Restricted Coverage in Wireless Networks

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***Abstract*—For wireless networks, coverage with different restrictions that can capture the practical requirements have received great research interests. We will study several restricted coverage problems. The first problem is about K-coverage , *i.e.*, how to deploy wireless nodes such that each target is covered by at least K wireless nodes. We study the problem restricted to linear-K-coverage where there is a line, all targets lie in one side of this line and all wireless nodes lie in the other side. Assume each wireless node is associated with a weight, the objective is to select a minimum weighted subset of nodes such that each target is K-covered. We propose a 3-approximation for this problem by exploring geometric properties. The second problem is called K-road-coverage. Given a road map in a two-dimensional area which contains a set of paths and a set of wireless nodes, the locations of nodes can either be arbitrary or fixed, the objective is to select a minimum number of wireless nodes such that each path can be K-covered. We will reduce the problem to K-coverage and apply the algorithmic results for K-coverage to solve it.**

Another line of this work is to investigate a well-motivated problem called strongly dominating set, which is intrinsically related to coverage. Given a wireless networking system represented by a digraph $G = (V, \overrightarrow{E})$. Each wireless node u has a covering disk centering at u with its radius equal to the transmission range of u. We then draw a directed edge $\overrightarrow{uv}$ in G if u's corresponding covering disk contains v. A subset $U \subseteq V$ of wireless nodes is a *strongly dominating set* if every wireless node in $V \setminus U$ has both an in-neighbor in U and an out-neighbor in U. The objective is to find a minimum size strongly dominating set. Our method can achieve an approximation factor of $(2 + \epsilon)$.

I. Introduction

Coverage has been playing a critical role for many challenges in wireless networks such as energy efficient routing, data aggregation, localization, fault tolerance, diagnosis. In many applications, we are often required to use some wireless nodes for monitoring. Taking an example for illustration, when we deploy some fire-alarm sensors to monitor a building, every place in the building should be in the sensing range of some sensor. In this work, we will investigate several coverage problems with different restrictions that can capture the practical requirements.

The first problem is about K-Coverage. Given a set $\mathcal{D}$ of unit disks with positive weight defined by a function $c : \mathcal{D} \mapsto N^+$ and a set P of targets in the plane. A subset $\mathcal{D}' \subseteq \mathcal{D}$ is said to be a K-cover of P if each target in P is covered by at least K disks in $\mathcal{D}'$. We study the problem restricted to linear K-coverage. Assume there is a line, all targets lie in one side of the line and all disks lie in the other side. The **Linear K-Coverage** problem seeks a minimum weighted K-cover of P. The problem is a restricted version of disk cover. Disk cover has been proved to be NP-hard even when all disks are unit disks and $K = 1$ [15]. The geometric disk cover can admit a constant-approximation, and tremendous work is done for various disk cover variants [3], [5], [6], [19]. However, The existing work focus on 1-coverage problem while we address the general K-coverage.

The second problem is called K-Road-Coverage. Given a set of paths and a set of wireless nodes, a path is covered if and only if any point on the path lies within the covering range of some wireless node(s). Here we assume the wireless nodes can collaborate with each other to cover all the targets. The locations of wireless nodes can either be arbitrary or fixed. The problem **K-Road-Coverage** is to find minimum number of wireless nodes such that all paths on the road map can be K-covered.

Since coverage is intrinsically related to dominating set, we also consider a problem called strongly dominating set. Given a wireless networking system represented by a digraph $G = (V, \overrightarrow{E})$. Each wireless node u has a covering disk centering at u with its radius equal to the transmission range of u. We then draw a directed edge $\overrightarrow{uv}$ in G if u's corresponding covering disk contains v. A subset $U \subseteq V$ of wireless nodes is a strongly dominating set if every wireless node $u \in V \setminus U$ has both an in-neighbor in U and an out-neighbor in U. The problem **Minimum Strongly Dominating Set (MSDS)** seeks a minimum size strongly dominating set.

The motivation for studying MSDS is described as follows. For a wireless networking system, to construct a backbone for routing and broadcasting, we need to ensure that for every wireless node v, there exist a node u such that both u and v are within the transmission ranges of each other, thus node v can both send and receive data from the whole network to the backbone. Traditional, this is modeled as selecting a dominating set in disk containment graph [25]. However, we observe that, to achieve the purpose, we actually can relax the requirement to that: for any wireless node v, there exists a node $u_1 \in U$ such that v can transmit its data to u_1 (u_1 is within of the transmission range of v), and at the same time, there exists another node $u_2 \in U$ such that v can be receive data from u_2 (v is within of the transmission range of u_2). These two wireless nodes u_1 and u_2 are not necessarily the same. This is actually to select a strongly dominating set in a *directed* version of disk containment graph.

Our main contributions are as follows. For the Linear K-Coverage problem. we present the first 3-approximation based on a dynamic programming technique and geometric

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properties. To the best of our knowledge, this is the first result to address the K-Coverage problem from a geometric perspective and the approximation ratio is independent of K. For the K-Road-Coverage problem, we will explore its relations to disk coverage, and apply the algorithmic results for disk coverage to solve it. For the problem MSDS, we can achieve an approximation factor of $(2+\epsilon)$. We also discuss the further treatment of this problem.

The rest of the paper is organized as follows. Section II is devoted to the presentation of our solution for Linear K-Coverage problem. In Section III, we study the K-Road-Coverage problem. Section IV deals with the MSDS problem. In Section V, we conduct a literature review for wireless coverage. We conclude our paper in Section VI.

