Stochastic Information Management for Voltage Regulation in Smart Distribution Systems

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***Abstract*—In this paper, we study distributed generation (DG) integration in smart grid, with a focus on the voltage regulation in smart distribution systems. To ensure the operation of a smart distribution system at an acceptable voltage level, voltage regulators are deployed at some strategic locations for voltage control. The two-way communication functionality of the smart distribution system is leveraged such that the voltage regulators are coordinated by a distribution substation. Based on the measurement reports from remote terminal units (RTUs) deployed at DG unit and load connection points, stochastic information management is performed by the distribution substation to address the randomness in renewable power generation and load demand. In this paper, we formulate a voltage regulation problem in the smart distribution system based on power flow analysis, while taking into account communication delays. We show that the problem can be represented as a partially observed Markov decision process (POMDP). Since voltage regulation is performed at a relatively low frequency to avoid excessive wear and tear on the voltage regulators, a large amount of measurements should be reported by the RTUs and processed by the distribution substation for optimal voltage regulation. In order to reduce the communication and computational overhead, we further investigate the voltage regulation problem and mathematically prove that a relatively small amount of information is sufficient for the distribution substation to make an optimal decision. The theoretical results are evaluated based on a case study of IEEE 13-bus test system with real DG power generation and demand data.**

I. Introduction

In the global trend of green economy, great concerns have been raised about the sustainability, cost-effectiveness, and environment-friendliness of electric power generation. To address these issues, penetration of renewable energy source based distribution generation (DG) has been increasing at a rapid rate over the last decade, thanks to government incentives, falling installation costs, and rising fossil-fuel prices. According to the International Energy Agency (IEA) forecast, electric power generation from renewable energy sources will nearly triple from 2010 to 2035, reaching 31% of the world's total power generation, with wind and solar generation providing 25% and 7.5%, respectively, of the total renewable power generation [1]. Various programs have been established world-wide to stimulate the development of renewable energy technology and the deployment of DG units. For instance, a feed-in tariff (FIT) program has been launched by Ontario Power Authority (OPA) in 2009, which provides standardized program prices and contracts for the deployment of DG units. Up to 2011, more than 2500 medium and large FIT projects have been approved to produce sufficient electricity to power 1.2 million homes [2]. By 2014, all coal-fired generation will be phased out in Ontario to improve air quality. Coming along with the potential economic and environmental benefits of DG are the power system operation stability issues due to the intermittency in DG output. The next generation power grid, also known as the smart grid, is expected to address these issues and adopt large-scale DG integration.

One of the most critical problems in power system operation with DG integration is voltage regulation. As renewable energy sources such as wind and solar are typically used to supply DG units and are intermittent in nature, the power output of DG units may vary from time to time. As a result, an unacceptably high (resp. low) voltage level may frequently occur at the DG unit and/or load connection points (i.e., buses) in a distribution system, which results in the overvoltage (resp. undervoltage) problem [3] [4]. For instance, over/under voltage which sustains for over 10 seconds is considered as long-duration over/under voltage in a distribution system and should be avoided [5]. In the current practice (e.g., according to the Ontario FIT program contract [2]), DG units are not required to contribute to grid voltage regulation. The power generation of DG units cannot be curtailed based on the contract to protect the owners' revenue. Therefore, the overvoltage and undervoltage problems are typically addressed in a conservative manner. The methodology is to cap the DG penetration level to ensure acceptable voltages at all buses of the distribution system under an extreme condition, such as maximum DG output and minimum demand.

In order to bring the DG penetration level to the theoretical limit, i.e., the thermal limit of distribution feeders (or lines), voltage regulators should be installed at strategic locations of the feeders for voltage regulation [3] [4]. In the future smart distribution system, the two-way communication functionality can be leveraged to facilitate voltage regulation. Remote terminal units (RTUs) are deployed at distribution system buses for measurement and reporting, based on which the distribution substation makes a control decision and notifies the voltage regulators to adjust system voltages. Wireless networks (e.g., ZigBee and WiFi networks) will be widely used in the future smart distribution systems because of the low deployment costs [6]. However, the communication delays become non-negligible in comparison with the wireline networks (e.g., fiber-optic networks) [7]. Without instant information on the power generation/demand on each of the distribution system buses, the randomness in DG output and load demand should be investigated under communication delays. Specifically, stochastic information management schemes should be developed to achieve optimal voltage regulation, while reducing the communication and computational overhead.

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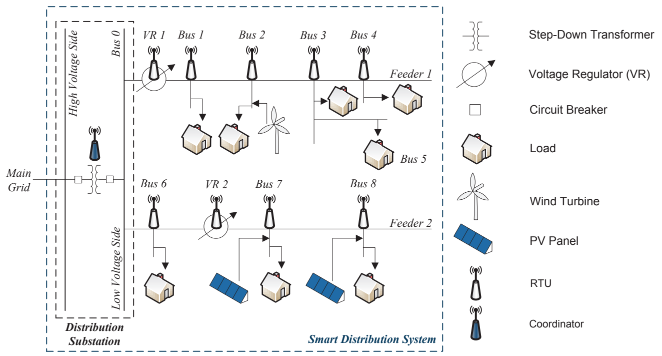


Fig. 1: One-line diagram of a smart distribution system.

In this paper, we formulate the optimal voltage regulation problem in smart distribution systems as a partially observed Markov decision process (POMDP) problem, which takes into account both power flow analysis and wireless communication delays. By investigating the problem based on stochastic analysis, we prove that a relatively small amount of information is sufficient for the distribution substation to make an optimal control decision, which significantly reduces the communication and computational overhead. A case study is carried out based on the IEEE 13-bus test system and real DG power generation and demand data. To the best of our knowledge, this is the first work in literature to formulate the optimal voltage regulation problem for DG integration in the smart grid as a POMDP problem while taking into account both power flow analysis and communication delays, and explicitly identify the sufficient information for optimal voltage regulation. The proposed research is expected not only to reduce the deployment/operation cost of communication networks and computational devices for utilities, but also to provide more stable services to electricity customers in terms of a closely regulated distribution system voltage which, in turn, stimulates the use of renewable energy sources.

II. Related Work

In literature, most research on voltage regulation is performed from a power system point of view. Estimation based voltage regulation schemes are proposed in early research works, in order to achieve the voltage regulation without pervasive distribution system monitoring [8] [9]. Recently, the RTU based voltage regulation schemes attract most attentions in research and development because of advances in communication technologies [3] [4]. In these research works, the communication delay is assumed to be negligible such that the distribution system buses can be monitored instantaneously and accurately. Power flow analysis is used to achieve voltage regulation in real distribution systems with DG integration. However, for future smart distribution systems with wireless networks, communication delays cannot be neglected. There is another stream of research which addresses the voltage regulation problem from a communication network point of view, where wireless links are selectively scheduled to control the voltage of each bus [10] [11]. The proposed schemes are more suitable for future smart microgrid applications where

TABLE I: Summary of important symbols.

