large bids over valuations. Unlike the unevenly distributed monetary credits for prioritized spectrum access, credit tokens can be carefully regenerated when needed for fair spectrum sharing, e.g., when new users become active. Each requester can be greedy in maximizing its utility regardless of interfering others. Hence, we frame the offerer selection problem as a non-cooperative game, in which each requester acts as a player.

As for each offerer $o \in \mathcal{O}$, an auction is held periodically until the offerer becomes a requester when short of spectrum. Each auction period consists of an initial auction phase and an online auction phase. The durations of these phases are offerer-dependent, which can be determined by channel availability, traffic load, and control overhead. For example, if the spectrum offered by a database-driven white space network is found to be locally available for a long time, the auction periods can be relatively long for minimal overhead. In the initial auction phase, offerer o maintains a set of interested requesters, denoted by $\mathcal{R}_o \subseteq \mathcal{R}$. At this point, offerer o has collected a set of bids, denoted by $\mathcal{B}_o$, which have been submitted by the requesters in $\mathcal{R}_o$ according to the set of valuations, denoted by $\mathcal{V}_o$. Given $\mathcal{B}_o$, offerer o selects a set of *winners*, denoted by $\mathcal{W}_o \subseteq \mathcal{R}_o$, to grant them the access to $\mathcal{G}_o$. In section IV, we will discuss the offerer's dynamic winner selection strategy that maximizes *social welfare*, denoted by S_o. The set of *losers* is denoted by $\mathcal{L}_o \triangleq \mathcal{R}_o - \mathcal{W}_o$. Due to spectrum reuse, multiple winners can share the same spectrum at different locations. In the online auction phase, interested requesters can dynamically arrive and depart between two successive initial auction phases. Hence, each $\mathcal{W}_o$ is subject to changes anytime in such an online multi-winner auction. For each winner, say $w \in \mathcal{W}_o$, offerer o computes a *price*, denoted by p_{ow}, as the amount of credit tokens that winner w needs to pay in a certain auction period. The set of prices is denoted by $\mathcal{P}_o$. Because bidding truthfulness is important for the maximization of social welfare, we study the spectrum sharing problem at each offerer as a truthful multi-winner auction. Besides, the maximization of *offerer's revenue*, denoted by Q_o, is also addressed given guaranteed system stability.

Essentially, the above system model can be considered as a hybrid game-auction coexistence framework. The requesters compete with each other in a non-cooperative game. The offerers hold truthful multi-winner auctions to allocate their surplus spectrum resource to requesters. The interactions between the requesters' offerer selection and bidding strategies and the offerers' winner selection and pricing strategies determine the stability, fairness, and efficiency of our coexistence framework.

IV. Auction Problem

In this section, we focus on the auction problem for a certain offerer, say $o \in \mathcal{O}$. The multi-offerer scenario will be studied in the context of a game. We propose the offerer's strategy that maximizes social welfare and guarantees bidding truthfulness.

A. Initial and Online Auctions

In each initial auction phase, offerer o selects winners $\mathcal{W}_o$ from interested requesters $\mathcal{R}_o$ to enable spectrum sharing among the selected winners. Here, $\mathcal{R}_o$ is given by the game problem in our game-auction framework. We assume that there is one channel available in $\mathcal{G}_o$, which can be shared by all the requesters in $\mathcal{R}_o$. Note that the formulation can be readily extended to a multi-channel case. The primary objective of credit-token-based auction problem is to maximize social welfare, which characterizes the efficiency of spectrum reallocation, i.e., coexistence fairness. Under-utilized spectrum needs to be reallocated to the networks who show the greatest interest in it, since all offerers and requesters are equal to share white-space spectrum for free. Hence, we borrow the concept of credit tokens from CT-CXP to enable fair spectrum sharing. But in section VI, we do consider the secondary objective to maximize offerer's revenue, which is more important in classical monetary auction problems. The social welfare achieved by offerer o, i.e., S_o, can be defined by

$$S_o \triangleq \sum_{r \in \mathcal{R}_o} x_{or} v_{or}, \tag{1}$$

where x_{or} is a binary indicator such that $x_{or} = 1$ (0) represents requester r is (is not) selected as a winner. The S_o is the sum of winners' valuations of $\mathcal{G}_o$. In a social welfare maximization (SWM) problem, offerer o finds a $\mathcal{W}_o$ that maximizes S_o.

Because all the winners in $\mathcal{W}_o$ share the same $\mathcal{G}_o$ at the same time, offerer o has to ensure that the winners do not interfere with each other. We assume that an interference graph, denoted by $\mathbf{I}_o \triangleq \{a_{ij}\}_{i,j\in\mathcal{R}_o}$, is generated at offerer o, where a_{ij} is a binary indicator such that $a_{ij} = 1$ (0) indicates requesters i and j are (are not) interfering neighbors. The interference graph can be generated by integrating neighbor sets, which are reported from interested requesters along with their bids [8]-[10][12]. Two neighbors are viewed to be interfering when at least one of them is interfered. Interfering neighbors cannot win together at the same offerer in the same auction period. Hence, the constraint for winner selection is expressed by

$$a_{ij}(x_{oi} + x_{oj}) \le 1, \text{ for } i, j \in \mathcal{R}_o. \tag{2}$$

We summarize the SWM problem at each offerer o as follows.

