

Flexauc: Serving Dynamic Demands in Spectrum Trading Markets with Flexible Auction

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Abstract—In spectrum trading markets, auctions are organized by spectrum holders (SHs) to distribute channels. As buyers, wireless service providers (WSPs) acquire channels to deploy services to end users. To optimize the profits, it is essential for the WSPs to determine their bidding strategies, which are affected by two key aspects: the service position to the end users and the auction schemes enforced by the SH.

In this paper, we jointly study the strategy of the SH in the auction design and the WSPs' strategies in the service provisions and biddings. The WSP's optimal strategy in the auction can be flexible in term of demands and valuations. To optimize social welfare and enable the WSPs to reveal truthful flexible demands, we design Flexauc, a novel auction mechanism for the SH. We prove theoretically that Flexauc not only maximizes the social welfare but also preserves other nice properties: truthfulness and computational tractability.

I. INTRODUCTION

Auctions are usually applied for spectrum redistribution to increase efficiency. In the spectrum trading markets, major wireless service providers (WSPs) purchase spectrum through auctions organized by the spectrum holders (SH). It is essential for the WSPs to determine proper bidding strategies in the auction to optimize the revenues. One key observation we have is that the WSPs' bidding strategies relate tightly to the service provisions to the end users. Both the WSPs' demands and valuations can be flexible in the auction due to their end users' heterogeneous demands and willingness to pay. Considering these flexibilities, it is therefore important for the SH to design auction schemes to not only optimize social welfare but also enable the buyers to reveal their truthful demands and valuations.

However, existing works failed to enable the WSPs' flexible bidding strategies. On one hand, there are no suitable existing auction schemes that are able to promote flexible biddings with computational efficiency. Most works rely on over-simplified assumptions or induce heavy overheads. Some of them [1]–[3] assume that one buyer can claim for at most one channel. Some others [4] [5] assume that a buyer can submit bids for multiple channels but win at most one. These assumptions indeed limit the scope of applications especially when a buyer is willing to win more than one channels. Although combinatorial auctions (e.g. [6]) can meet the requirements of flexible auction, it induces very heavy computational overheads. On the other

hand, as far as we know, no work has studied the relationship between users' willingness to pay and WSP's true evaluation of the channels. In the auction design, buyers and sellers have values in their minds for the commodity, which are defined as the true evaluations. It is a quantitative measurement of the satisfaction of consumption behavior. This concept is the core in any auction design. But no existing works study how an operator determines its evaluation of the channels.

In this paper, we aim to address the two problems mentioned above. Considering the service provision between the WSPs and their end users, we design a truthful auction mechanism for the SH to enable bidders' flexible strategies in terms of demands and valuations and also preserves computation efficiency. Specifically, we consider a three-layered spectrum trading scenario shown in Fig. 1 consisting of the SH, the WSPs and the end users. In the spectrum auction between the SH and the WSPs, the SH partitions the spectrum into channels with equal bandwidth for sale and designs the auction mechanism. The WSPs then bid the channels from the SH. In the service provision between the WSPs and the end users, the WSPs decide their optimal service prices for their end users and the users determine how much bandwidth they want to consume. Different from most existing works considering such a multi-layered spectrum trading markets [7] [8], in this paper, auction instead of pricing is conducted between the SH and the WSPs. The pricing mechanism is often used when the sellers knows precisely the value of selling resources. The auction mechanism on the other hand assumes no such information, which suits our scenario better.

In this paper, we design a novel auction mechanism called Flexauc for the WSPs' flexible bidding strategies. With Flexauc, the bidders can bid for any amount of demands with different valuations. It is an improved version of the combinatorial auction for identical items with significantly reduced computational overheads. In Flexauc, each WSP submits a series of bids representing his marginal benefit of the additional channels. A WSP is possible to win any number of channels and he will be satisfied with any possible result. We model the strategic behaviors in each layer and leverage backward induction to compute the WSPs' bids in the auction. We analyze the WSPs' true valuations for the channels based on their pricing strategies and users' possible responses in the

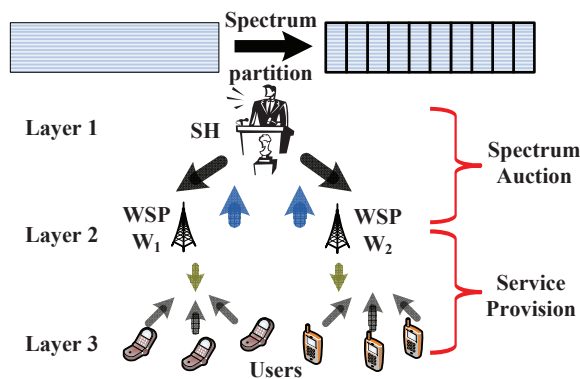


Fig. 1: A three-layered network scenario

service provision. Note that existing works usually assume the valuations are fixed and known inputs for the auction problem. In Flexauc, we propose two new payment mechanisms which are proved to be truthful and maximize social welfare.

In summary, the key conclusions and contributions of this work are summarized as follows.

- 1) As far as we know, this paper is the first to study the flexible auction mechanism, where the operators can flexibly decide the best quantity of channels to buy and the best bidding values, as well as the service price for end users.
- 2) By considering both the auction and service provision in wireless market, we propose a framework for the entire market. We design Flexauc, and show its advantages over previous mechanisms.
- 3) We present comprehensive simulation results to verify our theoretic conclusions.

The rest of this paper is organized as follows. We summarize the research literature in Section II. The detailed system model, concept definitions and design objectives are described in Section III. In Section IV, we discuss the spectrum partition, and elaborate auction mechanism, bidding behaviors and the demand response as well as pricing mechanism. We analyze the economic properties in Section V, specifically truthfulness, efficiency and time complexity. Section VI provides our numerical results to verify our design objectives and make comparison with previous works to show advantages of Flexauc. We conclude this paper in Section VII.