Symbol	Definition
$\mathcal{B}$	The set of buses
$\mathbf{K}_n$	The set of tap settings of all voltage regulators in time slot n
$K_n^{(r)}$	The tap setting of voltage regulator r in time slot n
$\mathcal{M}$	The set of voltage regulation periods
$\mathcal{R}$	The set of voltage regulators
$\mathbf{S}_t$	The set of all power injection states in time slot t
$S_t^{(b)}$	The state of power injection of bus b in time slot t
T	The duration of voltage regulation period
$\mathbf{U}_n$	The policy adopted by the coordinator in time slot n
$\mathbf{V}_t$	The set of voltage of all buses in time slot t
$V_t^{(b)}$	The voltage of bus b in time slot t
$\mathbf{X}_n$	The sufficient information state in time slot t
$\mathbf{Y}_n$	The available information state in time slot t
τ_b	The delay of measurement report from bus b
$\xi^{(b)}_{s_{t-1}^{(b)},s_t^{(b)}}$	The transition probability with respect to the power injection of bus b from state $s_{t-1}^{(b)}$ to state $s_t^{(b)}$

the voltages of the DG units can be controlled by the utilities. However, for DG integration in a smart distribution system where the generation curtailment is not possible, how to address the voltage regulation via voltage regulators is an open issue. Moreover, how to integrate power flow analysis into the voltage regulation problem is critical for practical applications and needs investigation.

III. System Model

In this section, we present the system model of a smart distribution system, for which the one-line diagram is shown in Fig. 1. As many symbols are used in this paper, Table I summarizes the important ones. Electric power is drawn from the main grid through a distribution substation. The step-down transformer in the distribution substation reduces the voltage from a transmission level (above 110 kV) to a distribution level (below 50 kV). Several feeders are connected to the low voltage side of the step-down transformer such that electric power can be delivered to customers (i.e., the electric loads). DG units such as wind turbines and photovoltaic (PV) panels are integrated at some points along the feeders. Consider a smart distribution system with $B+1$ buses and R voltage regulators. Denote the set of buses and voltage regulators as $\mathcal{B} = \{0, 1, 2, \cdots, B\}$ and $\mathcal{R} = \{1, 2, \cdots, R\}$, respectively, where bus 0 corresponds to the low voltage side of the step-down transformer while each of the other buses corresponds to a DG unit and/or load connection point. Let $\mathrm{R}_{i,j} + \mathrm{j}\mathrm{X}_{i,j}$ be the impedance of the feeder between two adjacent buses i and j. We define a mutual ordering among the buses with respect to the hop counts to the distribution substation. For each bus b, we denote the set of all downstream buses of bus b as $\mathbf{D}_b$, where the buses in $\mathbf{D}_b$ are connected along the same feeder (including the branches of the feeder) as bus b but have a higher hop count. For instance, in Fig. 1, we have $\mathbf{D}_2 = \{3, 4, 5\}$, $\mathbf{D}_3 = \{4, 5\}$, and $\mathbf{D}_7 = \{8\}$. Each voltage regulator is installed between a set of two buses. Denote the upstream and downstream buses of voltage regulator r ($r \in \mathcal{R}$) as b'_r and b''_r ($b'_r, b''_r \in \mathcal{B}$), respectively. For instance, we have $b'_2 = 6$

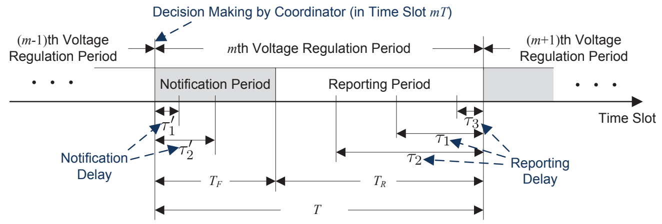


Fig. 2: An illustration of voltage regulation.

and $b''_2 = 7$ for voltage regulator 2 in Fig. 1. A coordinator (including both communication and computational devices) is deployed at the control room of distribution substation to coordinate the operations of all voltage regulators [7].

Consider a time slotted model for the smart distribution system, where time is partitioned into constant slots, each with a duration τ [12] [13]. The power generation and demand of each bus in each time slot are assumed to be a constant. Time slot 0 corresponds to the initial time slot of the system. As shown in Fig. 2, the voltage regulation is performed in a periodic manner, where the duration of each period is T time slots[1]. Consider $M+1$ voltage regulation periods in the set $\mathcal{M} = \{0, 1, 2, \cdots, M\}$, where the mth voltage regulation period begins with time slot mT. The statistics of the power generation and demand do not change significantly within the $M+1$ voltage regulation periods. A typical duration of each $M+1$ periods may vary from minutes to hours, depending on the weather condition and/or constraints on computational complexity. The voltage regulation decision is made by the coordinator at the beginning of each voltage regulation period (e.g., in time slot mT for the mth voltage regulation period). In the following, we introduce a model for each component in the smart distribution system. For notational clarity, we use upper and lower case letters to represent random variables and their corresponding realizations, respectively.

A. Power Generation and Demand

In the time slotted system, both power generation and demand processes can be modeled as Markov chains [12]–[16]. Instead of modeling them separately, we consider the power injection of each bus, which equals the amount of power generation minus demand [17]. Denote the state of power injection of bus b ($b \in \mathcal{B}$) in time slot t by $S_t^{(b)}$ ($S_t^{(b)} \in \mathcal{S}_b$), where $S_0^{(b)}$ is the initial state of bus b, and $\mathcal{S}_b$ represents a set of all power injection states. Without loss of generality, we consider a linear mapping between states and power injection. Given state $S_t^{(b)}$, the active and reactive power injection[2] by bus b in time slot t are, respectively, given by

$$P(S_t^{(b)}) = \alpha S_t^{(b)}, \; Q(S_t^{(b)}) = \beta S_t^{(b)} \tag{1}$$

where α and β are constants. The power injection states of all buses are periodically reported to the coordinator via RTUs to facilitate voltage regulation decision making.