Problem 1 (SWM) Find: x_{or} for all $r \in \mathcal{R}_o$;
Maximize: S_o;
Subject to: (2).

Given $\mathcal{R}_o$ and $\mathcal{V}_o$ from the game problem, Problem 1 is similar to the binary integer programming problem in [10], which was shown to be solvable. However, solving Problem 1 requires offerer o to know the true valuations $\mathcal{V}_o$, whereas offerer o only has a knowledge of the collected bids $\mathcal{B}_o$. Hence, bidding truthfulness, i.e., $\mathcal{B}_o = \mathcal{V}_o$, needs to be guaranteed.

In each online auction phase, the offerer's dynamic winner selection strategy also supports online updating of $\mathbf{I}_o$ and $\mathcal{W}_o$, which is initially given by solving Problem 1. Unlike a classical auction that always restricts winner selection at a specific time, the auction at offerer o allows online bidding and winning at any time. During this auction phase, there are three possible arrival or departure events at offerer o:

a. A requester arrives and does not cause harmful interference to any existing winner in $\mathcal{W}_o$;
b. A requester arrives and causes harmful interference to at least one existing winner in $\mathcal{W}_o$;
c. A loser in $\mathcal{L}_o$ departs.

The corresponding offerer's strategy is proposed as follows. For case a, the arrived requester, say w, can join in $\mathcal{W}_o$ directly to further increase S_o to $S_o + v_{ow}$ without waiting for the next auction period. For case b, the arrived requester cannot be added to $\mathcal{W}_o$, since it harms S_o due to the drop of v_{ow} for any interfered winner $w \in \mathcal{W}_o$. For case c, nothing needs to be done, since S_o is still maximized by the unchanged $\mathcal{W}_o$. We ignore the case that a winner in $\mathcal{W}_o$ departs, which decreases both social welfare and the winner's utility.

B. Bidding Truthfulness

The offerer's pricing strategy is another important issue in an auction problem. In the presence of untruthful bidding, social welfare cannot be maximized. Intuitively, two neighboring requesters, say i and j, submit untruthful bids $b_{oi} < v_{oi}$ and $b_{oj} < v_{oj}$ to offerer o, respectively, and $b_{oi} > b_{oj}$ while $v_{oi} < v_{oj}$. But requester i is the winner even though requester j needs the spectrum more, i.e., $v_{oi} < v_{oj}$.

A common way of guaranteeing bidding truthfulness, i.e., $\mathcal{B}_o = \mathcal{V}_o$, is to charge each winner w by a price $0 \le p_{ow} < b_{ow}$ and p_{ow} is not a function of b_{ow} so as to force requesters to report true valuations. The classical Vickrey-Clarke-Groves (VCG) auction [22] belongs to this category and is appropriate for our coexistence problem. The objective of a VCG auction is to maximize social welfare truthfully, which matches with our objective in (1). The major drawback of a VCG strategy is that seller's revenue can be very low (even zero) in some cases [10]. However, in our coexistence framework, spectrum reuse allows each offerer to collect considerable credit tokens from multiple winners, even without giving up the offered spectrum (by treating itself as a winner). This will be shown in section VIII. Furthermore, some extra credit tokens may be generated as a bonus to compensate offerer's loss of revenue when needed, which will be discussed in section VII.

Based on the VCG pricing strategy, the price (credit tokens per unit time) for each winner w, i.e., p_{ow}, is defined by

$$\begin{aligned} p_{ow} &\triangleq b_{ow} - \Big(\sum_{w'\in\mathcal{W}_o} b_{ow'} - f_w(\mathbf{b}_{-w})\Big) \\ &= f_w(\mathbf{b}_{-w}) - \sum_{w'\in\mathcal{W}_o-\{w\}} b_{ow'}, \text{ for } w \in \mathcal{W}_o, \end{aligned} \tag{3}$$

where $\mathbf{b}_{-w} = \mathcal{B}_o - \{b_{ow}\}$, and f_w is a certain pricing function defined for winner w. Here, f_w needs to satisfy

$$\sum_{w'\in\mathcal{W}_o} b_{ow'} - f_w(\mathbf{b}_{-w}) > 0, \tag{4}$$

which is used to guarantee $p_{ow} < b_{ow}$, and

$$\sum_{w'\in\mathcal{W}_o} b_{ow'} - f_w(\mathbf{b}_{-w}) \le b_{ow}, \tag{5}$$

which is used to guarantee $p_{ow} \ge 0$. The choice of f_w does not affect bidding truthfulness but determines system stability, which will be discussed in section VI. For each loser, say $l \in \mathcal{L}_o$, we assume that $p_{ol} = 0$. The revenue achieved by offerer o, i.e., Q_o, through the auction is defined by

$$Q_o \triangleq \sum_{w\in\mathcal{W}_o} p_{ow}. \tag{6}$$

In section VI, Q_o can be maximized subject to system stability.

In each online auction phase, $\mathcal{W}_o$ is updated by case a as discussed above. Hence, the offerer's pricing strategy needs to consider such dynamic change of $\mathcal{W}_o$. For case a, $\mathcal{R}_o$ and $\mathcal{W}_o$ are added with the newly joined winner, say w'. But its price $p_{ow'}$ can still be determined by (3) given a certain definition of $f_{w'}$. The price for an existing winner, say $w \in \mathcal{W}_o$, does not have to be changed for all the three cases.