II. RELATED WORK

We provide a summary of state-of-the-art auction mechanisms in this section. We explain that they are not suitable for the scenario where buyers may need more than one channels with different valuations. The existing models also provide no clues to how to reveal the true valuations of WSPs.

Single-seller multi-buyer auction with homogeneous channels has been studied extensively. VERITAS in [1] allowed users to buy channels based on their demands and spectrum owner to maximize revenue with spectrum reuse. In [2] the authors proposed a VCG auction to maximize the expected

revenue of the seller and a suboptimal auction to reduce the complexity for practical purpose. Double auction mechanisms are studied for the multi-seller multi-buyer case. McAfee mechanism [9] was proposed for trading homogeneous items in double auction. Many follow-up works have been done since then [3], [4], [10], [11].

Most of the state-of-the-art mechanisms assume that buyers can claim at most one channel. These mechanisms cannot satisfy the flexible demands from buyers. One recent mechanism for heterogeneous demands is designed for the cloud services [12]. Unlike our work, in [12], each bidder has only uniform unit valuations for any amount of demands. Another mechanism that enables the flexible demand is combinatorial auction [6] [13]. In this model, each buyer can submit a bid to flexibly claim time-channel combinations. The major concern for combinatorial auction is that in general cases, the problem is NP-hard. It means that the mechanism is not suitable for periodic auctions with too many buyers and channels.

In all these works, another over-simplified assumption is made that the buyers will consume the channels themselves for data transmission. So their satisfaction levels are quantitatively analyzed and considered in their utility functions. Usually, the satisfaction level is an input in auction problem, known as the true evaluation. But in our scenario, the operators provide services to users to make revenue. There are no prior true evaluations at the very beginning. The evaluations in fact depend on their users' willingness to pay. We theoretically derive the operators' true evaluations of channels by analyzing the service provision, i.e., the pricing mechanism and users' utility maximization.

In summary, none of the existing works provided a feasible auction model to enable flexible demand with polynomial time complexity. Most polynomial time auction models do not support flexible demand. Combinatorial auction supports flexible demand, but they are not computationally efficient. Flexauc preserves good properties of both types. It provides flexibility with polynomial time (more specifically, linear time) algorithms. We also show that in the three-layered market, how should the operators determine the best bids in the auction.

III. PRELIMINARIES AND PROBLEM FORMULATION

In this section, we first describe our general scenario, then the definition of the strategies, utility functions and concepts.

A. Problem Formulation

We consider a scenario of a macrocell with one SH and N WSPs. Each WSP deploys infrastructures within the coverage of the same macrocell. There are N_i users subscribing to the WSP W_i ($1 \leq i \leq N$). SH has a spectrum block of size $B \times C$ to sell. He partitions it into C channels, each with the same bandwidth B . Different users can have different evaluations on the same channel due to fading, multi-path effect and his own conditions.

W_i 's j 'th user U_{ij} requires a certain amount of bandwidth from W_i . Let the bandwidth requirement of U_{ij} , which is also the only strategy of a user, be w_j^i . U_{ij} can achieve a data

rate of $\alpha_i w_j^i \ln(1 + \frac{g_j^i}{w_j^i})$, where α_i is his evaluation of the data rate and $g_j^i = \frac{P_i H_j}{n_0}$.¹ P_i is the transmission power level of base stations of W_i . H_j is the path loss factor. n_0 is the power density of thermal noise. Let p_i be the unit price of the shared bandwidth charged by W_i . So U_{ij} 's payment is $p_i w_j^i$. We define U_{ij} 's utility to be

$$U_{ij}^u = \alpha_i w_j^i \ln(1 + \frac{g_j^i}{w_j^i}) - p_i w_j^i. \quad (1)$$

A WSP W_i 's bid is defined as $\{b_{i1}, \dots, b_{iC}\}$. It means that W_i is willing to pay b_{i1} for its first allocated channel, b_{i2} for his second channel, and so on. If W_i wins K_i channels, its utility will be the difference between the revenue from users and the cost of channels:

$$U_i = p_i \cdot \min\{\sum_{j=1}^{N_i} w_j^i, K_i B\} - \sum_{j=1}^{K_i} \tilde{b}_{ij}, \quad (2)$$

where \tilde{b}_{ij} is W_i 's payment to SH for the j -th channel. $\tilde{b}_{ij} \leq b_{ij}$ should hold as the payments for the channel should be less than the WSP's bid value. In the equation, p_i and $\{b_{ij}\}$, ($1 \leq j \leq C$) are W_i 's strategies. The auction results, K_i and $\{b_{ij}\}$, are determined by SH.

The utility of SH is his revenue from spectrum sales, which is defined as

$$U_{SH} = \sum_{i=1}^N \sum_{j=1}^{K_i} \tilde{b}_{ij}. \quad (3)$$

B. Definition of Concepts

In this part, we clarify some concepts we will use in this paper. Then we introduce the economic properties we would like to achieve.

Definition 1: Dominant Strategy: a dominant strategy of a player is the one that maximize its utility regardless of what other players' strategies are. Mathematically, if x_i is player i 's strategy, for any $x'_i \neq x_i$, and any strategy profile of others x_{-i} , we have $U_i(x_i, x_{-i}) \geq U_i(x'_i, x_{-i})$. If the inequality always holds, x_i is a strongly dominant one. Otherwise, x_i is a weakly dominant one.

Definition 2: Truthfulness: an auction is truthful if any player's true evaluation is his dominant strategy.

It means that given other players' strategy profile and the auction rules fixed, a player cannot improve its utility by submitting any bid that is different from its true bid (a vector of its true evaluation and budget).

When designing auction mechanisms, it is crucial to make them truthful. It is the most critical property and has been well accepted in the research literature [1].