Denote the set of all power injection states as $\mathbf{S}_t = \{S_t^{(b)} | b \in \mathcal{B}\}$. The evolutions of different states in $\mathbf{S}_t$ are assumed to be independent [12] [13]. Note that if the DG units and/or loads are very densely deployed, correlation may exist in the evolution of power injection states of some buses. However, an extension to model such correlation is straightforward by defining a joint state for each group of the correlated buses. For instance, a Markov chain based wind generation model with spatial correlation is developed in [16]. Given both power generation and demand Markov chains, it is straightforward to show that the evolution of $S_t^{(b)}$ over t follows the Markov property. Define the state transition probability with respect to the power injection of bus b as $\xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t} = \mathrm{Prob}(S_t^{(b)} = s_t^{(b)} | S_{t-1}^{(b)} = s_{t-1}^{(b)})$, where $s_{t-1}^{(b)}$ and $s_t^{(b)}$ are the realizations of $S_{t-1}^{(b)}$ and $S_t^{(b)}$, respectively. We use $\xi^{(b)}_{s^{(b)}_{-1}, s^{(b)}_0} = \mathrm{Prob}(S_0^{(b)} = s_0^{(b)})$ to denote the probability distributions of the initial power injection state.

B. Wireless Networks

RTUs are deployed at each of the buses as well as each of the voltage regulators in the smart distribution system. The RTUs of buses are in charge of reporting measurements (in terms of power injection states) to the coordinator. On the other hand, the RTUs of voltage regulators are used to receive control signals from the coordinator. A communication delay[3] of τ_b ($b \in \mathcal{B}$) time slots is associated with the measurement report of bus b. The delay is considered to be a worst-case delay to ensure data delivery. An effective retransmission scheme is assumed here, such that data packet losses in measurement reports are negligible. In other words, we account for data packet retransmissions as an increase in communication delay. The measurement report used by the coordinator in time slot t was generated by bus b in time slot $t - \tau_b$. Similarly, we denote the communication delay for the coordinator to notify voltage regulator r ($r \in \mathcal{R}$) by τ'_r.

C. Voltage Regulation

Denote the voltage (per unit) of all buses in time slot t by set $\mathbf{V}_t = \{V_t^{(b)} | b \in \mathcal{B}\}$, where $V_t^{(b)}$ represents the voltage of bus b. Without loss of generality, consider $V_t^{(0)} = 1$ (per unit) for normal operation of the main grid. Given certain line impedance, the voltage of each bus can be determined based on the active/reactive power injection and tap settings[4] of voltage regulators, via power flow analysis. For a typical distribution

[1]The choice of T is based on practical considerations. For a large T, the voltage regulation is performed at a relatively low frequency to avoid wear and tear of voltage regulators [4], at the cost of loosely regulated bus voltages, and vice versa.

[2]Typically, DG units are configured to operate at a unit power factor (i.e., without reactive power injection). Reactive power injection in a distribution system is mainly achieved by installing shunt capacitors [3] [4].

[3]The communication delay consists of several components such as channel access time, data transmission time, data buffering time, and data processing time. For a multi-hop network based on low-cost short-range communication devices such as ZigBee and WiFi devices, the end-to-end communication delay is a summation of the delays over all hops. The channel contention with the existing networks such as wireless local area networks operating on the same frequency band may further increase the communication delay.

[4]A tap is a connection point along a transformer winding which allows a change in the voltage boosting/bucking ratio.

system, the relation between the voltages of two adjacent buses i and j connected by a feeder (bus j is a downstream bus of bus i) is given by [3]

$$V_t^{(j)} = V_t^{(i)} - (FP_t^{(i,j)} - P(S_t^{(j)}))\mathrm{R}_{i,j} - (FQ_t^{(i,j)} - Q(S_t^{(j)}))\mathrm{X}_{i,j} \quad (2)$$

where $FP_t^{(i,j)}$ and $FQ_t^{(i,j)}$ are the active and reactive power flows, respectively, from bus i to bus j with respect to the power injection of all downstream buses of bus j, given by

$$FP_t^{(i,j)} = -\sum_{b \in \mathbf{D}_j} P(S_t^{(b)}),\ FQ_t^{(i,j)} = -\sum_{b \in \mathbf{D}_j} Q(S_t^{(b)}). \quad (3)$$

From (2), if the active and/or reactive power injection of bus j is large, $(FP_t^{(i,j)} - P(S_t^{(j)}))$ and/or $(FQ_t^{(i,j)} - Q(S_t^{(j)}))$ may become negative such that $V_t^{(j)}$ exceeds $V_t^{(i)}$, which results in the overvoltage problem [3]. Similarly, the undervoltage problem can be defined if the power injection is small.

In the mth voltage regulation period, the tap settings of the voltage regulators are determined by the coordinator in time slot mT, as shown in Fig. 2. Before each decision making of the coordinator, there is a reporting period with duration T_R for the RTUs to send their measurements to the coordinator. Then, the control decision is made by the coordinator and is sent to the voltage regulators in a notification period with duration T_F. Assuming $\max_{b \in \mathcal{B}} \tau_b \leq T_R$, $\max_{r \in \mathcal{R}} \tau'_r \leq T_F$, and $T_R + T_F = T$, the notification of control actions can be finished before the next round of measurement reports to ensure a stable system operation. An example is shown in Fig. 2 for a smart distribution system with four buses and two voltage regulators. The reporting delay from bus 0 to coordinator is considered to be negligible (i.e., $\tau_0 = 0$) since both of the elements are deployed within the distribution substation.

Denote the control decision made by the coordinator in time slot n ($n = mT, m \in \mathcal{M}$) by $\mathbf{K}_n = \{K_n^{(r)} | r \in \mathcal{R}\}$, where $K_n^{(r)} \in \mathcal{K}_r$ denotes the tap setting of voltage regulator r, and $\mathcal{K}_r$ is the set of all possible tap settings. For instance, the widely used McGraw-Edison single-phase voltage regulator is able to control the voltage of a feeder from 10% boost to 10% buck via 32 taps of approximately 0.625% per tap [18] [19]. In this case, we have $\mathcal{K}_r = \{-0.1, -0.09375, -0.0875, \cdots, 0.09375, 0.1\}$. Since the voltage regulation decisions are made by the coordinator periodically, we have a constant tap setting within each voltage regulator period, i.e., $\mathbf{K}_n = \mathbf{K}_{n+1} = \cdots = \mathbf{K}_{n+T-1}$. Since all previous actions are known by the coordinator itself, we can define the available information state for coordinator decision making as follows.

Definition 1 (Available Information State). *In time slot n, the information state available to the coordinator for decision making is given by*

$$\mathbf{Y}_n = \{S_i^{(b)}, K_j^{(r)} | b \in \mathcal{B}, i \in \{0, 1, 2, \cdots, n - \tau_b\}, r \in \mathcal{R}, j \in \{0, 1, 2, \cdots, n-1\}\}. \quad (4)$$

Because of the notification delay, a decision made by the coordinator is applied to voltage regulator r after τ'_r time slots. Therefore, the relation between the voltages of the upstream and downstream buses of voltage regulator r is given by $V_t^{(b''_r)} = V_t^{(b'_r)}(1 + K_{t-\tau'_r}^{(r)})$ [18]. Since the per unit bus voltages are close to one in typical distribution systems [3], we can approximate $V_t^{(b''_r)}$ by:

$$V_t^{(b''_r)} = V_t^{(b'_r)} + K_{t-\tau'_r}^{(r)}. \quad (5)$$

Note that the relation between the voltages of upstream and downstream buses is fully determined by the tap setting of a voltage regulator which is indeed a transformer [18].