II. Linear K-Coverage

In the Linear K-Coverage problem, we assume there is a horizontal line and all disks lie above this line and all targets lie below this line.

We begin with some terms and notations. We label all target points in P from left to right as $< p_1, p_2, \cdots, p_n >$, here $n = |P|$. Assume P_i is the set of targets from P lying to the left to p_i (including p_i).

Similar to [30], consider a disk $D \in \mathcal{D}$ intersecting a vertical line $x = x_p$, a disk $D' \in \mathcal{D}$ is said to be line-dominated by D *w.r.t.*the line $l_p : x = x_p$ (Figure 1) if one of following cases occurs.

1) D' does not intersect the line $l_p : x = x_p$;
2) The lowest endpoint of $D \cap l_p$ is below the lowest endpoint of $D' \cap l_p$;
3) $D \cap l_p$ and $D' \cap l_p$ have the same lowest endpoint, but D lies to the left of D'.

Note that when involving a disk's location, we refer to the location of the disk's center, later on we will keep this notation.

It's easy to verify that *line-dominating* is transitive: Suppose that D_1, D_2, D_3 are three disks in $\mathcal{D}$ and l_p is a vertical line. If D_1 line-dominates D_2 *w.r.t.*l_p and D_2 line-dominates D_3 *w.r.t.*l_p, then D_1 line-dominates D_3 *w.r.t.*l_p.

Definition 1: (skyline) Given a set of disks $\mathcal{D}' \subseteq \mathcal{D}$ and a target point $p \in P$, the skyline of $\mathcal{D}'$ at p is a sequence of K disks $D_1, D_2, \cdots, D_K$ from $\mathcal{D}'$ such that D_i line-dominates D_{i+1} *w.r.t.*the vertical line $x = x_p$, Moreover, D_K line-dominates all disks from $\mathcal{D}' \setminus \{D_1, D_2, \cdots, D_K\}$ *w.r.t.*the line $x = x_p$. Here x_p is the x-coordinate of the point p.

For any target $p_i \in P$, let $\mathcal{D}_i$ denote the set of disks in $\mathcal{D}$ covering p_i. Denote

$$\Gamma_i = \mathcal{D}_i \times \mathcal{D}_i \cdots \times \mathcal{D}_i.$$

For any $(D_1, \cdots, D_K) \in \Gamma_i$, let $\mathcal{C}_i(D_1, \cdots, D_K)$ denotes the collection of K-covers $\mathcal{D}'$ of P_i, which satisfies:

- $(D_1, \cdots, D_K)$ is the skyline of $\mathcal{D}'$ at p_i;
- $D_k : k = 1, \cdots, K$ can cover p_i.

Definition 2: (i-th startup cost) The i-th restart cost of a disk set $\mathcal{D}'$ (noted as $c_i(\mathcal{D}')$) is defined as follows: We proceed the target points from left to right. For each target point p_i, we can determine the skyline $\mathtt{SKY}_i$ of $\mathcal{D}'$ at p_i. For each disk $D \in \mathtt{SKY}_i$, if it does not appear in $\mathtt{SKY}_{i-1}$, then we add a cost of D. Otherwise, we will not add the cost.

If $\mathcal{C}_i(D_1, \cdots, D_K)$ is not empty, let $C_i(D_1, \cdots, D_K) \in \mathcal{C}_i(D_1, \cdots, D_K)$ be a K-cover of P_i with minimum i-th startup cost, and $c_i(D_1, \cdots, D_K)$ be the i-th startup cost of $C_i(D_1, \cdots, D_K)$; otherwise, set $C_i(D_1, \cdots, D_K)$ to null, and set $c_i(D_1, \cdots, D_K)$ to ∞.

For any $(D_1, \cdots, D_K) \in \Gamma_i$, we denote by $\Gamma'(D_1, \cdots, D_K)$ the set of $(D'_1, \cdots, D'_K)$ satisfying that $(D_1, \cdots, D_K)$ is the skyline of $\{D_1, \cdots, D_K\} \cup \{D'_1, \cdots, D'_K\}$ at p_i. We prove the following recursive relation. We do not consider the case when $c_i(D_1, \cdots, D_K) = \infty$.

***Theorem** 1:* $\forall i, \forall (D_1, \cdots, D_K) \in \Gamma_i$,

$$c_i(D_1, \cdots, D_K) = \min_{(D'_1, \cdots, D'_K) \in \Gamma'(D_1, \cdots, D_K)}$$
$$\left\{ c_{i-1}(D'_1, \cdots, D'_K) + c\Bigg(\Big\{ D_1, \cdots, D_K \Big\} \setminus \Big\{ D'_1, \cdots, D'_K \Big\} \Bigg) \right\}$$

Proof: **LHS $\geq$ RHS:** Let $D'_1, \cdots, D'_K$ be the skyline of $C_i(D_1, \cdots, D_K)$ at p_{i-1}. Let $\mathcal{D}''$ be the set of disks that appear in any of the previous $i-1$ skylines of $C_i(D_1, \cdots, D_K)$. Set

$$\mathcal{D}' = C_i(D_1, \cdots, D_K) \setminus \Bigg(\Big\{ D_1, \cdots, D_K \Big\} \setminus \mathcal{D}'' \Bigg)$$

We have $\mathcal{D}' \in \mathcal{C}_{i-1}(D'_1, \cdots, D'_K)$. Therefore, $c_{i-1}(D'_1, \cdots, D'_K) \leq c_{i-1}(\mathcal{D}')$. In addition, we have

$$c_{i-1}(\mathcal{D}') = c_i(D_1, \cdots, D_K) - c\Bigg(\Big\{ D_1, \cdots, D_K \Big\} \setminus \Big\{ D'_1, \cdots, D'_K \Big\} \Bigg).$$

Therefore, we have

$$c_i(D_1, \cdots, D_K) \geq c_{i-1}(D'_1, \cdots, D'_K) + c\Bigg(\Big\{ D_1, \cdots, D_K \Big\} \setminus \Big\{ D'_1, \cdots, D'_K \Big\} \Bigg).$$

LHS $\leq$ RHS: Suppose RHS achieves minimum at $(D'_1, \cdots, D'_K)$.