Definition 3: Social Efficiency: an auction is social efficient if the aggregate of all players' utilities is optimized. It shares a common meaning with Social Welfare. Here we consider the aggregate utility of the SH and WSPs, denoted by $S = \sum_{i=1}^N U_i + U_{SH}$. The users' utilities are not considered in

¹For simplification, we assume α_i depends on the WSP's service quality only. In general case, it should depend on individual user.

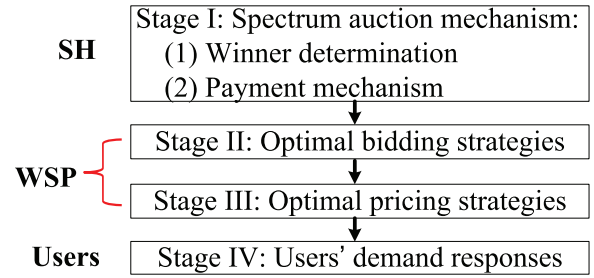


Fig. 2: A four-staged analysis framework

Social Welfare because there is no direct relationship between users and auction.

IV. STRATEGIES FORMING AND AUCTION DESIGN

In this section, we make theoretical analysis for this problem in the scenario shown in Fig. 2. The procedure can be divided into four stages. In the first stage, the SH decide channels to sell and the auction mechanism. In the second stage, we deduce the optimal bidding strategies of WSPs. In the third stage, we derive the optimal pricing strategy for service tariff. In the last stage, we analyze the best demand strategy for users. We adopt backward induction as analytical method. At the end of the section, we will discuss the impact of spectrum partition by SH.

A. Users' Demand Strategies

Given that W_i announces the price p_i and it is guaranteed that all the users' demands will be satisfied. Each user optimizes his utility by selecting his optimal demand strategy. For the ease of analysis, we make an approximation:

$$U_{ij}^u \approx \alpha_i w_j^i \ln \frac{g_j^i}{w_j^i} - p_i w_j^i. \quad (4)$$

Its first and second order derivatives are

$$\frac{\partial U_{ij}^u}{\partial w_j^i} = \alpha_i (\ln \frac{g_j^i}{w_j^i} - 1) - p_i. \quad (5)$$

and

$$\frac{\partial^2 U_{ij}^u}{\partial w_j^{i2}} = -\frac{\alpha_i}{w_j^i} < 0. \quad (6)$$

So the optimal demand strategy is

$$w_j^{i*} = g_j^i e^{-1 - \frac{p_i}{\alpha_i}}. \quad (7)$$

The error introduced by approximation decreases with p_i . Generally, the error is very small in our simulation by ITU and COST models [14].

B. WSPs' Pricing Strategies

In this stage, W_i decides the optimal price p_i based on the auction result and evaluation of users' demands.

To find the optimal price p_i^* that maximizes Eq. (2)², we need to discuss two cases.

²The decision variables p_i and $\{b_{i1}, \dots, b_{iC}\}$ can be separated. Now we try to obtain the optimal price based on the auction result and the bids.

Case (1): if $K_i B > \sum_{j=1}^{N_i} w_j^{i*}$, Eq. (2) can be simplified as

$$p_i \cdot \sum_{j=1}^{N_i} w_j^{i*} - \sum_{j=1}^{K_i} \tilde{b}_{ij}, \quad (8)$$

which has the only variable p_i . The unique solution $p_i^* = \alpha_i$ can be obtained. Physically, it means that if W_i has purchased abundant bandwidth, the optimal price that brings highest revenue is $p_i^* = \alpha_i$. Though part of its spectrum is unused, it is nonprofitable to offer a reduced price to stimulate more spectrum demand.

Case (2): if $K_i B \leq \sum_{j=1}^{N_i} w_j^{i*}$, the optimal $p_i^* \geq \alpha_i$ because $p_i \sum_{j=1}^{N_i} w_j^{i*} - \sum_{j=1}^{K_i} \tilde{b}_{ij}$ is monotonously decreasing in the region $p_i \in [1, \infty)$. Therefore, the highest utility is achieved when $\sum_{j=1}^{N_i} w_j^{i*} = K_i B$. We can obtain $p_i^* = \alpha_i (\ln \frac{G_i}{K_i B} - 1)$, where $G_i e^{-1 - \frac{p_i}{\alpha_i}} = \sum_{j=1}^{N_i} w_j^{i*}$ is the aggregated demand of W_i 's users. In this case, if W_i purchases more bandwidth, it can lower the price and increase the utility.

To make a summary, the optimal price is

$$p_i^* = \begin{cases} \alpha_i & \text{if } K_i B > G_i e^{-2} \\ \alpha_i (\ln \frac{G_i}{K_i B} - 1) & \text{Otherwise} \end{cases} \quad (9)$$

C. WSPs' Bidding Strategies

We analyze WSPs' bidding strategies in the auction. We take truthfulness and efficiency as the most desirable properties of an auction design.

In previous works, a buyer in the auction consumes his spectrum directly. So he bids for his desired quantity and offers his true evaluation. But a WSP indeed sells the spectrum to users, which makes the quantity and evaluation dynamic and flexible. In such case, the challenge is how to determine the best bidding strategies in terms of quantity and evaluation. We propose to decide the best strategies by the pricing scheme and users' demand responses.

Let us start with a simple case. Suppose W_i gets only one channel in the auction. His utility can be calculated by Eq. (1) as $U_i(K_i) = U_i(1)$. When W_i considers his bidding strategy, he cannot predict his payment $\{\tilde{b}_{ij}\}$ and K_i . A well-designed auction mechanism guarantees that the true value is the best strategy, which simplifies the strategy making process.