IV. Problem Formulation

Stochastic information management is performed by the coordinator for optimal voltage regulation, based on the measurement reports from RTUs and the stochastic models of power generation and demand. In this section, we formulate the optimal voltage regulation problem based on power flow analysis while taking into account the wireless communication delay. Let $\mathbf{v}_t = \{v_t^{(b)} | b \in \mathcal{B}\}$ denote the realization of bus voltages $\mathbf{V}_t$. With $T_F < T$, the notification of control actions can be completed within one voltage regulation period. Based on the power flow analysis in Subsection III-C, the value of $\mathbf{v}_t$ in time slot t ($t \in [n, n+T-1]$ with $n = mT$) is fully determined by the current power injection state $\mathbf{s}_t = \{s_t^{(b)} | b \in \mathcal{B}\}$ and the control actions in both current and previous voltage regulation periods, i.e., $\mathbf{k}_n = \{k_n^{(r)} | r \in \mathcal{R}\}$ and $\mathbf{k}_{n-1} = \{k_{n-1}^{(r)} | r \in \mathcal{R}\}$, respectively, which are the realizations of $\mathbf{K}_n$ and $\mathbf{K}_{n-1}$.

The objective is to minimize the voltage deviation with respect to the voltage rating at an acceptable level of wear and tear of the voltage regulators. Since the voltage deviation in time slot t is fully determined by $\mathbf{v}_t$, we can define the related cost function as $c : \mathbb{R}^{B+1} \to \mathbb{R}$, i.e., the cost associated with voltage deviation in time slot t is given by $c(\mathbf{v}_t)$. On the other hand, the wear and tear of voltage regulators are determined by the tap changing, which is the difference between current and previous tap settings [20]. We define the related cost function as $w : \prod_{r \in \mathcal{R}} \mathcal{K}_r \times \prod_{r \in \mathcal{R}} \mathcal{K}_r \to \mathbb{R}$, i.e., the cost associated with the wear and tear of voltage regulators in time slots $[n, n+T-1]$ is given by $w(\mathbf{k}_n, \mathbf{k}_{n-1})$. Since the control actions are determined by the coordinator periodically and the power injection state evolution is independent of the control actions, we can calculate the aggregated cost in time slots $[n, n+T-1]$ (which correspond to an entire voltage regulation period begins with time slot n) as

$$C(\mathbf{s}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) = \sum_{t=n}^{n+T-1} E[c(\mathbf{V}_t) | \mathbf{S}_n = \mathbf{s}_n] + w(\mathbf{k}_n, \mathbf{k}_{n-1}) \quad (6)$$

where the expectation is taken with respect to $\mathbf{V}_t$ because of the randomness in power injection state evolution.

We consider a deterministic policy which can be easily applied in smart distribution systems. Denote $\mathbf{U}_n$ as a policy adopted by the coordinator which maps the available information state ($\mathbf{Y}_n$) to a certain control action (i.e., tap settings) of the voltage regulators ($\mathbf{K}_n$). In other words, we have $\mathbf{K}_n = \mathbf{U}_n(\mathbf{Y}_n)$. Then, the optimal voltage regulation problem is formulated as

$$\textbf{(P1)} \quad \min_{\substack{\mathbf{U}_n \in \mathscr{U}, n=mT \\ m \in \mathcal{M}}} \sum_{\substack{n=mT \\ m \in \mathcal{M}}} E\left[C\left(\mathbf{S}_n, \mathbf{K}_n, \mathbf{K}_{n-1}\right)\right] \tag{7}$$

where the expectation is taken with respect to the randomness in $\mathbf{S}_n$, $\mathbf{K}_n$, and $\mathbf{K}_{n-1}$ to capture the power injection state evolution at each bus and the corresponding decision made by the coordinator. In (7), $\mathscr{U}$ represents the set of all admissible policies with respect to the capabilities of all voltage regulators, i.e., $\mathcal{K}_r$ ($r \in \mathcal{R}$).

Problem P1 is a special case of a partially observed Markov decision process (POMDP) problem where the coordinator is able to receive measurement reports from all buses, while the reports and control actions are delayed because of the wireless communication delay.

V. Sufficient Information State for Optimal Voltage Regulation

Two kinds of standard approaches can be used to address the POMDP problem P1, i.e., the information state approach and belief state approach [21]. The former generates a policy for each time slot which depends explicitly on the complete history, i.e., the available information state according to Definition 1 which grows with time without bound. The latter generates a policy for each time slot by utilizing the distribution of the states conditioned on all available information, then solving a dynamic program on a finite space of belief states. Both approaches require the measurement reports (from the RTUs) of all observations of the available information state, which results in significant communication and computational overhead. On the other hand, some recent advances in control theory show that generating an optimal policy for a POMDP problem may only require a relatively small amount of information, given the underlying system state evolution following a Markov process [22]. However, the general results for POMDP problems cannot be applied in this research since the voltage regulation is performed at a relatively low frequency (i.e., once every T time slots) to avoid wear and tear of voltage regulators, in contrast to the continuous control in each time slot. Moreover, the power injection state evolution is independent of the control actions of voltage regulators since generation curtailment is not an option, which may lead to a reduction in the size of sufficient information state, to be discussed in the following.

A. Definition of Sufficient Information State

In time slot n ($n = mT,\ m \in \mathcal{M}$), the sufficient information state can be considered as a subset of the available information state ($\mathbf{Y}_n$), which is sufficient for the coordinator to make an optimal decision on voltage regulation. Let $f_n(\cdot)$ ($n = mT,\ m \in \mathcal{M}$) be a sequence of functions, which maps the available information state to another (possibly smaller) set of information state. Then, the sufficient information state can be formally defined as follows.