Let

$$\mathcal{D}' = C_{i-1}(D'_1, \cdots, D'_K) \cup \Big\{ D_1, \cdots, D_K \Big\}.$$

First, $\mathcal{D}'$ is a K-cover of P_i and $(D_1, \cdots, D_K)$ is the skyline of $\mathcal{D}'$ at p_i. Then, $\mathcal{D}' \in \mathcal{C}_i(D_1, \cdots, D_K)$. Consequently, we have

$$c_i(D_1, \cdots, D_K) \leq c_i(\mathcal{D}') \leq c_{i-1}(D'_1, \cdots, D'_K) + c\Bigg(\Big\{ D_1, \cdots, D_K \Big\} \setminus \Big\{ D'_1, \cdots, D'_K \Big\} \Bigg).$$

■

***Theorem** 2:* $\min_{D_1, \cdots, D_K} c_n(D_1, \cdots, D_K) \leq 3c(OPT)$

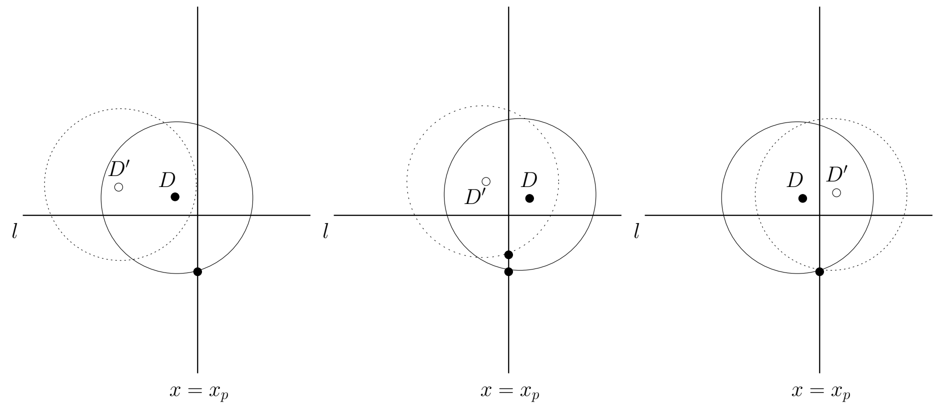

Fig. 1: [30] Three cases when disk D line-dominates D'.

Proof: Assume the skyline of OPT at p_n is $D_1, \cdots, D_K$, we will prove that $c_n(D_1, \cdots, D_K) \leq 3c(OPT)$. Since $OPT \in \mathcal{C}_n(D_1, \cdots, D_K)$, we have $c(OPT) \geq c_n(D_1, \cdots, D_K)$. Thus, we only need to prove that $c_n(OPT) \leq 3c(OPT)$. By Lemma 2, each disk can be counted at most 3 times in $c(OPT)$, thus the theorem holds. ■

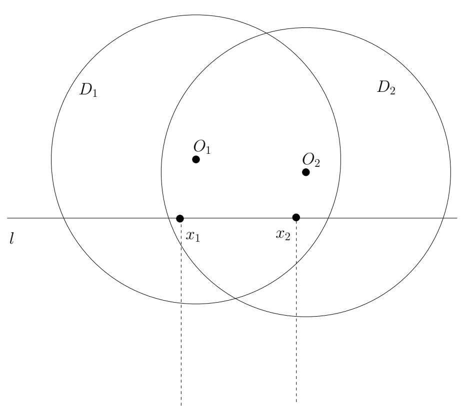

Fig. 2: The property of line-dominating.

The following lemma will serve as a technique basis for proving Lemma 2.

***Lemma** 1:* [27] Consider two disks D_1, D_2 lying above a horizontal line l, D_1 line-dominate D_2 *w.r.t.*$x = x_1$ and D_2 line-dominates D_1 *w.r.t.*$x = x_2$. Let O_1, O_2 be the centers of the disks D_1, D_2 respectively. Then O_1 lies to the left of O_2 *iff* $x_1 < x_2$ and vice versa. (Fig. 2).

***Lemma** 2:* For any disk set $\mathcal{D}$ which is a K-cover of P, each disk appears in the skylines non-consecutively for at most three times.

Proof: Assume to the contrary that some disk D appears in the skylines non-consecutively for 4 times. Then, there exists three target points p_1, p_2, p_3 (their x-coordinates are x_1, x_2, x_3 respectively) and D appears in the skyline for three times: (1) before x_1 (2) between x_1 and x_2, (3) between x_2 and x_3, and (4) after x_3. However, D does not appear in the skylines at x_1 or at x_2 or at x_3. We will derive contradiction.