First, we need to elaborate the true value in this problem. In previous auction works, the true values exist in the mind of buyers and sellers in advance. If a buyer is asked to pay the true value to win the object, his utility is zero. In this paper, we propose that the true value should be decided according to particular scenarios. We also define the true value as the one that makes a buyer's utility zero given he selects the best strategies in the later stages. We use b_{ik}^* to denote WSP's true value. So we get W_i 's true value for his first channel as:

$$b_{i1}^* = p_i^*(1) \cdot \min\{\sum_{j=1}^{N_i} w_j^{i*}(1), B\}. \quad (10)$$

$p_i^*(k)$ and $w_j^{i*}(k)$ stand for the optimal pricing strategy and demand strategy given W_i has k channels. Eq. (10) is also the

Algorithm 1 Flexible Auction: winner determination

- 1: Find the C largest bids from $\{b_{ij}\}, (i = 1, \dots, N, j = 1, \dots, C)$, with a max heap.
 - 2: Let $\{b_i^s\}, i = 1, \dots, C$ be the sorted array of the C largest bids in descending order.
 - 3: $K_i = 0$, for all $i = 1, \dots, N$.
 - 4: **for** $k = 1$ to C **do**
 - 5: $K_i = K_i + 1$, where b_k^s is submitted by W_i .
 - 6: **end for**
 - 7: return (W_i wins K_i channels, $i = 1, \dots, N$).
-

marginal benefit that the first channel can bring to W_i . If W_i pays more than this amount to get it, W_i definitely makes a loss.

Similarly, considering the marginal benefit of each additional channel for W_i , the true value for W_i 's k -th channel ($2 \leq k \leq C$) will be

$$b_{ik}^* = p_i^*(k) \cdot \min\{\sum_{j=1}^{N_i} w_j^{i*}(k), k B\} - p_i^*(k-1) \cdot \min\{\sum_{j=1}^{N_i} w_j^{i*}(k-1), (k-1) B\}. \quad (11)$$

By intuition, the optimal bids for any W_i are decreasing as the marginal benefit of the first several channels is higher than the latter ones. We can derive this relationship from Eq. (7)(9)(10)(11) to verify its correctness. We have the theorem:

Theorem 1: Any W_i 's bidding structure presents marginal decreasing property. Mathematically, $b_{i1}^* \geq b_{i2}^* \geq \dots \geq b_{iC}^*$.

The proof is provided in the Appendix. In fact this property facilitates the algorithm design for Flexauc, such that we can obtain linear-time algorithms.

D. Auction Design

The goal of auction design is two-fold. First, the auction mechanism should be truthful. Therefore, W_i bidding with his true values is the dominant strategy. Second, the auction should provide efficiency, which means the channels should be allocated to those bidders who evaluate them most. Therefore, SH's revenue can be increased.

The auction is a sealed-bid auction with one seller (SH) and multiple buyers (WSPs). The auction procedure consists of two parts: the winner determination (channel allocation) and payment mechanism. As the channels are identical, the channel allocation result is presented in the form of the number of winning channels $\{K_i\}$. The payment mechanism can be flexible. In this paper, we consider three different payment mechanisms. The first one is the well-known VCG mechanism [15]–[17]. The second one is a modified version of the uniform pricing mechanism which preserve truthfulness for multi-unit demands. Besides, we also design a partial uniform pricing mechanism.

Algorithm 2 Flexible Auction: payment mechanism (VCG)

```

1: Double the size of  $\{b_i^s\}$  to  $2C$ .
2: Find the  $2C$  largest bids from  $\{b_{ij}\}, (i = 1, \dots, N, j = 1, \dots, C)$ , with a max heap.
3: Store the sorted array of the  $2C$  largest bids in descending order in  $\{b_i^s\}, i = 1, \dots, 2C$ .
4: for  $i = 1$  to  $N$  do
5:   if  $K_i > 0$  then
6:      $losingBid = 0$ 
7:      $j = C + 1$ 
8:      $totalPayment(i) = 0$ 
9:     while  $losingBid < K_i$  do
10:      if  $b_j^s$  is not submitted by  $W_i$  then
11:         $totalPayment(i) = totalPayment(i) + b_j^s$ 
12:         $losingBid = losingBid + 1$ 
13:      end if
14:       $j = j + 1$ 
15:    end while
16:  end if
17: end for
18: return ( $W_i$  pays  $totalPayment(i), i = 1, \dots, N$ ).

```

1) *Winner determination*: The auction is a standard auction such that the C bids with highest values are selected as winning bids: $\sum_{i=1}^N K_i = C$. This can be easily achieved by selecting the largest C bids from all NC bids, which is done by building a maximum heap (Algorithm 1).

2) *Payment mechanism*: The payment mechanism is relatively independent of the previous winner determination part. Here we introduce three possible payment mechanisms, namely: (1) VCG mechanism, (2) modified uniform pricing mechanism, and (3) partial uniform pricing mechanism. The algorithms for payment mechanisms are following the notations in Algorithm 1.

The idea of VCG mechanism is highly abstract and can be applied to universal cases, independent of the form of bidding structure, items to be sold, and so on. With VCG mechanism, W_i 's payment is determined by the externality he exerts on other competing WSPs. In this auction, the externality is the sum of other WSPs' K_i highest losing bids. The VCG mechanism is designed as Algorithm 2. Still the same, SH needs to know the highest $2C$ bids only. Lines 1 to 5 do this job. Because even in the extreme case where one WSP wins all C channels, his externality is the following C largest bids. That means knowledge of largest $2C$ bids is enough.