Definition 2 (Sufficient Information State)**.** *Let $\mathbf{X}_n = f_n(\mathbf{Y}_n)$. Consider $\mathbf{X}_n$ as system state and define a new Markov decision process (MDP) with state transition probability $\text{Prob}\left(\mathbf{X}_{n+1} = \mathbf{x}_{n+1} | \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right)$, action space $\prod_{r \in \mathcal{R}} \mathcal{K}_r$, and cost function $\hat{C}(\mathbf{x}_n, \mathbf{k}_n, \mathbf{k}_{n-1})$ given by*

$$\hat{C}(\mathbf{x}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) = E\left[C(\mathbf{S}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) | \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right] \tag{8}$$

where the expectation is taken with respect to the randomness in $\mathbf{S}_n$, while $\mathbf{x}_n = f_n(\mathbf{y}_n)$ and $\mathbf{y}_n$ is a realization of $\mathbf{Y}_n$, given by

$$\begin{aligned} \mathbf{y}_n = \{ & s_i^{(b)}, k_j^{(r)} | b \in \mathcal{B}, i \in \{0, 1, 2, \cdots, n - \tau_b\}, \\ & r \in \mathcal{R}, j \in \{0, 1, 2, \cdots, n-1\}\}. \end{aligned} \tag{9}$$

Then, the MDP is called a sufficient information state MDP and $\mathbf{X}_n$ is called a sufficient information state, if the following conditions hold:

1) $\mathbf{X}_n$ satisfies the Markov property[5], i.e.,

$$\begin{aligned} & \text{Prob}\left(\mathbf{X}_{n+1} = \mathbf{x}_{n+1} | \mathbf{X}_{0:n} = \mathbf{x}_{0:n}, \mathbf{K}_{0:n} = \mathbf{k}_{0:n}\right) \\ & = \text{Prob}\left(\mathbf{X}_{n+1} = \mathbf{x}_{n+1} | \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right). \end{aligned} \tag{10}$$

Moreover, the above probability is independent of any specific POMDP policy $\mathbf{U}_n$ ($n = mT,\ m \in \mathcal{M}$);

2) The cost function $\hat{C}(\mathbf{x}_n, \mathbf{k}_n, \mathbf{k}_{n-1})$ is independent of any specific POMDP policy $\mathbf{U}_n$ ($n = mT,\ m \in \mathcal{M}$);

3) For all $n \geq 0$, we have

$$\begin{aligned} & \text{Prob}(\mathbf{S}_n = \mathbf{s}_n | \mathbf{X}_{0:n} = \mathbf{x}_{0:n}, \mathbf{K}_{0:n-1} = \mathbf{k}_{0:n-1}) \\ & = \text{Prob}(\mathbf{S}_n = \mathbf{s}_n | \mathbf{Y}_{0:n} = \mathbf{y}_{0:n}, \mathbf{K}_{0:n-1} = \mathbf{k}_{0:n-1}). \end{aligned} \tag{11}$$

Then, the following theorem holds with respect to the optimality of the sufficient information state MDP, which can be proved based on standard dynamic programming [23].

Theorem 1. *Consider POMDP problem P1 and a sufficient information state MDP based on Definition 2, we have*

$$\begin{aligned} & \min_{\substack{\mathbf{U}_n \in \mathscr{U}, n=mT \\ m \in \mathcal{M}}} \sum_{\substack{n=mT \\ m \in \mathcal{M}}} E[C(\mathbf{S}_n, \mathbf{K}_n, \mathbf{K}_{n-1})] \\ & = \min_{\substack{\hat{\mathbf{U}}_n \in \hat{\mathscr{U}}, n=mT \\ m \in \mathcal{M}}} \sum_{\substack{n=mT \\ m \in \mathcal{M}}} E[\hat{C}(\mathbf{X}_n, \mathbf{K}_n, \mathbf{K}_{n-1})] \end{aligned} \tag{12}$$

where $\hat{\mathbf{U}}_n$ is a sufficient information state MDP policy which maps the sufficient information state ($\mathbf{X}_n$) to a certain control action ($\mathbf{K}_n$) of the voltage regulators, i.e., $\mathbf{K}_n = \hat{\mathbf{U}}_n(\mathbf{X}_n)$, while $\hat{\mathscr{U}}$ represents the set of all admissible policies.

B. Sufficient Information State for Optimal Voltage Regulation

Based on Theorem 1, in order to find out the sufficient information state for optimal voltage regulation, we have to check each of the conditions in Definition 2 with respect to a certain $\mathbf{X}_n$. Moreover, as the voltage regulation decisions are made by the coordinator in a periodic manner, we only need

[5]In this paper, we use notation $\mathbf{A}_{i:j}$ to denote all elements in the set $\{\mathbf{A}_i, \mathbf{A}_{i+1}, \cdots, \mathbf{A}_j\}$. If $i > j$, we have an empty set $\mathbf{A}_{i:j}$.

to investigate the sufficient information state for the decision making time slot n ($n = mT, m \in \mathcal{M}$) in the following analysis. We first define $\mathbf{X}_n$ as follows

$$\mathbf{X}_n = \{S^{(b)}_{n-\tau_{\max}:n-\tau_b}, \mathbf{K}_{n-1} | b \in \mathcal{B}\} \tag{13}$$

where $\tau_{\max} = \max_{i \in \mathcal{B}} \tau_i$ is the maximum delay of voltage reports. For instance, we have $\tau_{\max} = \tau_2$ in Fig. 2. Denote the realization of $\mathbf{X}_n$ by $\mathbf{x}_n = \{s^{(b)}_{n-\tau_{\max}:n-\tau_b}, \mathbf{k}_{n-1} | b \in \mathcal{B}\}$. Then, the following lemma holds.

Lemma 1. *The information state $\mathbf{X}_n$ defined in (13) satisfies the Markov property, and the probability in (10) is independent of any specific POMDP policy $\mathbf{U}_n$ ($n = mT,\ m \in \mathcal{M}$).*

Proof: By definition of conditional probability, we have

$$\begin{aligned}&\mathrm{Prob}\left(\mathbf{X}_{n+1} = \mathbf{x}_{n+1} | \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right)\\ &= \frac{\mathrm{Prob}\left(\mathbf{X}_{n+1} = \mathbf{x}_{n+1}, \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right)}{\mathrm{Prob}\left(\mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right)}.\end{aligned} \tag{14}$$

For notation simplicity, we denote $\sum_1 = \sum_{s^{(b)}_{0:n-\tau_{\max}-1}, b \in \mathcal{B}}$ as a summation over all possible values of the variables in the set $\{s^{(b)}_0, s^{(b)}_1, \cdots, s^{(b)}_{n-\tau_{\max}-1} | b \in \mathcal{B}\}$, where each $s^{(b)}_t$ takes value from $\mathcal{S}_b$ ($b \in \mathcal{B}$). Similarly, we define $\sum_2 = \sum_{\mathbf{k}_{0:n-2}}$ and $\sum_3 = \sum_{s^{(b)}_{n-\tau_b+1:n}, b \in \mathcal{B}}$. Based on the marginal distributions of all variables not in the summations $\sum_1$, $\sum_2$, and $\sum_3$, the denominator of (14) is given by

$$\begin{aligned}&\mathrm{Prob}\left(\mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right) = \mathrm{Prob}(\mathbf{K}_{n-1:n} = \mathbf{k}_{n-1:n},\\ &\qquad S^{(b)}_{n-\tau_{\max}:n-\tau_b} = s^{(b)}_{n-\tau_{\max}:n-\tau_b} | b \in \mathcal{B})\\ &= \sum\nolimits_1 \sum\nolimits_2 \sum\nolimits_3 \mathrm{Prob}(\mathbf{K}_{0:n} = \mathbf{k}_{0:n}, S^{(b)}_{0:n} = s^{(b)}_{0:n} | b \in \mathcal{B})\\ &= \sum\nolimits_1 \sum\nolimits_2 \sum\nolimits_3 [\mathrm{Prob}(\mathbf{K}_{0:n} = \mathbf{k}_{0:n} | S^{(b)}_{0:n} = s^{(b)}_{0:n}, b \in \mathcal{B})\\ &\qquad \cdot \mathrm{Prob}(S^{(b)}_{0:n} = s^{(b)}_{0:n} | b \in \mathcal{B})].\end{aligned} \tag{15}$$