For the K skyline-disks at $x_i (i = 1, 2, 3)$, assume L_i disks lie to the right of D and R_i disks lie to the right of D. Then, we have $L_1 + R_1 = L_2 + R_2 = L_3 + R_3 = K$. However, for any target point before x_1, by Lemma 1, at least $L_1 + L_2 + L_3$ disks line-dominates D, we have $L_1 + L_2 + L_3 < K$ as D appears in the skyline. For any target point between x_1 and x_2, by Lemma 1, at least $R_1 + L_2 + L_3$ disks line-dominates D, we have $R_1 + L_2 + L_3 < K$ as D appears in the skyline. For any target point between x_2 and x_3, by Lemma 1, at least $R_1 + R_2 + L_3$ disks line-dominates D, we have $R_1 + R_2 + L_3 < K$ as D appears in the skyline. For any target point after x_3, by Lemma 1, at least $R_1 + R_2 + R_3$ disks line-dominates D, we have $R_1 + R_2 + R_3 < K$ as D appears in the skyline. Thus, we have $(L_1 + L_2 + L_3) + (R_1 + L_2 + L_3) + (R_1 + R_2 + L_3) + (R_1 + R_2 + R_3) < 4K$. This means that $(L_1 + 3R_1) + 2(L_2 + R_2) + (3L_3 + R_3) < 4K$. However, we have $(L_1 + 3R_1) + 2(L_2 + R_2) + (3L_3 + R_3) = 4K + 2R_1 + 2L_3 > 4K$. This causes contradiction, which finishes the proof. ■

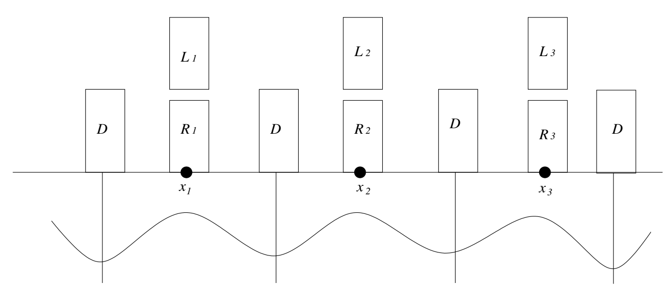

Fig. 3: Illustration for Lemma 2.

III. K-ROAD COVERAGE

In the K-Road Coverage problem, we assume that the feasible locations of all wireless nodes are continuous, *i.e.*, nodes can be deployed anywhere on the two-dimensional area. We further assume that all nodes have the same sensing range of exactly one.

We first describe the discretization process. We partition the deployment region by using a set of vertical lines $a_i : x = i \cdot \ell : i \in \mathbb{Z}$ and horizontal lines $b_j : y = j \cdot \ell : j \in \mathbb{Z}$, where $\ell < 1$ is grid length (grid granularity) and $\mathbb{Z}$ denotes the set of all integers. We call i, j as the index of vertical line a_i and horizontal line b_j respectively. We label the crossing point of a vertical line a_i and horizontal line b_j as grid points $p_{i,j}$. Let $P_\ell = \{p_{i,j} : i, j \in \mathbb{Z}\}$ be the set of all grid points. See Figure 4 for an illustration.

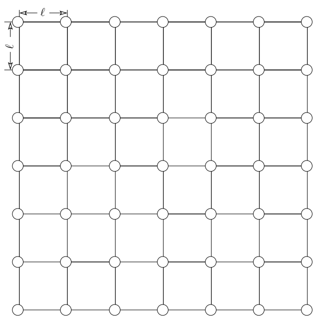

Fig. 4: Discretization: P_ℓ is the set of all grid points (white circles).

We then state the discrete version:

Discrete K-Road Coverage: assume the feasible locations of all wireless nodes are restricted to P_ℓ. Given a set of paths, the objective is to find a minimum number of nodes such that each path is K-covered.

Observe that the new problem differs from the original one in only one additional restriction: nodes can only lie in the grid points from P_ℓ. For simplicity, we call the original problem continuous K-Road-Coverage. The optimum solution (assumed to be $\mathcal{S}_D^*$) for the discrete version is a feasible solution for the original continuous K-Road-Coverage problem. We will show that by properly choosing grid length ℓ, we can ensure that $\mathcal{S}_D^*$ is a 3-approximation solution for the original problem.

Let us introduce some geometric facts:

Lemma 3: Given a sector OO_1O_2 centered at O with Central angle $120°$ and radius one, assume O_1OA and O_2OA are two sectors centered at O_1 and O_2 respectively, here A is the middle point of the arc $\widehat{O_1O_2}$ let u be any point lying in the intersection area of two sectors, then the distance between u and any other point in sector OO_1O_2 is at most one (Figure 5).

Proof: Any point v in sector OO_1O_2 lies either in the area OO_1AC (the union of the sector O_1OA centered at O_1 and sector OO_1A centered at O) or in the area OO_2AB (the union of the sector O_2OA centered at O_2 and sector OO_2A centered at O). Thus, the line segment uv must be wholly contained in any one of the two areas. As both areas have diameter one, the length of uv is at most one. This finishes the proof. ■

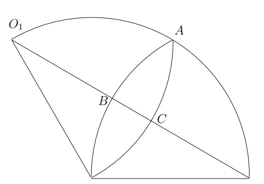

Fig. 5: The illustration of Lemma 3 and Lemma 4.

Lemma 4: Given a sector OO_1O_2 centered at O with Central angle $120°$ and radius one, if the grid size $\ell \leq 0.189$, there exists a grid point such that the distance between the grid point and any point in Sector OO_1O_2 is at most one.