Lemma 1 The pricing strategy of each offerer o for $o \in \mathcal{O}$ defined by (3) guarantees bidding truthfulness, i.e., $\mathcal{B}_o = \mathcal{V}_o$, regardless of f_w for $w \in \mathcal{W}_o$.

Proof For each winner w, its auction benefit $v_{ow} - p_{ow}$ is not a function of its bid b_{ow}. Hence, bidding with $b_{ow} = v_{ow}$ is always the best strategy for each requester so as to avoid either higher chance to lose due to $b_{ow} < v_{ow}$ or higher chance to pay $p_{ow} > v_{ow}$ due to $b_{ow} > v_{ow}$. □

V. Game Problem

In this section, we model the game problem for an entire system, in which multiple offerers and multiple requesters coexist. We propose the requester's strategy accordingly.

In the non-cooperative game where each requester acts as a player, if two requesters are interfering neighbors and bid at the same offerer, at most one of them can obtain positive utility. Hence, there exist conflicts of interest among interfering requesters. Each requester r selects preferred offerers $\mathcal{D}_r$ from $\mathcal{O}_r$ that maximizes utility, i.e., U_r, which is defined by

$$U_r \triangleq \sum_{o\in\mathcal{O}_r} y_{or}U_{or} = \sum_{o\in\mathcal{O}_r} y_{or}(x_{or}v_{or} - p_{or}), \quad (7)$$

where y_{or} is a binary indicator such that $y_{or} = 1$ (0) represents requester r chooses (does not choose) offerer o, and $U_{or} \triangleq x_{or}v_{or} - p_{or}$. In a requester's utility maximization (RUM) problem, requester r wants to find $\mathcal{D}_r$ who provide the most valuable spectrum and charge the lowest prices to maximize U_r. To clearly study the move actions of requester r in the game, we assume that requester r selects at most one offerer at a time. The constraint for offerer selection is expressed by

$$\sum_{o\in\mathcal{O}_r} y_{or} \leq 1. \quad (8)$$

We write the RUM problem at each requester r as follows.

Problem 2 (RUM) Find: y_{or} for all $o \in \mathcal{O}_r$;
Maximize: U_r;
Subject to: (8).

Given the values of x_{or} and p_{or} from the auction problem at offerer o for $o \in \mathcal{O}_r$, Problem 2 can be solved directly via brute force, since $x_{or}v_{or}$ and p_{or} defining U_r are viewed as constants. But, solving Problem 2 requires requester r to know auction results from other offerers, which is hard to implement in practice. The related issues will be discussed in section VII.

VI. System Stability

In the game problem, each requester solving Problem 2 may switch from one preferred offerer to another. This can in return trigger the affected offerer solving Problem 1 to change the sets of winners and prices in the auction problem. The changes of winners and prices may again trigger more requesters to switch between different offerers. Hence, such chain reactions can result in an unstable system. In this section, we describe the offerer's pricing strategy that guarantees system stability.

Each offerer o observes a population of interested requesters $\mathcal{R}_o$. The size of each population dynamically changes as requesters switch between different offerers. According to the Lyapunov's direct stability theorem in nonlinear control theory, if there exists a *Lyapunov function* on a region containing a *critical point*, which is set as the origin, then the origin is a stable critical point and all solutions originating in the region approach the origin asymptotically [23]. Hence, if we can find a critical point and a Lyapunov function on a region of the state space $\mathbf{R}$, where a state is defined by $\mathbf{r} = \{\mathcal{R}_o \mid o \in \mathcal{O}\} \in \mathbf{R}$, the coexistence system is guaranteed to be stable.

The gain of utility is the incentive of each requester to take a switching action, i.e., a change of preferred offerer. If every requester in the game cannot find a better offerer, the system converges to an equilibrium point, which is a critical point. The total utility of requesters in $\mathcal{R}$, denoted by U^L, is

$$U^L \triangleq \sum_{o\in\mathcal{O}} U_o^L = \sum_{r\in\mathcal{R}} U_r, \quad (9)$$

where $U_o^L \triangleq \sum_{r\in\mathcal{R}_o} U_r = \sum_{r\in\mathcal{R}_o} U_{or}$. The U^L characterizes the incentive of requesters to take switching actions. A higher value of U^L means less incentive of requesters. We show that $\hat{U}^L - U^L$ is a Lyapunov function under a carefully designed pricing function f_w, where $\hat{U}^L$ is the upper bound of U^L.

In (7), each U_r depends on $x_{or}v_{or} - p_{or}$ for $o \in \mathcal{O}_r$. We know that $p_{ol} = 0$ for $l \in \mathcal{L}_o$, while each p_{ow} for $w \in \mathcal{W}_o$ depends on the choice of f_w as needed in (3). Hence, each offerer o can define a pool of f_w for $w \in \mathcal{W}_o$ to control the value of U_o^L. For a winner w, one possible definition of f_w is

$$f_w(\mathbf{b}_{-w}) \triangleq \alpha_w \sum_{w'\in\mathcal{W}_o^{-w}} v_{ow'}, \quad (10)$$

where $\mathcal{W}_o^{-w}$ represents the set of winners obtained by solving Problem 1 given that winner w is not present, and $\alpha_w \in (0, 1]$ is a control factor. The physical meaning of p_{ow} given by (3) and (10) is the harm caused by winner w's presence to other requesters in $\mathcal{R}_o$. In the rest of this section, we show how this definition of f_w for every winner w ensures system stability.

