Smart Spectrum Access Algorithms in Mobile TV White Space Networks for Utility Maximization

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The demand for mobile computing services has grown significantly and rapidly in the last few years. This has led to a serious challenge on the quality of experience for mobile users. The wireless network consists of consumer devices that communicate simultaneously on a shared and severely-limited spectrum. In 2010, the Federal Communication Commission (FCC) has unleashed 500MHz of newly available broadcast TV Whitespace for opportunistic unlicensed use, i.e., the unlicensed secondary users are allowed to coexist and use the licensed spectrum. In fact, the mobile TV white space network is a wireless cognitive radio network, where the overall network utility is adversely affected not only by interference due to the broadcast nature of the wireless medium but also by the absence of the intelligence for spectrum opportunity. Thus, this new technology innovation requires an optimal resource allocation in a collaborative and smart manner. With uncontrolled resource allocation, secondary users may cause overwhelming interference to the primary users, and the network operating point can be highly unfair.

We study a utility maximization framework for spectrum sharing among unlicensed secondary users and licensed primary users in mobile TV white space networks. All the users maximize the network utility by adapting their signal-to-interference-plus-noise ratio (SINR) assignment and transmit power subject to power budget constraints and additional interference temperature constraints for the secondary users, which is given by

$$\begin{array}{ll} \text{maximize} & U(\boldsymbol{\gamma}) \\ \text{subject to} & \mathbf{w}_l{}^\top \mathbf{p} \le \bar{p}_l, \quad l = 1, \ldots, L, \\ & \mathbf{e}_l{}^\top \mathbf{q} \le \bar{q}_l, \quad l = 1, \ldots, L, \\ & \mathbf{q} = \mathbf{F}\mathbf{p} + \mathbf{n}, \\ & \gamma_l = p_l / q_l, \quad l = 1, \ldots, L, \\ \text{variables:} & \boldsymbol{\gamma}, \mathbf{p}, \mathbf{q} \end{array} \tag{1}$$

where the utility function U is in terms of the SINR $\boldsymbol{\gamma}$, and $\mathbf{p}$ and $\mathbf{q}$ denote respectively the transmit power and the interference temperature vectors constrained by affine constraint set. Note that all the parameters mentioned in this abstract are nonnegative constants related to the channel conditions, and $\boldsymbol{\gamma}$, $\mathbf{p}$ and $\mathbf{q}$ are the unknown to be solved. Hence, the attainable network utility of all the users depends on the joint wireless resource control of the SINR, the transmit power and the interference temperature. However, the main challenges of solving the wireless utility maximization problems in (1) come from 1) the tight coupling between the power budget and the interference temperature; and 2) the nonlinearity and nonconvexity of the SINR and individual utility functions. These are nonconvex problems that are notoriously difficult to solve, and designing scalable algorithms with low complexity to solve them optimally is even harder.

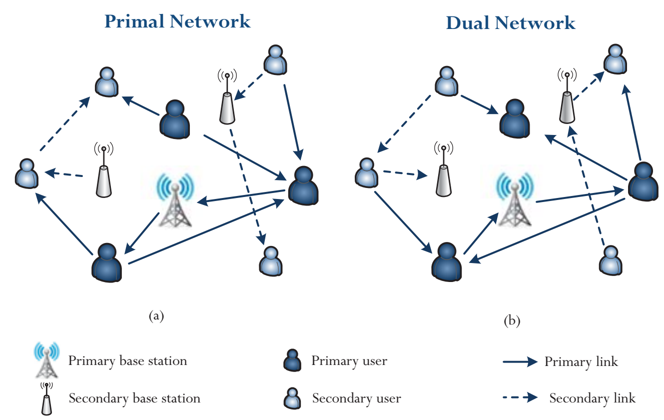

Fig. 1. Illustration of a cognitive network showing that the link directions are reversed in the primal network and the dual network. (a) The primal network. (b) The dual network.

By exploiting *the nonnegative matrix theory*, we first reformulate the utility maximization problem as an equivalent optimization problem involving spectral radius constraints and with SINR as its only variable, given by

$$\begin{array}{ll} \text{maximize} & U(\boldsymbol{\gamma}) \\ \text{subject to} & \rho\big(\operatorname{diag}(\boldsymbol{\gamma})(\mathbf{F} + \frac{1}{\bar{p}_l}\mathbf{n}\mathbf{w}_l^\top)\big), \le 1, \; l = 1, \ldots, L, \\ & \rho\big(\mathbf{F}\operatorname{diag}(\boldsymbol{\gamma}) + \frac{1}{\bar{q}_l}\mathbf{n}\mathbf{e}_l^\top\big) \le 1, \; l = 1, \ldots, L, \\ \text{variables:} & \boldsymbol{\gamma}. \end{array} \tag{2}$$