The general idea of uniform pricing is to charge each channel the same price as the channels are identical. It increases the buyers' acceptance since there is no price discrimination. The uniform price charged is also called the market clearing price. However, in general, uniform pricing is not truthful for multi-unit demand [18]. To guarantee the truthfulness, the clearing price should be selected independent of the winners' bids. Here we introduce a modification of the traditional uniform pricing scheme. Instead of choose the highest losing bids or the

Algorithm 3 Flexible Auction: payment mechanism (Uniform pricing)

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1: (Given  $C < N$ )
2:  $maxLoserBid = 0$  // Store the highest bid from a bidder who does not win any channel
3: for  $i = 1$  to  $N$  do
4:   if  $b_{i1} < b_C^s$  and  $b_{i1} > maxLoserBid$  then
5:      $maxLoserBid = b_{i1}$ 
6:   end if
7: end for
8: return ( $W_i$  pays  $totalPayment(i) = maxLoserBid \times K_i, i = 1, \dots, N$ ).

```

Algorithm 4 Flexible Auction: payment mechanism (Partial uniform pricing)

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1: for  $i = 1$  to  $N$  do
2:   if  $K_j < C$  then
3:      $maxLoserBid(i) = b_{j(K_j+1)}$ 
4:   else
5:      $maxLoserBid(i) = 0$ 
6:   end if
7: end for
8: Find the largest two elements from the array  $maxLoserBid$ . Let them be  $maxLoserBid1$  and  $maxLoserBid2$ .
9: for  $i = 1$  to  $N$  do
10:  if  $maxLoserBid(i) < maxLoserBid1$  then
11:     $totalPayment(i) = maxLoserBid1 \times K_i$ 
12:  else
13:     $totalPayment(i) = maxLoserBid2 \times K_i$ 
14:  end if
15: end for
16: return ( $W_i$  pays  $totalPayment(i), i = 1, \dots, N$ ).

```

lowest winning bids as the clearing price, we use the highest bids from the bidder who loses all his bids. This modified version of uniform pricing only works under the condition that $C < N$. When $C < N$, we can then find a WSP who does not win a channel at all. The modified uniform pricing algorithm is given by Algorithm 3. In the rest of this paper, we mean "uniform pricing" by the modified version.

Motivated by the previous two mechanisms, we design the partial uniform pricing, which preserves both their advantages. By partial uniform pricing, a winning WSP pays for each of his channels the same amount of money. His unit price is determined by others' highest losing bid. But different WSPs can have different unit prices. There are two advantages of partial uniform pricing compared with uniform pricing. On one hand, this mechanism will generate revenue for SH no less than that of VCG mechanism. An intuitive explanation is given here. By partial uniform pricing mechanism, W_i pays the amount of K_i multiples others' highest losing bid. By VCG mechanism, W_i pays the amount of the sum of others' highest K_i bids. On the other hand, this mechanism does not require

the condition $C < N$ since we always have a losing bid as W_i 's clearing price. The partial uniform pricing mechanism is shown in Algorithm 4.

We can combine any of the three payment mechanisms with the standard winner determination part to get a complete auction rule. The auction is truthful with any of the three mechanisms. The auction also maximizes social welfare. These two property will be proved later.

E. Spectrum Partition

From the previous part, we show that the payment mechanism can affect SH's revenue. Not surprisingly, the value of C also does. A significant and interest question is: if SH has a whole spectrum block initially, how many channels should he partition it into in order to maximize his own revenue $U_{SH}(C)$ or social welfare $S(C)$? Note that the size of spectrum block BC is constant.

For the question of maximization of SH's own revenue, there seems no beautiful answer. Intuitively C should not be too small or too large. A small C means that SH sells bigger piece of channels to fewer WSPs. Considering that WSPs' marginal evaluations on spectrum bandwidth are decreasing, it may be good for SH to further divide the big spectrum piece to smaller ones and sell them to more WSPs. A large C means there are lots of bids presented, which increases the burden of buyers and incurs high computational overheads.

To maximize the social welfare, we can take any of the three payment mechanisms. Because the affects of payment can be canceled out by the summation of WSPs' and SH's utilities. Maximization of social welfare is equivalent of allocating the channels to those who evaluate them highest, which is exactly what we have done in the winner determination mechanism.

Besides, C also affects the social welfare. If $C \rightarrow \infty$, it is a standard water filling problem similar as power control in multi-antenna Gaussian channels (i.e., [19]). There is a well-known algorithm which runs iteratively to obtain the numerical solution. The optimal spectrum partition is a water filling problem with the constraint of granularity of C . So we can conclude that the social welfare increases with C generally. Strictly, we have the property that $S(kC) \geq S(C)$ under the same auction rule, where k is any positive integer. Because after the further partition of current channels, SH can allocate k times of number of channels to current winning WSPs, which leads to the same social welfare. So the further partition of channels does not decrease social welfare. Due to the marginal effect of WSPs' evaluations, the improvement of social welfare by further partition gets smaller while the overhead gets larger. SH can select an appropriate C by considering all factors.

To make a summary, there is no closed-form solution for optimal C to maximize SH's revenue or social welfare though it does exist. When knowing possible service positioning strategies of the WSPs, SH can jointly optimize the decision of C and the auction design to obtain the highest revenue. In practice, SH can select best C according to his purpose, computational overheads and other concerns, such as the

technical requirements of wireless communication standards and channel aggregation cost. If circumstances allowed, one simple approach can be that SH iteratively doubles C and stops when $|\frac{S(2C)-S(C)}{S(C)}|$ is less than a predefined threshold. We will leave the detailed discussions as our future work.

V. ECONOMIC PROPERTIES AND TIME COMPLEXITY

In this section, we prove the properties of Flexauc: truthfulness and efficiency. We also analyze its time complexity.

A. Truthfulness

Theorem 2: Flexauc is truthful with any of the three payment mechanisms such that W_i 's best bidding strategy is $\{b_{i1}^*, \dots, b_{ik}^*, \dots, b_{iC}^*\}$.

The proof is provided in the Appendix.

B. Efficiency

Theorem 3: Flexauc maximizes social welfare.

The proof is provided in the Appendix.

C. Time Complexity

We now analyze the running time of the algorithms.