We now focus on an offerer o to study the change of U_o^L as a requester arrives or departs. In each initial auction phase, there are three possible cases for the arrival or departure of a requester at offerer o solving Problem 1:

1. A requester w' arrives and becomes a winner without harming the existing winners in $\mathcal{W}_o$ (original);
2. A requester w' arrives and becomes a winner by forcing a set of existing winners, $\mathcal{W}^l \subseteq \mathcal{W}_o$ (original), to be losers;
3. An existing loser l in $\mathcal{L}_o$ departs.

Note that the three cases described here affect the solution to Problem 1, while the three cases listed in section IV are used to support extra online auction after solving Problem 1. We will discuss the impact of online auction on system stability later. According to (7), we know that $U_{ow} = v_{ow} - p_{ow} > 0$ for $w \in \mathcal{W}_o$, while $U_{ol} = 0$ for $l \in \mathcal{L}_o$. Hence, a loser always has greater incentive to take a switching action than a winner does. We assume that each requester always bids for the channel with the currently highest valuation. Once the requester wins, it can stick to this channel if no new resource becomes available within this auction period, since the channel is mostly preferred and the requester's contribution to social welfare is maximized. Hence, we ignore the counterintuitive case that an existing winner departs. We will show that social welfare and requester' utility are maximized under such a strategy. If a loser cannot win a channel when its bid is much lower than the winning bids, it can quit the system, since its utility is always zero and cannot be improved.

Now, we prove system stability formally. Several Lemmas are presented to use the Lyapunov's direct stability theorem.

Lemma 2 At each offerer o for $o \in \mathcal{O}$, for case 1 and case 3, $U_o^L = \sum_{w \in \mathcal{W}_o} U_{ow}$ is increasing. For case 2, U_o^L is increasing when $\alpha_{w'}$ in (10) satisfies

$$\check{\alpha}_{w'} \leq \alpha_{w'} < \hat{\alpha}_{w'},$$

where $\check{\alpha}_{w'}$ is a lower bound making $p_{ow'} = 0$, and $\hat{\alpha}_{w'}$ is an upper bound given by $\mathcal{V}_o$ and α_w for $w \in \mathcal{W}_o - \mathcal{W}^l$.

Proof For both case 1 and case 3, we consider the worst case where $\alpha_{w'} = 1$. For case 1, $U_{ow'} > 0$ and the values of U_{ow} are non-decreasing for $w \in \mathcal{W}_o$. For case 3, if $l \in \mathcal{W}_o^{-w}$ for a certain $w \in \mathcal{W}_o$, the value of p_{ow} is decreased due to the decreased $f_w(\mathbf{b}_{-w})$. Then, the value of U_{ow} is increased. If $l \notin \mathcal{W}_o^{-w}$ for a certain $w \in \mathcal{W}_o$, the value of U_{ow} is unchanged. Hence, U_o^L is increasing for case 1 and case 3.

For case 2, we assume the worst case that all the new losers in $\mathcal{W}^l$ drop in $\mathcal{W}_o^{-w}$ and no existing requesters in $\mathcal{W}_o^{-w}$ are removed for $w \in \mathcal{W}_o - \mathcal{W}^l$ (including w'). In (10), the maximized size of $\mathcal{W}_o^{-w}$ causes the maximal possible drop of U_o^L. For the new winner w', the utility gain $\Delta U_{ow'} = (\sum_{w \in \mathcal{W}_o - \mathcal{W}^l} v_{ow} + v_{ow'}) - \alpha_{w'}(\sum_{w \in \mathcal{W}_o - \mathcal{W}^l} v_{ow} + \sum_{l \in \mathcal{W}^l} v_{ol})$. For each remaining winner $w \in \mathcal{W}_o - \mathcal{W}^l$, the utility gain $\Delta U_{ow} = v_{ow'} - (1+\alpha_w) \sum_{l \in \mathcal{W}^l} v_{ol}$. For each new loser $l \in \mathcal{W}^l$, the utility gain $\Delta U_{ol} = -(\sum_{w \in \mathcal{W}_o - \mathcal{W}^l} v_{ow} + \sum_{l \in \mathcal{W}^l} v_{ol}) + f_l(\mathbf{b}_{-l})$. Hence, to guarantee that U_o^L is always increasing, we make the total utility gain $\Delta U_o^L > 0$, namely

$$\Delta U_o^L = \Delta U_{ow'} + \sum_{w \in \mathcal{W}_o - \mathcal{W}^l} \Delta U_{ow} + \sum_{l \in \mathcal{W}^l} \Delta U_{ol} > 0. \quad (11)$$