Proof: In Figure 5, the disk with its diameter BC must be wholly contained in the intersection area of two sectors O_1OA and O_2OA. We have $|BC| = 2 - \sqrt{3} = 0.268$. If the grid size is small enough, *e.g.*, $\ell \leq 0.189$, we have $\sqrt{2}\ell \leq 0.268$, then there exist intersections between grid points and any disk of diameter 0.268. Thus, there exists a grid point u in the intersection area of two sectors O_1OA and O_2OA. By Lemma 3, the distance between u and any other point in sector OO_1O_2 is at most one. ■

Lemma 5: A node O at an arbitrary location can be replaced by up to 3 wireless nodes at grid points such that O's covering disk can be wholly contained by the union of 3 nodes' covering disks. This means that the coverage multiplicity of any path will not decrease after the replacement. Here P_ℓ is the set of all grid points from a partition scheme with $\ell = 0.189$.

Proof: We partition u's covering disks into 3 sectors, each with Central angle $120°$. By Lemma 4, for each sector, there exists a grid point, such that any point in the sector can be covered by a wireless node lying at the chosen grid point. Therefore, we can replace the node u by up to 3 wireless nodes lying at grid points. ■

Theorem 3: Suppose that there exists a polynomial time ρ-approximation algorithm for the discrete K-Road-Coverage problem with $\ell = 0.189$, then there exists a polynomial time 3ρ-approximation algorithm for the original K-Road-Coverage problem.

Proof: Suppose $\mathcal{S}^*$ is an optimal solution for the continuous K-Road-Coverage problem. If we can construct a feasible solution $\mathcal{S}$ for the discrete K-Road-Coverage problem satisfying $|\mathcal{S}| \leq 3 \cdot |\mathcal{S}^*|$, then we have $|\mathcal{S}_D^*| \leq |\mathcal{S}| \leq 3 \cdot |\mathcal{S}^*|$, here $\mathcal{S}_D^*$ is an optimal solution for the discrete K-Road-Coverage with $\ell = 0.189$. then $\mathcal{S}_D^*$ is a 3-approximation solution for the original continuous K-Road Coverage problem. We next describe the process of constructing $\mathcal{S}$.

By Lemma 5, any node v in S^* can be replaced by 3 nodes lying at grid points such that the coverage multiplicity of any path will not decrease. We perform the replacement for all nodes in S^* and assume $\mathcal{S}$ is the union of the selected wireless nodes. Clearly, all paths can be K-covered by $\mathcal{S}$, *i.e.*, $\mathcal{S}$ is a feasible solution for the discrete K-Road-Coverage problem, and $|\mathcal{S}| \leq 3 \cdot |\mathcal{S}^*|$ holds. Thus, we have finished the construction.

If there exists a polynomial time ρ-approximation solution $\mathcal{S}_D$ for the discrete K-Road-Coverage problem with $\ell = 0.189$, then $\mathcal{S}_D$ is a feasible solution for the continuous K-Road-Coverage problem and at the same time we have $|\mathcal{S}_D| \leq \rho\,|\mathcal{S}_D^*| \leq 3\rho \cdot |\mathcal{S}^*|$. Therefore, there exists a polynomial time 3ρ-approximation solution for the continuous K-Road-Coverage problem. This finishes the proof. ■

Next, we describe the process of Reducing discrete K-Road-Coverage to K-Coverage. In the discrete K-Road-Coverage problem, there is a set of paths and a set of wireless nodes. Assume that all covering nodes have discrete locations.

The objective is to select minimum number of wireless nodes such that each path is K-covered. We will reduce it to K-Coverage problem and show that making all paths K-covered is equivalent to making some points on the paths K-covered.

As we know, the coverage range of each node is a disk. For each node, we draw the corresponding boundary circle. Let the set of paths consists of a set of d road segments. Then the paths will be divided into sub-segments by these circles. The total number of sub-segments is bounded by $O(md)$ where m is the number of wireless nodes and d is the number of road segments. Observe that each sub-segment is an atomic part in terms of coverage, *i.e.*, the sub-segment is either totally covered by a node or not covered by a node at all. We call each sub-segment as an equivalence class. Let the *coverage multiplicity* of any point be defined as the number of wireless nodes that covers this point. Then, all points in an equivalence class have the same coverage multiplicity. We will show that, for a general K, K-covering all these sub-segments corresponds to K-covering those representative points of equivalence classes :

***Lemma** 6:* For the discrete K-Road-Coverage problem, the set L of sub-segments is K-covered if and only if the set of representative points is K-covered.

Proof: Let P be the set of representative points. We first prove that K-covering P is necessary for K-covering all sub-segments L. This is obvious since any point on a sub-segment is K-covered when L is K-covered.

Next, we prove that K-covering P is sufficient for K-covering all sub-segments L. Recall that the resultant sub-segment set L is obtained by drawing a circle centered at each feasible wireless nodes. This means that each sub-segment is either entirely covered by a wireless node or totally outside of the covering range of a wireless node. Any sub-segment cannot have any intersection (except the two end points) with any circle(s); otherwise, it would be further divided into smaller sub-segments. This causes contradiction. Hence, if a representative point of a sub-segment is K-covered, the entire sub-segment is K-covered. ■

Based on Lemma 6, we reduce the problem of K-covering all sub-segments to the problem of K-covering all representative points of all sub-segments while the candidate wireless node set is the same. The new problem is equivalent to K-Coverage problem. We can apply the algorithmic results of K-coverage such as [2] to solve the original K-Road Coverage problem.