For Algorithm 1, building heap takes $O(N)$. Heap adjustment takes $O(\log N)$. The complexity of Algorithm 1 is $O(C \log N)$. For Algorithm 2, lines 1 to 5 take $O(C \log N)$. Lines 6 to 19 take $O(NC)$. So the complexity of Algorithm 2 is $O(NC)$. Both Algorithms 3 and 4 take $O(N)$ time.

It means that Flexauc with three payment mechanisms induces the computational overhead $O(NC)$, $O(N + C \log N)$ and $O(N + C \log N)$ respectively. Since SH receives in total NC bids from WSPs, the overhead is definitely acceptable.

VI. NUMERICAL RESULTS

Here we present simulation results to verify our conclusions, evaluate the performance and compare it with existing mechanisms. The experiment environment is MATLAB. We evaluate WSPs' bidding strategies and show its marginal effect and truthfulness, then WSPs' optimal pricing strategies. The comparison of three payment mechanism and influence of C are also presented. We also compare our mechanism with previous auction mechanism. A frequently used scheme restricts each WSP to submit only one bid and SH to make a 0/1 allocation. We call it OneBid auction. We show that Flexauc always outperforms OneBid.³ All results have been averaged over an abundance of cases with randomly generated parameters.

The default settings of parameters are $C = 5$, $B = 50$ (MHZ), $N = 10$, N_i is randomly distributed within $[500, 1000]$ for $i = 1, \dots, N$. $\{\alpha_i\}$ are equally distributed in $[0.2, 0.4]$. The transmission range to the base station is randomly chosen in $[500, 1000]$ (meter) for any user with a uniform distribution. Assume that 75% of users are indoor and their service requirement is originated from indoor environment, and 25% from outdoor environment.

³Generally speaking, if each W_i submit his bid as $\{b_{i1} > 0, 0, \dots, 0\}$, then Flexauc degrades to OneBid. So the solution space of OneBid auction is contained within that of Flexauc, which guarantees its sub-optimality.

TABLE I: Truthfulness of WSPs' bids.

N, C	$U_i^j(u) < U_i^j(t)$	$U_i^j(u) = U_i^j(t)$	$U_i^j(u) > U_i^j(t)$
10, 5	0.1012	0.8988	0
10, 10	0.1331	0.8669	0
10, 20	0.1721	0.8279	0

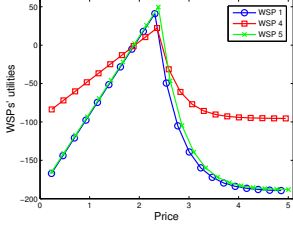


Fig. 3: WSPs' optimal pricing strategies.

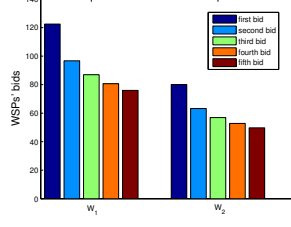


Fig. 4: WSPs' bids structure follows the marginal effect.

We define the attenuation factors as the multiplicative inverses of the path-losses based on the ITU and COST models [14].

- 1) From base station to outdoor user U_{ij} :

$$H_j = 10^{-4.9} \left(\frac{r}{1000} \right)^{-4} f^{-3} 10^{-S/10};$$

- 2) From base station to indoor user U_{ij} :

$$H_j = 10^{-3.7} \left(\frac{r}{1000} \right)^{-3} 10^{-S/10} 10^{-\frac{18.3n(\frac{n+2}{n+1}-0.46)}{10}};$$

The other default values of parameters are as follows: $P_m = 1$ (watt), $n_0 = -204$ (dB/Hz), r (in meters) is the transmission range, $f = 2000$ (MHz) is the carrier frequency, $n = 20$ is the number of floors in the path, S is the log-normal shadowing factor with the standard deviation of 8 (dB),.

We generate 100 cases with random radio parameters and randomly selected payment mechanism. In each case, we do 100 times of random selection of one WSP (either winner or loser) and make a random adjustment of his true bidding values (while still keeping the marginal effect of bids). Table I shows the truthfulness of the bids. $U_i^j(u)$ and $U_i^j(t)$ are WSP i 's utilities when he bids untruthfully and truthfully respectively. The values are the probabilities of the three cases with different N and C values. We see that in any case, $U_i^j(u) > U_i^j(t)$ never happens, which supports the truthfulness of our auction. With more WSPs and channels, the probability that $U_i^j(u) < U_i^j(t)$ is rising. It means that in such cases, cheating is more likely to reduce the cheater's utility as the competition becomes severe. No matter the WSP is a winner or loser, he has no incentive to bid untruthfully.

Fig. 3 shows the WSPs' utilities under different pricing strategies. We select the three winning WSPs after the auction and calculated their utilities under the prices of $[0.1p_i^*, 0.2p_i^*, \dots, 2p_i^*]$. Setting higher or lower prices may

lead to their overall losses. It verifies the correctness of WSPs' optimal pricing strategies.

Fig. 4 presents two WSPs' bids structure. The results consist with our theoretical analysis of diminishing marginal value of obtained channels. In this figure, W_2 's first bid value is small than W_1 's fourth bid. If W_2 wins one channel, then W_1 must win at least four channels.

Fig. 5, 6 and 7 compare the three pricing mechanisms in the auction. We have several observations. First, partial uniform pricing always generate revenue for SH no worse than VCG and uniform pricing. Uniform pricing does not outperform VCG all the time and vice versa. Second, the performance gaps of the three mechanisms increase with C . When 10 WSPs bid for 9 channels, uniform pricing generates sub-optimal results for SH in most cases.

When $C > N$, uniform pricing does not work any more. We compare the remaining two policies. Again Fig. 8 verifies the advantages of partial uniform pricing under the two cases of $C = 20$ and $C = 30$.