We can control $\alpha_{w'}$ (and α_w for $w \in \mathcal{W}_o - \mathcal{W}^l$) to make

$$\Delta U_o^L \triangleq (\hat{\alpha}_{w'} - \alpha_{w'})(\sum_{w \in \mathcal{W}_o - \mathcal{W}^l} v_{ow} + \sum_{l \in \mathcal{W}^l} v_{ol}) > 0,$$

where $\hat{\alpha}_{w'}$ is a function of $\mathcal{V}_o$ and α_w for $w \in \mathcal{W}_o - \mathcal{W}^l$ based on (11). Hence, we have $\alpha_{w'} < \hat{\alpha}_{w'}$. Such an $\alpha_{w'}$ always exists. Even for the worst case that U_o^L is already maximized by $p_{ow} = 0$ for $w \in \mathcal{W}_o$ (original) before the arrival of winner w', a solution to make $\Delta U_o^L > 0$ is $\alpha_w = \check{\alpha}_w$ making $p_{ow} = 0$ for $w \in \mathcal{W}_o - \mathcal{W}^l$ (including w'), since $v_{ow'} > \sum_{l \in \mathcal{W}^l} v_{ol}$. Hence, U_o^L can be increasing for case 2. □

Furthermore, we show that ΔU_o^L is non-increasing after certain actions of requesters in the game.

Lemma 3 At each offerer o for $o \in \mathcal{O}$, after $p_{ow} = 0$ for all $w \in \mathcal{W}_o$ and $\mathcal{W}_o$ is unchanged, $U_o^L = \sum_{w \in \mathcal{W}_o} U_{ow}$ reaches its upper bound, say $\hat{U}_o^L$, i.e., $\Delta U_o^L = 0$.

Proof A rational requester always selects an offerer who achieves the currently highest valuation, so each winner w at offerer o maximizes $x_{ow} v_{ow}$. If winner w can also minimize p_{ow} at the same time, U_{ow} is maximized. Fortunately, after certain switching actions of requesters in $\mathcal{R}$, we can have $p_{ow} = 0$ for $w \in \mathcal{W}_o$. As more losers quit the coexistence system due to too low bids and valuations, there is higher chance that $\mathcal{W}_o^{-w} = \mathcal{W}_o - \{w\}$ occurs. For each winner w, when $\mathcal{W}_o^{-w} = \mathcal{W}_o - \{w\}$, we have $p_{ow} = 0$ according to (3) and (10) for $\alpha_w = 1$. In the end, all the remaining requesters in the system are winners, and all the losers have quitted due to zero utility. Once $\mathcal{W}_o$ keeps unchanged, i.e., social welfare is maximized at offerer o, and $p_{ow} = 0$ for all $w \in \mathcal{W}_o$, U_o^L is maximized to $\hat{U}_o^L \triangleq \sum_{w \in \mathcal{W}_o} v_{ow}$, and thus $\Delta U_o^L = 0$. □

Then, we discuss the change of U_o^L as a requester arrives or departs in each online auction phase.

Lemma 4 At each offerer o for $o \in \mathcal{O}$, online auction does not decrease U_o^L, and guarantees $\Delta U_o^L = 0$.

Proof In the three cases described in section IV, case a increases U_o^L, which can be proved as case 1 in Lemma 2. For case b and case c, $\mathcal{W}_o$ is not changed. The prices for existing winners in $\mathcal{W}_o$ do not have to be changed. Then, U_o^L is not changed by case b and case c. Hence, online auction does not decrease U_o^L. After all the losers quit the coexistence system due to zero utility, we still have $\Delta U_o^L = 0$. □

Because a higher value of U^L indicates less incentive of requesters to change preferred offerers, we can expect that the coexistence system gradually approaches and finally reaches an equilibrium point. We now prove this conjecture formally using the Lyapunov's direct stability theorem.

Theorem 1 For certain $\mathcal{O}$ and $\mathcal{R}$, the coexistence system is defined by Problem 1 at each offerer o for $o \in \mathcal{O}$ and Problem 2 at each requester r for $r \in \mathcal{R}$. The offerer's pricing strategy is defined by (3) and (10). Given an initial point, say $\mathbf{r}_0 = \{\mathcal{R}_o \mid o \in \mathcal{O}\} \in \mathbf{R}$, when (11) holds, the coexistence system converges to a stable equilibrium point, say $\mathbf{r}^* \in \mathbf{R}$, where U^L is maximized and $\Delta U_o^L = 0$ for all $o \in \mathcal{O}$.

Proof First, we show that $\mathbf{r}^*$ is a critical point. Because $\Delta U_o^L = 0$ for all $o \in \mathcal{O}$ at $\mathbf{r}^*$, then $\Delta U^L \triangleq \sum_{o \in \mathcal{O}} \Delta U_o^L = 0$ at $\mathbf{r}^*$, which accords with the definition of critical point. We set $\mathbf{r}^*$ as the origin of a region.

Next, we prove that $\hat{U}^L - U^L$ is a Lyapunov function, where $\hat{U}^L \triangleq \sum_{o \in \mathcal{O}} \hat{U}_o^L$. If $\hat{U}^L - U^L$ is indeed a Lyapunov function, it needs to satisfy the following conditions [23]:

- To show $\hat{U}^L - U^L = 0$ at the origin, i.e., $\mathbf{r}^*$, and $\hat{U}^L - U^L > 0$ for other $\mathbf{r} \in \mathbf{R}$;
- To show $-\frac{dU^L}{d\mathbf{p}} > 0$, where $\mathbf{p}$ is any vector radiating from the origin, i.e., $\mathbf{r}^*$;
- To show $-\frac{dU^L}{dt} < 0$, where t is the time.