IV. MINIMUM STRONGLY DOMINATING SET

In this section, we present a $(2+\epsilon)$-approximation algorithm for the problem MSDS.

A forward dominating set is a subset U_1 of U such that for each node u, there exist a node $v \in U_1$ and $\overrightarrow{uv} \in \overrightarrow{E}$. A backward dominating set is a subset U_2 of U such that for each node u, there exist a node $v \in U_2$ and $\overrightarrow{vu} \in \overrightarrow{E}$. Observe that, any strongly dominating set is the union of a forward dominating set and backward dominating set. Our method for selecting a strongly dominating set can be divided into two phases. First, we select a forward dominating set U_1. Second, we select a backward dominating set U_2. We finally output the union $U_1 \cup U_2$ of two sets (deleting the redundant nodes).

Thus, we have the following main theorem.

***Theorem** 4:* Suppose that there exists a polynomial a-approximation algorithm for the Minimum Geometric Hitting Set and there exists a polynomial b-approximation algorithm for the Minimum Disk Cover, then there is polynomial $(a+b)$-approximation algorithm for the strongly dominating set problem in a disk containment graph.

We observe that the problem forward dominating set selection is equivalent to the problem Minimum Geometric Hitting Set. There exists a polynomial $(1+\epsilon)$-approximation algorithm for Minimum Geometric Hitting Set [18] by using a Local Search method.

On the other hand, the backward dominating set selection is equivalent to the problem Minimum Disk Cover. For the problem Minimum Disk Cover, there exists a polynomial $(1+\epsilon)$-approximation algorithm [24] by using a Local Search method as well.

To summarize, our method is described as follows. Given a wireless networking system represented by a digraph $G = (V, \overrightarrow{E})$. Fix the parameter k, we begins with the node set $U_1 = V$, which clearly is a forward dominating set of G. We then replace any point subset with size at most k by a point subset of size at most $k-1$, if this replacement still results in a forward dominating set. We will keep replacing until no further possible replacement. Second, we begins with the node set $U_2 = V$, which clearly is a backward dominating set. We then replace any node subset with size at most k by a subset of size at most $k-1$, if this replacement still results in a backward dominating set. We will keep the processing until no further possible replacement. The union of $U_1 \cup U_2$ has been proved to be a strongly dominating set and furthermore, we have the following theorem.

***Theorem** 5:* The problem MSDS admits a polynomial time $(2+\epsilon)$-approximation solution.

Discussions: The communication topology of a wireless network can be modeled by a disk containment graph [8], [25] where there exists an edge between two nodes if and only if they are within each other's communication ranges. We assume all nodes have disparate communication ranges. In the disk containment graph, each node u is associated with a covering disk centering at u with its radius r_u equal to the transmission range of u. We connect a pair of nodes if both nodes' covering disks contain the other, *i.e.*, the corresponding two nodes u and v are within the transmission ranges of each other. A subset U of nodes is a dominating set if each node is either in U or adjacent to a node in U. The dominating set problem has been studied extensively in disk intersection graph model [8]. In a disk intersection graph, there is an edge between two nodes if and only if their corresponding communication disks intersect with each other. Clearly, disk containment graph is different from disk intersection graph and the techniques for dominating set selection in disk intersection graph cannot be applied here. Additionally, most of the existing works focused on uniform communication ranges while in practice the nodes may have disparate communication ranges.

Let us introduce a new concept called *restricted dominating set*. Given a disk containment graph $G = (V, E)$, a subset $U \subset V$ is a restricted dominating set if every node $v \in V$ is either $v \in U$ or $\exists u \in U$ with $r_u \geq r_v$ such that $uv \in E$.

The following theorem reduces the minimum dominating set in disk containment graph to its restricted version.

Theorem 6: Suppose that there exists a polynomial algorithm for selecting the restricted dominating set of minimum size, then there is polynomial 6-approximation algorithm for selecting the minimum dominating set in a disk containment graph.

Proof: Consider a disk containment graph $G = (V, E)$, assume U^* is the optimum solution for minimum dominating set, we will construct a restricted dominating set U_R based on U^* and satisfying: $|U_R| \leq 6 \cdot |U^*|$. Then it is easy to verify that the optimum solution U_R^* for minimum restricted dominating set is 6-approximation for the minimum dominating set problem since $|U_R^*| \leq |U_R| \leq 6 \cdot |U^*|$.

We next construct U_R based on U^*. First we define a one-to-one mapping between nodes and disks:

Definition 3: For each node $v \in V$, we define its corresponding disk D_v as the disk centering at v and with r_v as the radius.

Observe that any node lies inside at least one disk in $\cup_{u \in U^*} D_u$ since U^* is a feasible dominating set. Let $U_R = U^*$ initially. Thus, if we add disks to U_R to ensure that: for each disk D_u : $u \in U^*$, all nodes inside D_u satisfy the restricted property, then the resulted solution U_R is a restricted dominating set. We check the disk one by one. For each disk $D_u : u \in U^*$, we will add minimal nodes to U_R such that all nodes inside D_u satisfy the restricted property. Note that the nodes inside D_u with communication radius no larger than r_u (including u) already satisfy the restricted property.