Fig. 9 gives numerical results for optimal C . We see that the larger C is, the higher social welfare it achieves. When C is large enough, the increment of social welfare gets smaller. If SH concerns more about social welfare, he can select, for example, $C = 30$. We also see that when C is too small ($C \leq 2$), both the social welfare and SH's revenue is small. When C is large enough ($C \geq 30$), both the social welfare and SH's revenue get converged. With partial uniform pricing, the converged revenue is higher than that of VCG. If SH concerns more about his revenue, he can choose partial uniform pricing and select large enough C .

Fig. 10 compares Flexauc with OneBid auction. In OneBid auction, both VCG and partial uniform pricing degrade to uniform pricing. So we do not have other payment mechanism for comparison under OneBid. We compare OneBid with Flexauc(VCG). We find that when C is small enough ($C \leq 3$), the two auction almost perform the same in both SH's revenue and social welfare. When C is larger, Flexauc(VCG) outperforms OneBid and the gaps keep increasing. If $C \geq N$, each WSP will obtain a channel under OneBid. But it must not happen for two reasons. First, there is no clearing price for this case to guarantee the truthfulness. Second, if $C > N$, definitely there are $C - N$ channels wasted. A rational SH will not make $C > N$ under OneBid auction.

VII. CONCLUSION

The WSPs face dynamic and diverse users' demands which impact their decision on how much bandwidth to purchase and how much money to pay for the SH. Previous spectrum auction studies do not pay attention to the tight relationship between bidding strategies and service provisions. Existing auction mechanism cannot be directly applied in this scenario or with efficient computational performance. In this paper, we analyze the end users' demands response, WSPs' optimal pricing and bidding strategies, SH's auction design and discuss the spectrum partition. We propose Flexauc as the solution

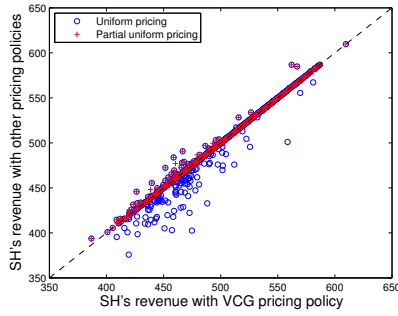


Fig. 5: Comparison of three pricing policies when $C = 3$.

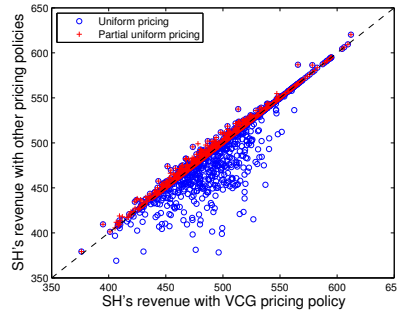


Fig. 6: Comparison of three pricing policies when $C = 5$.

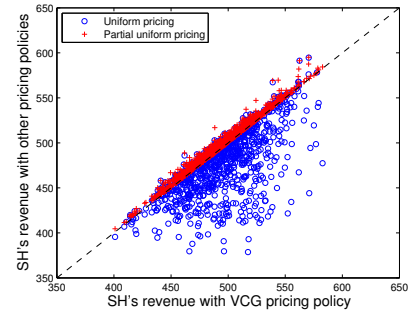


Fig. 7: Comparison of three pricing policies when $C = 9$.

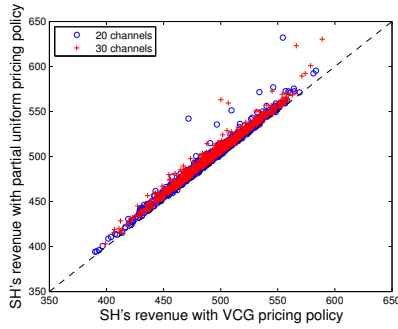


Fig. 8: Comparison of VCG and partial uniform pricing when $C > N$.

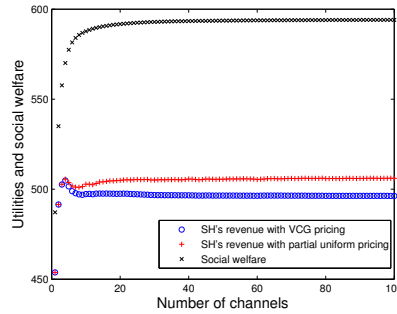


Fig. 9: Influences of C on SH's revenue and social welfare.

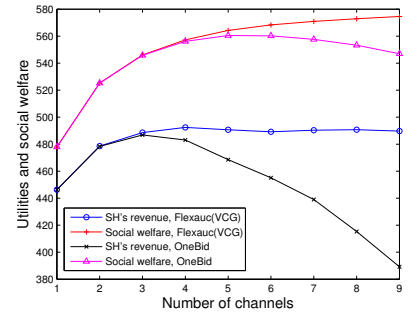


Fig. 10: Comparison of Flexauc(VCG) and OneBid mechanisms.

framework. Flexauc consists of a standard winner determination part and a flexible payment mechanism. There are three payment mechanisms studied and compared. All of them are truthful and maximize social welfare. The computational overhead of Flexauc is linear to the input size of the bids. We conduct comprehensive numerical simulation to verify our conclusions.