According to Lemma 3, U^L can reach its upper bound $\hat{U}^L$. Clearly, $\hat{U}^L - U^L = 0$ at $\mathbf{r}^*$ and $\hat{U}^L - U^L > 0$ for other $\mathbf{r} \in \mathbf{R}$. Hence, condition 1 is satisfied. According to Lemma 2, U^L is increasing in time. Hence, condition 3 is satisfied. We know that $\mathbf{r}^*$ is the highest point in terms of U^L, so U^L is decreasing via $\mathbf{p}$ radiating from $\mathbf{r}^*$. Hence, condition 2 is satisfied. Then, we can say that $\hat{U}^L - U^L$ is a Lyapunov function.

According to the Lyapunov's direct stability theorem, we conclude that $\mathbf{r}^*$ is a stable critical point and the coexistence system starting from any $\mathbf{r}_0$ approaches $\mathbf{r}^*$ asymptotically. □

We now relax the definition of f_w in (10) for the offerer's pricing strategy to a general setting. In this case, each offerer o can dynamically select a f_w for each winner w from a pool of definitions to maximize its revenue, i.e., Q_o, given guaranteed system stability by Theorem 1. Besides Problem 1, each offerer o can solve an offerer's revenue maximization (ORM) problem, which is written as follows.

Algorithm 1 Offerer o's strategy for $o \in \mathcal{O}$

1: **check** the availability of surplus spectrum $\mathcal{G}_o$ for the current auction period (via geolocation database or spectrum sensing), and broadcast it to requesters $\mathcal{R}$
2: **collect** bids $\mathcal{B}_o$ (i.e., valuations $\mathcal{V}_o$ of $\mathcal{G}_o$) from interested requesters $\mathcal{R}_o$, and update interference graph $\mathbf{I}_o$
3: **solve** Problem 1 to select a set of non-interfering winners $\mathcal{W}_o$ according to $\mathcal{B}_o$ and $\mathbf{I}_o$
4: **solve** Problem 3 to select a pricing function f_w for each winner w for $w \in \mathcal{W}_o$
5: **compute** a price p_{ow} for each winner w according to (3), and obtain prices $\mathcal{P}_o$ and a bonus $\tilde{p}_o$
6: **allocate** $\mathcal{G}_o$ to each winner w, and broadcast winning/losing notification along with $\mathcal{W}_o$ and $\mathcal{P}_o$
7: **while** in the online auction phase **do**
8:     **collect** a bid from an online requester r
9:     **if** case a in section IV **then**
10:         **allocate** $\mathcal{G}_o$ to requester r, charge a price p_{or}, and broadcast updated notification
11:     **end if**
12: **end while**

Problem 3 (ORM)

$$\begin{aligned} \text{Find:} \quad & f_w \text{ for all } w \in \mathcal{W}_o; \\ \text{Maximize:} \quad & Q_o; \\ \text{Subject to:} \quad & (4), (5), (11). \end{aligned}$$

Therefore, our game-auction coexistence framework addresses the trade-offs among social welfare and offerer's revenue in the auction and requester's utility in the game.

Corollary 1 The coexistence system that is defined by Problem 1 and Problem 3 at each offerer and Problem 2 at each requester is guaranteed to be stable.

Proof At each offerer, the choice of pricing functions is constrained by (4), (5), and (11). In the general setting, Lemma 2 still hold due to (11), and Lemma 3 still hold due to increasing and bounded total utility (bounded total price) by (4) and (5). Hence, the coexistence system defined in Theorem 1 is stable. Because solving Problem 3 at each offerer does not change the winner selections at offerers and the offerer selections at requesters, system stability is still guaranteed. □

VII. Implementation Discussion

In this section, we summarize the offerer's and requester's strategies, and discuss related implementation issues.

The proposed offerer's strategy as described in Algorithm 1 can be tailored to support the offering procedure defined in CT-CXP, including offering advertisement, renting request, and resource allocation. Specifically, line 1 implements offering advertisement. Line 2 enables renting request. Lines 3 to 6 realize not only the basic resource allocation required in CT-CXP but also spectrum reuse. Lines 7 to 12 define the extra online auction. The pricing in lines 5 and 10 guarantees bidding truthfulness, which is neglected in CT-CXP. A bonus of credit tokens, denoted by $\tilde{p}_o$, can be added to each offerer o's revenue Q_o, which is used to encourage offerer o to participate in spectrum etiquette and maximize social welfare.

Algorithm 2 Requester r's strategy for $r \in \mathcal{R}$

1: **collect** announcement messages about under-utilized spectrum from offerers $\mathcal{O}$
2: **check** the availability of offered spectrum for the current auction period locally (via geolocation database or spectrum sensing), and obtain available offerers $\mathcal{O}_r$
3: **solve** Problem 2 to select a set of preferred offerers $\mathcal{D}_r$
4: **submit** a bid $b_{or} = v_{or}$ to each offerer o for $o \in \mathcal{D}_r$, and wait for winning/losing notification (online requester)
5: **if** winning at offerer o **then**
6:     **access** $\mathcal{G}_o$ directly
7: **else**
8:     **try** other offerers, or wait for offerer o's next auction period (regular requester)
9: **end if**

As one possible definition of credit token bonus, $\tilde{p}_o$ can be proportional to the achieved social welfare S_o. In this case, the maximization of social welfare further contributes to the maximization of offerer's revenue. We plan to explore more options of the bonus design as part of our future work.