Thus, we only need to focus on the nodes inside disk D_u with communication radius larger than r_u, let $V(u)$ be the set of such nodes. We begin with $U_R(u) = V(u)$ and then delete nodes from $U_R(u)$ as long as each nodes in $V(u)$ satisfy the restricted property with respect to $U_R(u)$ after the deletion. Assume the output is $U_R(u)$ when our deletion process cannot proceed any more. We prove that for each node $u \in U^*$, $|U_R(u)| \leq 5$. Otherwise, there exist two nodes $v, w \in U_R(u)$ such that $\angle vuw \leq 60^\circ \Rightarrow \|vw\| \leq \max\{uv, uw\} \leq r_u \leq \min\{r_v, r_w\}$. Thus, we have $vw \in E$ and we can delete one node in $\{v, w\}$ with smaller communication range from $U_R(u)$. This contradicts that $U_R(u)$ is the output after our deletion process can not proceed any more.

Finally, for each node $u \in U^*$, we add $U_R(u)$ to U_R. Clearly, the resulted U_R is a feasible solution for restricted minimum dominating set, and $U_R = \left(\bigcup_{u \in U^*} R_u\right) \cup U^*$, thus $|U_R| \leq |U^*| + \sum_{u \in U^*} |R_u| \leq |U^*| + \sum_{u \in U^*} 5 \leq 6|U^*|$. ■

Thus, if we could find a constant approximation algorithm for selecting the minimum restricted dominating set, we would achieve a constant approximation algorithm for the original problem, *i.e.*, the minimum dominating set problem in disk containment graph.

V. Literature Review

In order to evaluate the quality of coverage of the wireless network, Meguerdichian *et.al* [16] formulated the coverage problem under two extreme cases: the best case coverage problem and the worst case coverage problem. They proposed centralized optimal algorithms for both problems. Mehta *et.al* [17] improved these algorithms and made them more computationally efficient.

There were some work which focused on the distributed version of coverage problem formulated in [16]. Li *et.al* [26] showed that the best case coverage path can be constructed using edges that belong to the relative neighborhood graph of the wireless node set. Meguerdichian *et.al* [16] implied that a variant of the localized exposure algorithm presented in [21] can be used to solve the worst case coverage problem locally. Another localized algorithm with more practical assumptions was proposed by Huang *et.al* [13]. Huang *et al.* [13] studied the problem of determining if the area is sufficiently K-covered, for the general coverage problem. In [4], Huang *et al.* further extended this problem to three-dimensional wireless networks and proposed a solution The connected K-coverage problem was addressed in [29] in which Zhou *et.al* studied the problem of selecting a minimum set of wireless nodes which are connected and each point in a target area is covered by at least K distinct wireless nodes. Xing *et.al* [10] explored the problem concerning energy conservation while maintaining both desired coverage degree and connectivity. Some studies focused on the relationship of the coverage multiplicity, the number of wireless nodes, and the sensing radius. Kumar *et.al* [22] considered the problem of determining the appropriate number of wireless nodes that are enough to provide K-coverage of an area when covering nodes are allowed to sleep during most of their lifetime. In [23], Wan *et. al* analyzed the probability of the K-coverage when the sensing radius or the number of wireless nodes changes while taking the boundary effect into account. Recently, Yun *et al.* [28] studied deployment patterns to achieve full coverage and K-connectivity with different ratios of the communication range to the sensing range of wireless nodes for homogeneous wireless networks.

Coverage problem can be reduced to disk cover problem in most cases. Disk cover is a classical geometric *set cover* problem. It is NP-hard [15] even for unit disks, while there exist some constant-approximation algorithms for it, in contrast to the fact that the general set cover problem is not approximable within $O(\log n)$, where n is the size of the input instance [20]. For the problem *continuous disk cover* with minimum cardinality, where the disk center locations may be chosen at any point in the plane, it admits a PTAS by using a grid-shifting strategy [11], [12]. For the problem *min-cost disk cover*, [9], [30] proposed $(4+\epsilon)$-approximation algorithm for unit disk graph (UDG), which is the best result so far. Note that, a sequence of results for *min-weight dominating set* in UDG [1], [7], [14], imply algorithms for the problem disk cover with the same approximations.

An independent branch for the disk cover problem that receives great research interest is called *discrete unit disk cover*, and there have been a series of work done for it [3], [5], [6], [19]. Specifically, Brönnimann and Goodrich [3] presented a deterministic ϵ-net based algorithm where the constant factor is not specified. Calinescu *et al.* [5] gave a

102-approximation algorithm. Narayanappa and Vojtechovsky [19] improved the approximation ratio to 72. Carmi *et al.* [6] gave a 38-approximation algorithm by solving a subproblem where the points are located below a line and to be covered by a subset of disks of above the line.

VI. Conclusion

We presented several approximation algorithms for three mutually related coverage problems. Note that for the problem linear K-coverage, this is the first time in the literature that the geometric properties for K-coverage with was explored, and the approximation algorithm was obtained. A possible future research direction is whether we can achieve constant approximation for the general K-Coverage problem based on the geometric properties.

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References

[1] Ambuhl, C., Erlebach, T., Mihal ak, M., and Nunkesser, M. Constant-Factor Approximation for Minimum-Weight (Connected) Dominating Sets in Unit Disk Graphs. *Lecture Notes in Computer Science 4110* (2006), 3.

[2] Bansal, N., and Pruhs, K. Weighted geometric set multi-cover via quasi-uniform sampling. In *Algorithms–ESA 2012*. Springer, 2012, pp. 145–156.