To proceed further, we studied a special case that maximizes the egalitarian fairness of all the SINRs as the utility (also known as the max-min weighted SINR), given by

$$\begin{array}{ll} \text{maximize} & \min\limits_{l=1,\ldots,L} \dfrac{\gamma_l}{\beta_l} \\ \text{subject to} & \rho\big(\operatorname{diag}(\boldsymbol{\gamma})(\mathbf{B}_l)\big) \le 1, \quad l = 1, \ldots, L, \\ & \rho\big(\mathbf{D}_l \operatorname{diag}(\boldsymbol{\gamma})\big) \le 1, \quad l = 1, \ldots, L, \\ \text{variables:} & \boldsymbol{\gamma}, \end{array} \tag{3}$$

where the nonnegative matrices $\mathbf{B}_l$ and $\mathbf{D}_l$ are defined as

$$\mathbf{B}_l = \mathbf{F} + \frac{1}{\bar{p}_l}\mathbf{n}\mathbf{w}_l^\top \;\text{ and }\; \mathbf{D}_l = \Big(\mathbf{I} + \frac{1}{\bar{q}_l - n_l v_l}\mathbf{n}\mathbf{e}_l^\top\Big)\mathbf{F}, \;\; \forall l.$$

Cognitive Radio Network Duality

$$
\begin{array}{c|lcl}
 & \text{Primal network} & & \text{Dual network} \\
\begin{array}{c}\text{Power}\\\text{Budget}\\\text{Constraint}\end{array} &
\left\{\begin{array}{l}
\mathbf{p}^\star = \mathbf{x}\big(\operatorname{diag}(\boldsymbol{\gamma}^\star)(\mathbf{F}+(1/\bar{p}_i)\mathbf{n}\mathbf{w}_i^\top)\big) \\
\quad = \big(\mathbf{I}-\operatorname{diag}(\boldsymbol{\gamma}^\star)\mathbf{F}\big)^{-1}\operatorname{diag}(\boldsymbol{\gamma}^\star)\mathbf{n} \\
\quad \mathbf{w}_l^\top\mathbf{p}^\star \le \bar{p}_l
\end{array}\right. &
\begin{array}{c}\leftrightarrow\\ \\ \leftrightarrow\end{array} &
\begin{array}{l}
\mathbf{s}^\star = \operatorname{diag}(\boldsymbol{\gamma}^\star)\mathbf{y}\big(\operatorname{diag}(\boldsymbol{\gamma}^\star)(\mathbf{F}+(1/\bar{p}_i)\mathbf{n}\mathbf{w}_i^\top)\big) \\
\quad = \big(\mathbf{I}-\operatorname{diag}(\boldsymbol{\gamma}^\star)\mathbf{F}^\top\big)^{-1}\operatorname{diag}(\boldsymbol{\gamma}^\star)\mathbf{w}_i \\
\quad \mathbf{n}^\top\mathbf{s}^\star \le \bar{p}_i
\end{array} \\
\begin{array}{c}\text{Interference}\\\text{Temperature}\\\text{Constraint}\end{array} &
\left\{\begin{array}{l}
\mathbf{q}^\star = \mathbf{x}\big((\mathbf{I}+\frac{1}{\bar{q}_j-n_jv_j}\mathbf{n}\mathbf{e}_j^\top)\mathbf{F}\operatorname{diag}(\boldsymbol{\gamma}^\star)\big) \\
\quad = \big(\mathbf{I}-\mathbf{F}\operatorname{diag}(\boldsymbol{\gamma}^\star)\big)^{-1}\mathbf{n} \\
\quad \mathbf{e}_l^\top\mathbf{q}^\star \le \bar{q}_l
\end{array}\right. &
\begin{array}{c}\leftrightarrow\\ \\ \leftrightarrow\end{array} &
\begin{array}{l}
\mathbf{t}^\star = \operatorname{diag}(\boldsymbol{\gamma}^\star)^{-1}\mathbf{y}\big((\mathbf{I}+\frac{1}{\bar{q}_j-n_jv_j}\mathbf{n}\mathbf{e}_j^\top)\mathbf{F}\operatorname{diag}(\boldsymbol{\gamma}^\star)\big) \\
\quad = \big(\mathbf{I}-\mathbf{F}^\top\operatorname{diag}(\boldsymbol{\gamma}^\star)\big)^{-1}\mathbf{w}_j \\
\quad \mathbf{n}^\top\mathbf{t}^\star \le \bar{q}_j
\end{array}
\end{array}
$$

Fig. 2. The cognitive radio network duality illustrates the connection between the primal and the dual networks in terms of both the Perron right and left eigenvectors of the nonnegative matrices associated with the spectral radius constraints in (2).

Note that (3) is a special case of (1) that has a non-smooth concave objective function. The egalitarian SINR fairness in (3) is solved by leveraging *the nonlinear Perron Frobenius theory* [1]. We first state the key theorem in [1].