The proposed requester's strategy as described in Algorithm 2 can support the requesting procedure defined in CT-CXP. Specifically, lines 1 to 2 correspond to offering advertisement. Lines 3 to 4 define renting request. A naive way of implementing line 3 would require each requester to know the exact value of achieved utility at each offerer to solve Problem 2, which is hard to achieve in practice. In a more effective approach, each requester can solve Problem 2 to maximize expected utility, which is computed based on a certain probability distribution. Alternatively, each requester can keep trying other offerers and switch to a new offerer only after it has won and has improved utility at the offerer. Lines 5 to 9 deal with resource allocation. The interactions between Algorithms 1 and 2 guarantee system stability, which is also neglected in CT-CXP.

Because credit tokens can be flexibly generated or renewed for various purposes, e.g., fair spectrum sharing in our coexistence framework, the legitimacy of credit tokens in circulation should be ensured by a third-party entity. As guided by CT-CXP, coexisting networks can follow some spectrum etiquette to achieve efficient spectrum reallocation.

The credit-token-based spectrum etiquette framework relies on a common control channel [21] to enable the exchange of control messages between collocated offerers and requesters. For heterogeneous CR networks with geolocation capability, Internet connection to a common geolocation database can be used for coordinated channel reservation [4]. For better real-time performance, e.g., online auction bidding and winning, each coexisting network may be equipped with an extra radio with common radio access technology for local exchange of control messages. A contention-based MAC, such as carrier sense multiple access (CSMA), can be used to deal with control message collisions, especially in any offerer's initial auction phases. The message collisions can be mitigated by asynchronous offerers and prolonged auction periods. Each

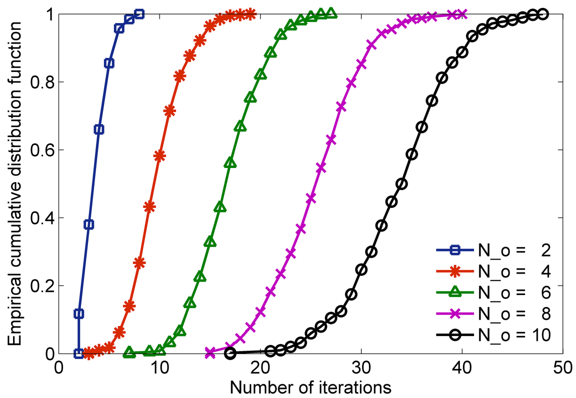

Fig. 2. System convergence speed.

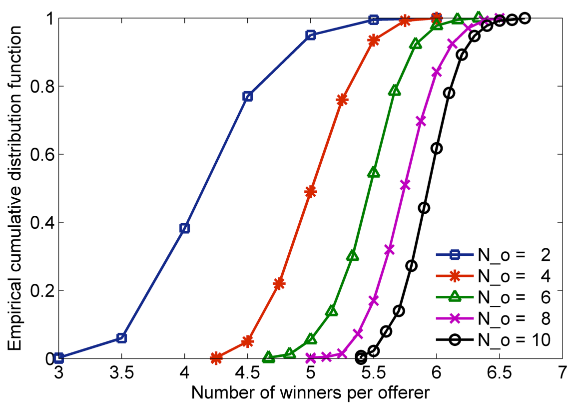

Fig. 3. Spectrum utilization efficiency.

offerer can collect bids not only in the current initial auction phase but also in previous auction periods.

VIII. Simulation Results

In this section, we evaluate our coexistence framework by simulation. In order to clearly show the convergence process, we temporarily fix the number of offerers, i.e., $N_o \triangleq |\mathcal{O}|$, and the number of requesters, i.e., $N_r \triangleq |\mathcal{R}| = 10 \times N_o$. Any change of $\mathcal{O}$ or $\mathcal{R}$ results in a new coexistence scenario. Each spectrum valuation $v_{or} \in [0,1]$ for $o \in \mathcal{O}$ and $r \in \mathcal{R}$ and each interference indicator $a_{ij} \in \{0,1\}$ for $i,j \in \mathcal{R}$ are randomly generated. An interference graph $\mathbf{I}_o = \{a_{ij}\}_{i,j\in\mathcal{R}_o}$ is created at each offerer o according to the current $\mathcal{R}_o$. The value of v_{or} is set to zero when offerer o is not in $\mathcal{O}_r$. Each offerer runs Algorithm 1, and each requester runs Algorithm 2. The offerer's pricing strategy is defined by (3) and (10). To characterize the distributed nature of CR networks, we assume that interested requesters arrive at each offerer according to a Poisson process. The coexisting networks are asynchronous.

First, we evaluate the convergence of proposed coexistence framework. We are interested in the speed of convergence in terms of the number of iterations for a convergence process. Here, an "iteration" represents a switching action of any requester changing offerer selection in the game. The time duration of an iteration depends on arrival process, the durations of offerer-dependent auction periods, and the complexity of requester-dependent offerer selection. For certain fixed N_o, we

TABLE I
Average frequency of online winning.

num. offerers	4	6	8	10
frequency	24.85%	30.58%	36.02%	38.89%

TABLE II
Impact of control message collisions.

num. offerers	4	6	8	10
num. iterations	51.82%	48.53%	43.45%	42.59%
social welfare	-4.35%	-3.07%	-2.59%	-2.02%

randomly generated multiple starting points for the coexistence system and count how many iterations it takes for the system to reach a stable point. The empirical cumulative distribution function (ECDF) of the counted number of iterations is illustrated in Fig. 2. We can see that in a practical case where N_o is not large, the number of iterations per offerer is small enough. When online auction is enabled in Algorithm 1, the average frequency of online winning (i.e., case a in section IV) that occurs in a convergence process is shown in Tab. 1. We can see that the durations of considerable iterations can be shorten, since waiting time for an online iteration is always shorter than that for a regular iteration, which is equal to the duration of an entire auction period.