Based on the initial distribution and transition probabilities with respect to the power injection state, we have

$$\mathrm{Prob}(S^{(b)}_{0:n} = s^{(b)}_{0:n} | b \in \mathcal{B}) = \prod_{t=0}^{n} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t}. \tag{16}$$

For a specific and deterministic POMDP policy $\mathbf{U}_n$ ($n = mT,\ m \in \mathcal{M}$), we have

$$\begin{aligned}&\mathrm{Prob}(\mathbf{K}_{0:n} = \mathbf{k}_{0:n} | S^{(b)}_{0:n} = s^{(b)}_{0:n}, b \in \mathcal{B})\\ &= \prod_{\substack{j=mT, m \in \mathcal{M}\\ j \le n}} I(\mathbf{k}_j = \mathbf{U}_j(\mathbf{y}_j))\end{aligned} \tag{17}$$

where $I(\mathrm{E})$ is an indication function which equals 1 if event E is true and 0 otherwise. Note that only the control actions in time slots $j = mT$ ($m \in \mathcal{M}$) are considered because of the periodic voltage regulation mechanism.

Next, we show that some of the factors in the two multiplication operations of (16) and the probability in (17) are independent of the variables in summation $\sum_3$ (i.e., the variables in the set $\{s^{(b)}_{n-\tau_b+1:n} | b \in \mathcal{B}\}$). If $\xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t}$ depends on at least one of the elements in $\{s^{(b)}_{n-\tau_b+1:n} | b \in \mathcal{B}\}$, we have $t-1 \ge n-\tau_b+1$ or $t \ge n-\tau_b+1$. Therefore, we can conclude that if $t \le n-\tau_{\max}$, $\xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t}$ ($b \in \mathcal{B}$) is independent of any variable in $\{s^{(b)}_{n-\tau_b+1:n} | b \in \mathcal{B}\}$. On the other hand, $\mathbf{y}_j$ ($j \le n$) is independent of the variables in $\{s^{(b)}_{n-\tau_b+1:n} | b \in \mathcal{B}\}$ by definition. Therefore, the probability in (17) is independent of any variable in $\{s^{(b)}_{n-\tau_b+1:n} | b \in \mathcal{B}\}$. Based on the arguments, we can simplify (15) to

$$\begin{aligned}&\mathrm{Prob}\left(\mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right)\\ &= \sum\nolimits_1 \sum\nolimits_2 \prod_{\substack{j=mT, m \in \mathcal{M}\\ j \le n}} I(\mathbf{k}_j = \mathbf{U}_j(\mathbf{y}_j))\\ &\quad \cdot \prod_{t=0}^{n-\tau_{\max}} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t} \cdot \sum\nolimits_3 \prod_{t=n-\tau_{\max}+1}^{n} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t}\\ &= \Theta \cdot \left[\sum\nolimits_3 \prod_{t=n-\tau_{\max}+1}^{n} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t}\right]\end{aligned} \tag{18}$$

where $\Theta = \sum_1 \sum_2 \prod_{\substack{j=mT, m \in \mathcal{M}\\ j \le n}} I(\mathbf{k}_j = \mathbf{U}_j(\mathbf{y}_j)) \cdot \prod_{t=0}^{n-\tau_{\max}} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t}$. The second equality in (18) holds since all variables in the set of square brackets (with the smallest index $t = n - \tau_{\max} + 1 - 1$ for $s^{(b)}_t$) are independent of the variables to be summed over in Θ (with the largest index $t = n - \tau_{\max} - 1$ for $s^{(b)}_t$). Similarly, we can simplify the numerator of (14) to

$$\begin{aligned}&\mathrm{Prob}\left(\mathbf{X}_{n+1} = \mathbf{x}_{n+1}, \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right)\\ &= \Theta \cdot \left[\sum\nolimits_4 \prod_{t=n-\tau_{\max}+1}^{n+1} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t}\right]\end{aligned} \tag{19}$$

where $\sum_4 = \sum_{s^{(b)}_{n-\tau_b+2:n+1}, b \in \mathcal{B}}$. Substitute (18) and (19) into (14) and removing the common factors, we have

$$\begin{aligned}&\mathrm{Prob}\left(\mathbf{X}_{n+1} = \mathbf{x}_{n+1} | \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right)\\ &= \frac{\sum_4 \prod_{t=n-\tau_{\max}+1}^{n+1} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t}}{\sum_3 \prod_{t=n-\tau_{\max}+1}^{n} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t}}.\end{aligned} \tag{20}$$

Using a similar method, we can verify that the left hand side of (10) equals the right hand side of (20). Moreover, the probability in (20) is independent of any specific POMDP policy $\mathbf{U}_n$ ($n = mT,\ m \in \mathcal{M}$) since the associated common factors are removed, which completes the proof. ■

For the cost function defined in (8), we have the following lemma:

Lemma 2. *The cost function $\hat{C}(\mathbf{x}_n, \mathbf{k}_n, \mathbf{k}_{n-1})$ is independent of any specific POMDP policy $\mathbf{U}_n$ ($n = mT,\ m \in \mathcal{M}$).*