Theorem 1 (Krause's theorem [1]): Let $\|\cdot\|$ be a monotone norm on $\mathbb{R}^L$. For a concave mapping $f:\mathbb{R}_+^L \to \mathbb{R}_+^L$ with $f(\mathbf{z}) > 0$ for $\mathbf{z} \ge \mathbf{0}$, the following statements hold. The conditional eigenvalue problem $f(\mathbf{z}) = \lambda\mathbf{z}$, $\lambda \in \mathbb{R}$, $\mathbf{z} \ge \mathbf{0}$, $\|\mathbf{z}\| = 1$ has a unique solution $(\lambda^\star, \mathbf{z}^\star)$, where $\lambda^\star > 0$, $\mathbf{z}^\star > \mathbf{0}$. Furthermore, $\lim_{k\to\infty}\mathbf{z}(k+1)$ converges geometrically fast to $\mathbf{z}^\star$ (the *unique* fixed point $\mathbf{z} = f(\mathbf{z})/\|f(\mathbf{z})\|$), where

$$\mathbf{z}(k+1) = \frac{f(\mathbf{z}(k))}{\|f(\mathbf{z}(k))\|} \tag{4}$$

regardless of the initial point.

Coming back to solving (3), the optimal solution is computed by

$$p_l(k+1) = \frac{\beta_l}{\mathsf{SINR}_l(\mathbf{p}(k))}p_l(k)$$

which is normalized by an infinity norm

$$\mathbf{p}(k+1) \leftarrow \frac{\mathbf{p}(k+1)}{\max\limits_{l=1,\ldots,L}\left\{\dfrac{\mathbf{w}_l^\top\mathbf{p}(k+1)}{\bar{p}_l}, \dfrac{\mathbf{e}_l^\top\mathbf{F}\mathbf{p}(k+1)}{\bar{q}_l - n_lv_l}\right\}}$$

in each iteration. From Theorem 1, this is a tuning-free distributed algorithm with a geometric convergence rate.

Then, we solve the general utility maximization by developing a novel *cognitive radio network duality* [2] to decouple the SINR assignment, the transmit power and the interference temperature allocation. As illustrated in Figure 1, two different networks, respectively a primal network and a dual network, can be constructed to attain an identical SINR performance but with all the link directions reversed. Hence, a feasible SINR for one is also feasible for the other. We can characterize analytically the power and the interference temperature of the primal and the dual networks, as shown in Figure 2.

Furthermore, the power and the interference temperature in the primal and the dual networks can be jointly optimized to solve the utility maximization problem formulated for the primal network subject to the power budget and the interference temperature constraints. By applying the cognitive radio network duality, a projected subgadient method that solves (2) can be made distributed by connecting the gradients of the spectral radius functions in (2) with the power and the interference temperature in both the primal and the dual networks (cf. Figure 2). Let $\mathbf{p}$ and $\mathbf{q}$ denote respectively the power and the interference temperature in the primal network, and $\mathbf{s}$ and $\mathbf{t}$ denote respectively the power and the interference temperature in the dual network. The gradients $\mathbf{g}$ of the spectral radius functions in (2) can be written in terms of $\mathbf{p}$, $\mathbf{q}$, $\mathbf{s}$ and $\mathbf{t}$, which are, respectively, given by

$$g_l = p_l\left(\frac{1}{\gamma_l n_l}\right)s_l \text{ and } g_l = q_l\left(\frac{\gamma_l}{n_l}\right)t_l,\ \ l = 1,\ldots,L.$$

This leads to a utility maximization algorithm that leverages the egalitarian fairness power control as a submodule to maintain a desirable separability in the SINR assignment between the secondary and primary users.

This algorithm has the advantage that it can be distributively implemented, and the method converges relatively fast. As shown in Figure 3, we also develop a system implementation of this cognitive radio network protocol by exploiting the the Microsoft White Space Finder API [3] to build a White Space Finder-Resource Allocator to control the TV Whitespace wireless resources.

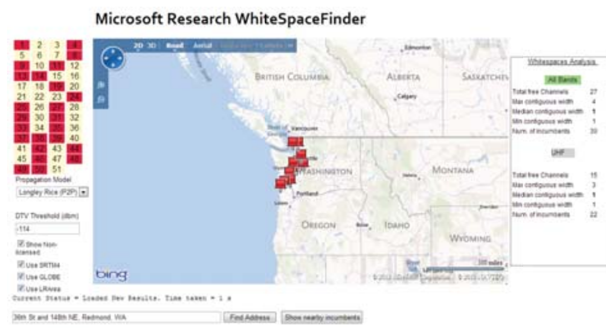

Fig. 3. Illustration of the Microsoft White Space Finder API.

References

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