Second, we evaluate the spectrum utilization of proposed coexistence framework. Because spectrum reuse is achieved by a multi-winner auction at each offerer, spectrum utilization can be characterized by the number of winners per offerer, whose ECDF is illustrated in Fig. 3 for certain fixed N_o. We can see that the proposed framework always outperforms regular CT-CXP that only allows one winner at a time. In the proposed coexistence framework, however, each offerer can accommodate 5 to 6 requesters on average at the same time so as to achieve more efficient spectrum sharing.

Third, we evaluate the optimality of proposed coexistence framework. We start from discussing the total utility of requesters, i.e., U_o^L, and the total revenue, i.e., Q_o, achieved at each offerer o. The average values of U_o^L and Q_o (excluding extra bonus $\tilde{p}_o$) in a convergence process are illustrated in Fig. 4. We can see that U_o^L is always increasing before reaching an upper bound, which verifies Lemma 2 and Lemma 3. Because the offerer's pricing strategy is defined by (3) and (10) here, Q_o drops to zero in the end due to the leave of losing requesters. However, each offerer can still collect considerable credit tokens by accommodating multiple winners, compared to the average spectrum valuation '0.5'. The optional bonus of credit tokens can further improve offerer's revenue by a fraction of maximized social welfare. Other definitions of pricing functions can also be developed to improve offerer's revenue. Although it has been proved in Theorem 1 that the coexistence system always converges to a stable equilibrium solution in a distributed manner, such a distributed solution may not be unique and can be a local optimal solution due to multiple possible stable points. Hence, we need to verify the optimality of distributed solution compared to the centralized

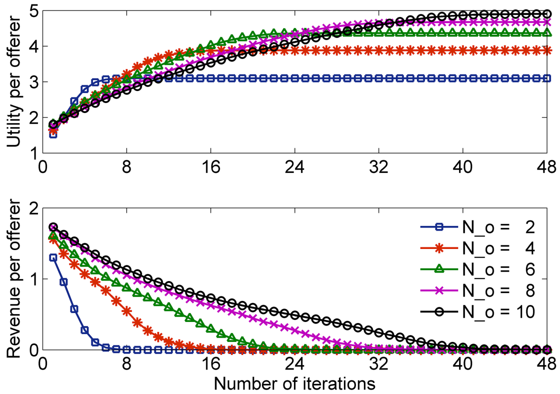

Fig. 4. Average utility and revenue per offerer.

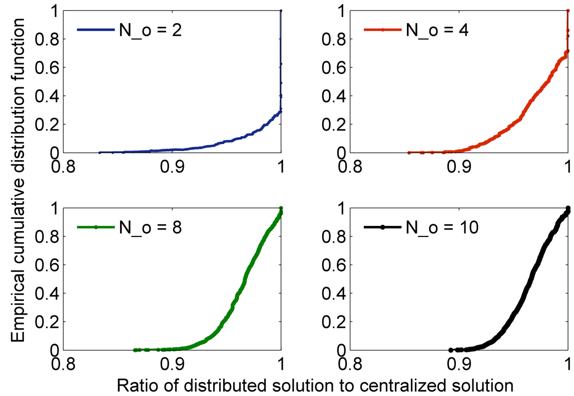

Fig. 5. Optimality of social welfare maximization.

solution, which is given by a certain centralized infrastructure that maximizes total social welfare, i.e., $\sum_{o\in\mathcal{O}} S_o$, globally. The ratio of distributed solution to centralized counterpart in terms of $\sum_{o\in\mathcal{O}} S_o$ is illustrated in Fig. 5 for certain fixed N_o. We can see that the stable points given by our distributed system are usually very close to the optimal point.

Fourth, we evaluate the negative impact of control message collisions. Here, the common control channel is accessed by IEEE 802.11 CSMA/CA MAC. The average increase in the number of iterations and the average decrease in social welfare caused by control message collisions in a convergence process are shown in Tab. 2. We can see that the number of iterations increases largely, but the convergence of coexistence framework is not harmed due to retransmission and rebidding. The average value of social welfare slightly decreases owing to the collisions of expected winners' bidding messages.

IX. Conclusion

In this paper, we have addressed spectrum sharing among distributed heterogeneous CR networks with equal priority through a credit-token-based spectrum etiquette framework. Specifically, a game-auction coexistence framework has been proposed to extend IEEE 802.16h CT-CXP in many aspects, including optimization trade-offs, spectrum reuse, online auction winning, bidding truthfulness, and system stability. Our simulation results have shown that the proposed coexistence framework always converges to a near-optimal distributed solution without a centralized coexistence infrastructure, and it improves heterogeneous coexistence jointly in truthfulness, coexistence fairness, and spectrum utilization.

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