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APPENDIX

A. Proof of Theorem 1

Proof: By Eq. (10)(11), we have

$$\sum_{k=1}^{K_i} b_{ik}^* = p_i^*(K_i) \cdot \min\left\{\sum_{j=1}^{N_i} w_j^{i*}(K_i), K_i B\right\}. \quad (12)$$

Substitute Eq. (9) into it, then we get

$$\sum_{k=1}^{K_i} b_{ik}^* = \min\left\{\left(\ln\left(\frac{G_i}{K_i B}\right) - 1\right) K_i B, G_i e^{-2}\right\}. \quad (13)$$

Define function

$$f(K_i) = \left(\ln\left(\frac{G_i}{K_i B}\right) - 1\right) K_i B \quad (14)$$

Because

$$\frac{\partial f(K_i)}{\partial K_i} = B\left(\ln\left(\frac{G_i}{K_i B}\right) - 2\right) \geq 0 \quad (15)$$

and

$$\frac{\partial^2 f(K_i)}{\partial K_i^2} = -\frac{B}{K_i} < 0. \quad (16)$$

$G_i e^{-2}$ is a constant and the operator min keeps the convexity. So Eq. (12) is a convex function.

Note that $f(K_i)$ is positive for all $K_i = 1, 2, \dots$. By $f(K_i)$'s continuity, we have

$$\begin{aligned} f(0) &= \lim_{K_i \rightarrow 0^+} f(K_i) = \lim_{x = \frac{1}{K_i} \rightarrow +\infty} \frac{B\left(\ln\left(\frac{G_i}{B}\right) - 1\right)}{x} \\ &= \lim_{x \rightarrow +\infty} \frac{B}{x} = 0. \end{aligned} \quad (17)$$

By Eq. (12)'s convexity, we have

$$2 \sum_{k=1}^{K_i+1} b_{ik}^* \geq \sum_{k=1}^{K_i} b_{ik}^* + \sum_{k=1}^{K_i+2} b_{ik}^* \quad (18)$$

for all $K_i = 0, 1, 2, \dots$. After simplification, we obtain $b_{i1}^* \geq b_{i2}^* \geq \dots \geq b_{iC}^*$.

B. Proof of Theorem 2

Proof: To prove the truthfulness, we need to show that for any $1 \leq i \leq N$ and $1 \leq k \leq C$, $\{b_{i1}^*, \dots, b_{ik}^*, \dots, b_{iC}^*\}$ weakly dominates any other $\{b_{i1}, \dots, b_{ik}, \dots, b_{iC}\}$.

First we show that by replacing only b_{ik} with b_{ik}^* (if $b_{i(k-1)} \leq b_{ik}^* \leq b_{i(k+1)}$), the new strategy $b_{i1}, \dots, b_{ik}^*, \dots, b_{iC}$ weakly dominates the original strategy $b_{i1}, \dots, b_{ik}, \dots, b_{iC}$. By original strategy W_i wins k_i channels and by new one he wins k_i^* channels. We discuss the three possible cases:

- 1) Case 1: $k_i = k_i^*$. W_i wins the same number of channels by both strategies. By any of the three mechanisms, his

payment is determined by other WSPs' bids. So W_i utilities are the same by both strategies.

- 2) Case 2: $k_i < k_i^*$. It means W_i wins more channel by new strategy. His payment will be no more than the marginal benefit of the additional channel(s). So his utility is improved or keep the same by the new strategy. New strategy dominates original one.
- 3) Case 3: $k_i > k_i^*$. By new strategies, W_i wins less channels. W_i 's bid on k -th channel does matter. b_{ik} is one of the highest C biddings but b_{iK}^* is not. As both b_{ik} and b_{ik}^* are larger than b_{ij} ($j = k+1, k+2, \dots, C$), so b_{ij} is not one of the highest C biddings. So $k_i = k_i^* + 1$. That means the only difference is that original strategy gets k -th channel and new strategy does not. According to the payment mechanisms, by original strategy, W_i pays more than the marginal benefit b_{iK}^* to get the k -th channel. So New strategy dominates original one.

Then for any strategy $\{b_{i1}, \dots, b_{iC}\}$, we can adjust it in reverse order repeatedly like bubble sort algorithm: $\{b_{i1}, \dots, b_{iC}\} \rightarrow \{b_{i1}, \dots, \min\{b_{iC}^*, b_{i(C-1)}\}\} \rightarrow \{b_{i1}, \dots, \min\{b_{iC}^*, b_{i(C-1)}\}, \dots\} \rightarrow \dots \rightarrow \{b_{i1}^*, \dots\} \rightarrow \dots \rightarrow \{b_{i1}^*, b_{i2}^*, \dots\} \rightarrow \dots$ until it becomes $\{b_{i1}^*, \dots, b_{iC}^*\}$. It can be achieved by no more than $\frac{C(C+1)}{2}$ adjustments. During the adjustments, the new strategies dominant the old ones. So $\{b_{i1}^*, \dots, b_{iC}^*\}$ dominates any $\{b_{i1}, \dots, b_{iC}\}$. ■

C. Proof of Theorem 3

It is well-known that VCG mechanism maximizes social welfare. We provide a simple proof here.

Proof: An auction rule \mathbf{R}^* is efficient if it maximizes social welfare,

$$\mathbf{R}^*(\mathbf{x}) \in \arg \max_{\mathbf{R}} \sum E_j x_j, \quad (19)$$

where $x_j = 0, 1$. By VCG mechanism, the payment of bidder i is

$$P_i = W(0, \mathbf{x}_{-i}) - W_{-i}(\mathbf{x}). \quad (20)$$

Bidder i 's utility is

$$\begin{aligned} E_i - P_i &= E_i + W_{-i}(\mathbf{x}) - W(0, \mathbf{x}_{-i}) \\ &= W(\mathbf{x}) - W(0, \mathbf{x}_{-i}). \end{aligned} \quad (21)$$

So maximization of his own utility is equivalent to maximization of social welfare. ■

We know that the uniform pricing auction and partial uniform pricing auction distinguish from VCG auction only in the payment part. The payment effect can be canceled out by the summation of WSPs' and SH's utilities. So auctions with the two payment mechanism also maximize social welfare. It means an auction maximizes social welfare as long as it allocates items to those who evaluate them most. By motivating WSPs to bid truthfully and selecting the highest C bids, Flexauc indeed maximizes social welfare.