Proof: By definition, we have

$$\begin{aligned}\hat{C}(\mathbf{x}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) = \sum_c c\mathrm{Prob}(C(\mathbf{S}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) = c |\\ \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n)\end{aligned} \tag{21}$$

where the summation is performed with respect to all possible values of costs. By definition, we have

$$\begin{aligned} &\text{Prob}\left(C(\mathbf{S}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) = c | \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right) \\ &= \frac{\text{Prob}\left(C(\mathbf{S}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) = c, \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right)}{\text{Prob}\left(\mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right)} \end{aligned} \quad (22)$$

where the denominator is identical to (18). Moreover, we can simplify the numerator to

$$\begin{aligned} &\text{Prob}\left(C(\mathbf{S}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) = c, \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n\right) = \Theta \\ &\cdot \left[\sum\nolimits_3 \prod_{t=n-\tau_{\max}+1}^{n} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t} I(C(\mathbf{s}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) = c) \right] \end{aligned} \quad (23)$$

where the equality holds based on similar arguments as in deriving (18) while $\mathbf{k}_n$ and $\mathbf{k}_{n-1}$ in $C(\mathbf{s}_n, \mathbf{k}_n, \mathbf{k}_{n-1})$ are independent of the variables in $\sum_2$. Substitute (18) and (23) into (22) and removing common factors, we have

$$\begin{aligned} &\text{Prob}(C(\mathbf{S}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) = c | \mathbf{X}_n = \mathbf{x}_n, \mathbf{K}_n = \mathbf{k}_n) \\ &= \frac{\sum_3 \prod_{t=n-\tau_{\max}+1}^{n} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t} I(C(\mathbf{s}_n, \mathbf{k}_n, \mathbf{k}_{n-1}) = c)}{\sum_3 \prod_{t=n-\tau_{\max}+1}^{n} \prod_{b \in \mathcal{B}} \xi^{(b)}_{s^{(b)}_{t-1}, s^{(b)}_t}}. \end{aligned} \quad (24)$$

We can see that, the right hand side of (24) is independent of any specific POMDP policy $\mathbf{U}_n$ ($n = mT,\ m \in \mathcal{M}$), which completes the proof. ■

Then, the main result of sufficient information state for optimal voltage regulation is given by the following theorem.

Theorem 2. *In time slot n ($n = mT, m \in \mathcal{M}$), the sufficient information for the coordinator to make an optimal decision is given by (13).*

Proof: Based on Lemma 1 and Lemma 2, conditions 1) and 2) of sufficient information state in Definition 2 hold for $\mathbf{X}_n$ in (13). By definition, we can show that condition 3) also holds with respect to $\mathbf{X}_n$. Therefore, $\mathbf{X}_n$ is a sufficient information state for POMDP problem P1. ■

Note that the delay in a notification period does not affect the sufficient information state according to Theorem 2. The main reason is that the control action of voltage regulators does not affect the power injection state evolution. Moreover, the size of sufficient information state increases as the communication delay increases because of the increment in $\tau_{\max}$. Intuitively, since bus voltages are mutually correlated based on power flow analysis, the historic states of the buses with low communication delays can reveal the voltages of the buses with high communication delays, for which the most recent states cannot be reported to the coordinator on time.

VI. A Case Study

In this section, we present a case study to evaluate how the sufficient information state affects the performance of voltage regulation in a smart distribution system with DG integration. The case study is based on IEEE 13-bus test system and real DG power generation and demand data. A linear approximation is used for the power injection state evolution to simplify the calculation of optimal voltage regulation policy.

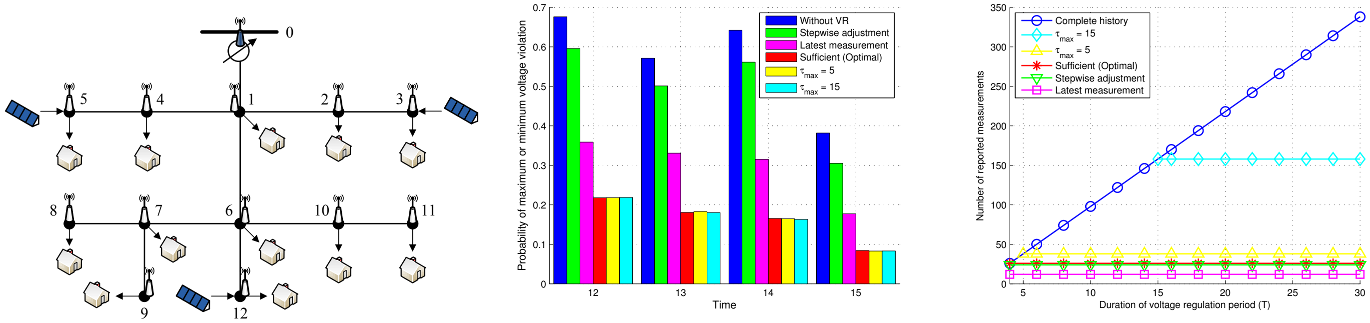
A. System Configuration

An illustration of the IEEE 13-bus test system is shown in Fig. 3(a), where the parameters of feeders are given by IEEE standard [24]. Note that the buses in Fig. 3(a) are re-indexed according our system model in Section III. The transformer between buses 2 and 3 of the original system is not considered for simplicity and the switch between buses 6 and 10 is closed. Real data is used for DG power generation and demand. A uniform load condition is considered for the IEEE 13-bus test system [3], where the demand on each bus follows the same distribution and equals the aggregated demand of 600 households. The statistics of demand for each household at different times of a day is obtained from the smart meter readings of two residences subscribed to Waterloo North Hydro in the Laurelwood neighborhood of Waterloo Region in Canada [25]–[27]. Three PV farms (with aggregated PV panels) are installed on buses 3, 5, and 12, respectively, with an AC rating 9.24 MW for each of them. The statistics for the generation data of PV farms are obtained from National Renewable Energy Laboratory (NREL) PVWatts$^{\text{TM}}$ site specific calculator for Toronto (100 km away from Waterloo Region) [28] [29]. Here, we choose a relatively large capacity for each PV farm to better demonstrate the dynamics of system voltage at different times of a day. Accordingly, we consider relatively large maximum and minimum voltage violation tolerance ranges, given by 115% and 85% of the voltage rating, respectively [30]. The McGraw-Edison single-phase voltage regulator is used [19].

The duration of each time slot (τ) is 1 second. As an example, multi-hop communications are used by RTUs for measurement reports. The (worst-case) communication delay for each hop is set to 1 second to ensure that one-hop data transmission can be finished via wireless communications (e.g., via ZigBee and WiFi communications). In this case, we have $\tau_{\max} = 4$ since at most four hops are involved for the measurement report from any of the buses in Fig. 3(a). Note that the delay requirement is more elastic as compared with that of the traditional power systems. Typically, the operational data of distribution substations is reported to the utilities once every 2 seconds in traditional power systems [7]. The voltage regulator is deployed at the distribution substation, i.e., between buses 0 and 1. The duration of each voltage regulation period is 30 s (i.e., $T = 30$), with a negligible notification delay from the coordinator (i.e., $\tau'_1 = 0$). The initial tap setting of the voltage regulator is 0 without voltage boosting or bucking.

B. Linear Approximation

In Section V, the sufficient information state is discussed for optimal voltage regulation. The results are general for all kinds of smart distribution systems. Since the derivation of an optimal voltage regulation policy (with respect to $\hat{\mathbf{U}}_n$) is out of the scope of this paper, we consider a simple example in



(a) System configuration. (b) Probability of voltage violation. (c) Communication overhead.