[3] Br"onnimann, H., and Goodrich, M. Almost optimal set covers in finite VC-dimension. *Discrete and Computational Geometry 14*, 1 (1995), 463–479.

[4] C. Huang, Y. C. T., and Lo, L. The coverage problem in threedimensional wireless sensor networks,. In *Proceedings of IEEE GLOBECOM* (2004).

[5] Călinescu, G., Mandoiu, I., Wan, P., and Zelikovsky, A. Selecting forwarding neighbors in wireless ad hoc networks. *Mobile Networks and Applications 9*, 2 (2004), 101–111.

[6] Carmi, P., Katz, M., and Lev-Tov, N. Covering points by unit disks of fixed location. *Algorithms and Computation* (2007), 644–655.

[7] Dai, D., and Yu, C. A $5 + \epsilon$-approximation algorithm for minimum weighted dominating set in unit disk graph. *Theoretical Computer Science 410*, 8-10 (2009), 756–765.

[8] Du, D.-Z., and Wan, P.-J. Connected dominating set: Theory and applications.

[9] Erlebach, T., and Mihalák, M. A (4+ ε)-Approximation for the Minimum-Weight Dominating Set Problem in Unit Disk Graphs. *Approximation and Online Algorithms* (2010), 135–146.

[10] G. Xing, X. Wang, Y. Z. C. L. R. P., and Gill, C. Integrated coverage and connectivity configuration for energy conservation in sensor networks. *ACM Transactions on Sensor Networks, vol. 1, pp. 36-72* (2005).

[11] Gonzalez, T. Covering a set of points in multidimensional space* 1. *Information processing letters 40*, 4 (1991), 181–188.

[12] Hochbaum, D., and Maass, W. Approximation schemes for covering and packing problems in image processing and VLSI. *Journal of the ACM (JACM) 32*, 1 (1985), 130–136.

[13] Huang, C., and Tseng, Y. The coverage problem in a wireless sensor network. In *ACM International Workshop on Wireless Sensor Networks and Applications (WSNA)* (2003).

[14] Huang, Y., Gao, X., Zhang, Z., and Wu, W. A better constant-factor approximation for weighted dominating set in unit disk graph. *J. Comb. Optim 1382* (2008), 6905.

[15] Johnson, D. The NP-completeness column: an ongoing guide. *Journal of algorithms 13*, 3 (1992), 502–524.

[16] Meguerdichian, S., Potkonjak, F. K. M., and Srivastava, M. B. Coverage problems in wireless ad-hoc sensor networks. In *Proc. INFOCOM 139-150* (2001).

[17] Mehta, D.P. Lopez, M., and Lin, L. Optimal coverage paths in ad-hoc sensor networks. In *Proceedings of IEEE ICC 03, 507-511* (2003).

[18] Mustafa, N., and Ray, S. PTAS for geometric hitting set problems via local search. In *Proceedings of the 25th annual symposium on Computational geometry* (2009), ACM, pp. 17–22.

[19] Narayanappa, S., and Vojtechovskỳ, P. An improved approximation factor for the unit disk covering problem. In *Proc. Can. Conf. on Comp. Geom* (2006), Citeseer.

[20] Raz, R., and Safra, S. A sub-constant error-probability low-degree test, and a sub-constant error-probability PCP characterization of NP. In *Proceedings of the twenty-ninth annual ACM symposium on Theory of computing* (1997), ACM, pp. 475–484.

[21] Rourke, J. O. Computational geometry in c. *Cambridge University Press, New York* (1998).

[22] S. Kumar, T. L., and Barlogh, J. On k-coverage in a mostly sleeping sensor network. In *Proceedings of MobiCom pp. 144-158.* (2004).

[23] Wan, P., and Yi, C. Coverage by randomly deployed wireless sensor networks,. *IEEE Transactions on Information Theory, vol. 52, pp. 2658-2669* (2006).

[24] Wan, P.-J., Xu, X., and Wang, Z. Wireless coverage with disparate ranges. In *Proceedings of the Twelfth ACM International Symposium on Mobile Ad Hoc Networking and Computing* (2011), ACM, p. 11.

[25] Wang, L., Wan, P.-J., and Yao, F. Minimum cds in multihop wireless networks with disparate communication ranges. *IEEE Transactions on Mobile Computing 12*, 5 (2013), 909–916.

[26] X. Li, P. W., and Frieder, O. Coverage in wireless ad-hoc sensor networks. *IEEE Transaction on Computers, vol. 52, no. 6, pp. 753-763* (Jun. 2003).

[27] Xu, X., Tang, S., Mao, X., and Li, X.-Y. Distributed gateway placement for cost minimization in wireless mesh networks. In *Distributed Computing Systems (ICDCS), 2010 IEEE 30th International Conference on* (2010), IEEE, pp. 507–515.

[28] Yun, Z., Bai, X., Xuan, D., Lai, T., and Jia, W. Optimal Deployment Patterns for Full Coverage and k-Connectivity ($k \leq 6$) Wireless Sensor Networks. *IEEE/ACM TRANSACTIONS ON NETWORKING 18*, 3 (2010).

[29] Z. Zhou, S. D., and Gupta, H. Connected k-coverage problem in sensor networks. In *Proceedings of ICCCN* (2004).

[30] Zou, F., Wang, Y., Xu, X., Li, X., Du, H., Wan, P., and Wu, W. New approximations for minimum-weighted dominating sets and minimum-weighted connected dominating sets on unit disk graphs. *Theoretical Computer Science* (2009).