Fig. 3: A case study based on the IEEE 13-bus test system with DG integration.

this case study which is based on a linear approximation of the power injection state evolution. Specifically, we approximate the state of power injection at each bus to be a continuous variable which evolves as $S_{t+1}^{(b)} = S_t^{(b)} + \Delta_b\tau + \sigma_b$, where Δ_b is the rate of power injection state change and σ_b is a white Gaussian noise process with zero mean. The approximation is valid given the duration of each time slot (τ) is small such that the power injection state of each bus does not change much and the duration of voltage regulation period (T) is small such that the statistics of power injection do not change significantly. In our analysis and simulations, the values of Δ_b and σ_b are obtained based on historic measurements and updated on an hourly basis. We neglect the wear and tear of the voltage regulator and consider a quadratic objective function $c(\mathbf{v}_t) = \sum_{b\in\mathcal{B}} |v_t^{(b)} - v_0|^2$, where v_0 denotes the voltage rating. Intuitively, the negative impact of over/under voltage appears to be more severe for a larger voltage deviation from the voltage rating. For computational simplicity, we approximate the tap setting of the voltage regulator to be a continuous variable within the maximum voltage boosting/bucking range. In operation, each control action is discretized to the tap setting of voltage regulator. Under the approximations, the optimal voltage regulation problem is equivalent to a traditional linear-quadratic regulator (LQR) problem and the optimal control action (in terms of $\hat{\mathbf{U}}_n$ as defined in Theorem 1) is a linear function of the power injection states in $\mathbf{X}_n$. The parameters of the optimal controller can be calculated based on existing convex optimization methods [21] [31].

C. Numerical Results

Three kinds of voltage regulation schemes are considered for comparison. The stepwise adjustment scheme adjusts the tap setting of voltage regulator in a step-by-step manner based on the maximum/minimum voltage of the smart distribution system during each voltage regulation period, without investigating the statistics of DG power generation and demand [3]. The latest measurement scheme does not take the communication delays into account. Each bus only reports the most recent state of power injection, and the coordinator uses the available information to solve an LQR problem. The sufficient information state scheme is based on the controller designed in Subsection VI-B with $\tau_{\max} = 4$. For illustration purpose, we also show the results with respect to a distribution system without a voltage regulator.

The probability of maximum or minimum voltage violation at different times of a day is shown in Fig. 3(b). We consider five consecutive voltage regulation periods for each voltage regulation scheme, given the first voltage regulation period begins at each of the time indices in the x-axis. The simulation results are obtained via an average over 100 simulation runs, based on randomly generated DG power generation and demand according to the statistics obtained from real data. As we can see, the probability of maximum or minimum voltage violation is high without a voltage regulator because of the integration of DG (i.e., PV farms) in the smart distribution system and the randomness in power generation and demand. The probability is reduced by using the stepwise adjustment scheme since the voltage regulator dynamically boosts (resp. bucks) voltage whenever the minimum (resp. maximum) voltage of the smart distribution system exceed the regulatory limit. However, the reduction is not significant since tap setting of voltage regulator is adjusted after over/under voltage happens. Moreover, the step-by-step adjustment may not be fast enough to capture the fast dynamics of bus voltages. The latest measurement scheme allows tap-changing beyond one-step based on the solution of the LQR problem and can reduce the probability of maximum or minimum voltage violation significantly. The probability can be further reduced if the coordinator utilizes more information according to the sufficient information state.

The sufficient information state scheme is optimal in a sense that the probability of maximum or minimum voltage violation cannot be further reduced by utilizing more information state. As an example, we manually set $\tau_{\max} = 5$ and $\tau_{\max} = 15$ in (13) (which is equivalent to adding one and eleven reported measurements, respectively, from each bus within each voltage regulation period) and recalculate the optimal policy by solving the corresponding LQR problem. The results are shown in Fig. 3(b). We can see that, the probabilities of maximum or minimum voltage violation for $\tau_{\max} = 5$ and $\tau_{\max} = 15$ are both consistent with that for the sufficient information state scheme (i.e., $\tau_{\max} = 4$). In addition, based on the optimal control, the probability of maximum or minimum voltage violation decreases from

12:00noon to 15:00pm mainly because of the decrement in PV power generation. At 12:00noon, the power output of PV farms is large and exceeds the demand of some buses which results in significant overvoltage. On the other hand, the PV power generation and demand become more balanced at 15:00pm so that the voltages of buses can be better regulated.

Fig. 3(c) demonstrates the communication overhead of different voltage regulation schemes, which is measured based on the number of reported measurements within the five consecutive voltage regulation periods. The duration of each voltage regulation period (in terms of T) varies from the minimum possible value 4 (since $\tau_{\max} = 4$ and $\tau_1' = 0$) to 30. We can see that, the latest measurement scheme achieves the lowest communication overhead since only one measurement needs to be reported by each bus within each voltage regulation period. On the other hand, since two measurements (i.e., the maximum and minimum voltages) should be reported by each bus within each voltage regulation period, the communication overhead of stepwise adjustment scheme doubles that of latest measurement scheme. The sufficient information state scheme requires slightly more measurement reports than that of the stepwise adjustment scheme. However, the increment is not significant since some of the buses (i.e., buses 8, 9, and 11) which have a communication delay equal to $\tau_{\max}$ only need to send one measurement report within each voltage regulation period, according to the definition of sufficient information state. The communication overhead of a complete history scheme and the schemes with $\tau_{\max} = 5$ and $\tau_{\max} = 15$ respectively, is also shown. Although the optimal controller can also be obtained based on these three schemes, the communication overhead is high and may increase with the duration of a voltage regulation period. Note that the computational overhead of optimal voltage regulation is also determined by the number of reported measurements, which is associated with the computational complexity in obtaining the optimal controller [21] [22]. By using a relatively small amount of reported measurements in comparison with the complete history scheme and the schemes with $\tau_{\max} = 5$ and $\tau_{\max} = 15$ respectively, the sufficient information state scheme is able to reduce the computational complexity while achieving optimal voltage regulation in the smart distribution system.

VII. Conclusions

In this paper, we investigate the stochastic information management for voltage regulation in smart distribution systems with DG integration. The optimal voltage regulation problem is formulated as a POMDP problem by considering both power flow analysis and communication delays. We mathematically prove a sufficient information state for the optimal voltage regulation problem, based on which the optimal control can be established. The theoretical results are evaluated by simulation based on an IEEE 13-bus test system and real DG power generation and demand data. Numerical results indicate that, the voltage regulation scheme based on sufficient information state has a relatively low communication and computational overhead and can significantly reduce the probability of maximum or minimum voltage violation, in comparison with the existing schemes without considering DG power generation and demand statistics and/or communication delays. Future work includes the derivation and complexity analysis of an optimal voltage regulation policy for general smart distribution systems without linear approximation, and an extension of our current study to stochastic communication delays.

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