Wireless Device-to-Device Hypergraph Optimization

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I. Introduction

With the increasing demand for local traffic load, the technique of Device-to-Device (D2D) communication has received great attention [1]. Using the same spectrum as that in cellular communication, D2D communications can increase the overall network spectral efficiency. Meanwhile, D2D communications can potentially generate interference into the existing cellular network if the radio resources are not allocated properly. Thus, much attention has been paid to manage the interference in D2D networks.

It has been well known that graph theory is a useful tool to solve the resource allocation problem in wireless communications. However, the edge in traditional graph method is not accurate in modeling the interference relationship as the cumulative effect of the interference from several vertices may constitute a strong interferer [2]. Hence, for the sake of a better Quality-of-Services (QoS), it is necessary to take into account the accumulation impact of multiple interference sources. And hypergraph, a generalization of an undirected graph in which the hyperedges are arbitrary subsets of the given set of vertices, is more accurate than the traditional graph. In a given arbitrary network, the throughput using the graph method can be improved using an appropriate hypergraph method. Therefore, the hypergraph is adopted as the efficient method to solve the interference problem.

II. System Model and Problem Formulation

As shown in Fig. 1, we consider an uplink transmission scenario which consists of N cellular UEs, M D2D pairs and K channels. In our system, the D2D pairs can share the same channel with the cellular UEs. We denote a cellular UE by U_n, and a D2D pair by D_m. The transmission power of cellular UEs and D2D pairs is denoted by P^d and P^c,respectively. In addition, the channel is modeled as a Rayleigh fading channel, and thus channel gains contain the path loss and the normalized small-scale fading.

Without loss of generality, we assume that a D2D pair and cellular UE can be allocated at most one channel for data transmission, and a channel can be only allocated to at most one cellular UE. Since different communication links can perform their individual data transmission, the key issue of channel allocation is to find the channel assignment solution for both the cellular UEs and D2D pairs for the optimal total system capacity. We denote the channel allocation matrix for the cellular UEs and the D2D pairs by $A = [\alpha_{n,k}]$ and $B = [\beta_{n,k}]$, respectively. If the channel k is allocated to U_n, $\alpha_{n,k} = 1$, and otherwise, $\alpha_{n,k} = 0$. And so is $\beta_{n,k}$.

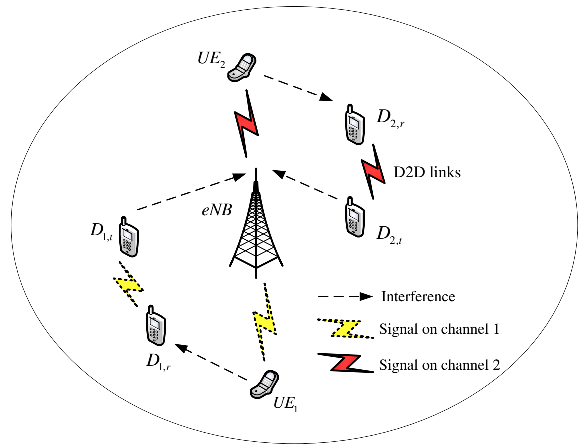

Fig. 1. System model for D2D communications underlaying cellular network when sharing uplink resource.

Under these assumptions, the optimization problem can be described as

$$\max \sum_{k=1}^{K} W \left[\sum_{n=1}^{N} \log_2(1+\gamma_n^c)\alpha_{n,k} + \sum_{m=1}^{M} \log_2(1+\gamma_m^d)\beta_{m,k} \right] \quad (1)$$

where W denotes the bandwidth of the channel, γ_n^c and γ_m^d are the signal-to-noise-and-interference-ratio (SINR) of U_n and D_m, respectively.

It's known that the optimal solution for the allocation problem aforementioned is a Non-Deterministic Polynomial-Time Hard (NP-hard) problem, and graph coloring can be used as an approximate but efficient solution. Thus, we formulate the channel resource as colors, the cellular UEs and the D2D pairs are formulated as vertices in the plane. Therefore, the channel allocation problem can be transformed as the coloring of the vertices with fixed colors.

III. Hypergraph Based Allocation

In this section, we propose a hypergraph based channel sharing scheme for D2D communications underlaying cellular networks.

A. Hypergraph Construction

We here define two kinds of interferers. The first kind is the *independent interferers*, which are close to the UE and bring down the SINR independently. The second kind is the *cumulative interferers* which will threaten the QoS when combining their interference. We construct the hypergraph as the following steps:

1) Neighbor List Foundation: The neighbor list of the UEs should be determined for potential interference. This can be realized by comparing the interference with an interference threshold γ to verify whether they are in the neighbor list. This step is to eliminate the interferers of each UE with slight impact, and enhance the efficiency of the algorithm.

2) Independent Interferer Recognition: After finding the neighbor list, we search the independent interferer for each UE, and construct the corresponding two-verticed edge. This step is to eliminate the severe interference from two different UEs sharing the same channel.We follow the pairwise comparison with an interference threshold δ to pick out the independent interferers, and then form a two-verticed hyperedge.

3) Cumulative Interferer Recognition: After all the independent interferers are determined, the cumulative interference from more than one UEs in the neighbor list need to be determined. We select a certain number of members in the neighbor list, and compare the interference with an interference threshold η to verify whether they are cumulative interferers. After all these steps, we construct the hypergraph H.

B. Hypergraph Coloring Algorithm

After the hypergraph construction, the cellular UEs and D2D pairs find their neighbor lists, and the hyperedges are formed to model the interference, and the hypergraph H can be colored. A color in the hypergraph corresponds to a channel, and the coloring of vertices is equivalent to the channel allocation to the D2D pairs and cellular UEs, while the vertices contained in the same hyperedge cannot be colored by the same color.

So far, some coloring algorithms have been proposed to color a hypergraph efficiently. The algorithm mentioned in [3] is a greedy algorithm to color the hypergraph which the number of colors is sufficient. However, in the OFDM system, the condition may not be fulfilled. In the light of the observations, we propose to modify the greedy algorithm mentioned in [3] to meet the need in OFDM network.

Definition 1: In a hypergraph $H(X, D)$, *strong deletion* of a vertex $x \in X$ from H is the removing of all the edges containing x from D and removing of x from X.

The basic idea of the modified greedy algorithm is to find a good ordering of the vertices by first decomposing hypergraph H using the degrees of the vertices. First, select the vertex with the minimum degree from the induced subhypergraph, and delete the vertex strongly to form the new induced subhypergraph. Then the vertex selected is labeled. Next, after labeling all the vertices, the vertices is to be colored in label order with a color selected from the available color set of this vertex, instead of the lowest numbered color. Besides, when the available color set is empty, which might happen when colors are not sufficient, leave the vertex uncolored. The whole procedure stops until all the vertices are examined and the coloring is determined.

IV. Simulation Results

In this section, we provide the simulation results of the proposed channel allocation scheme. Simulations are in comparison with conventional graph-based method, and No D2D scheme in which all the UEs are forced to work in cellular mode even though some may be suitable for D2D communication.

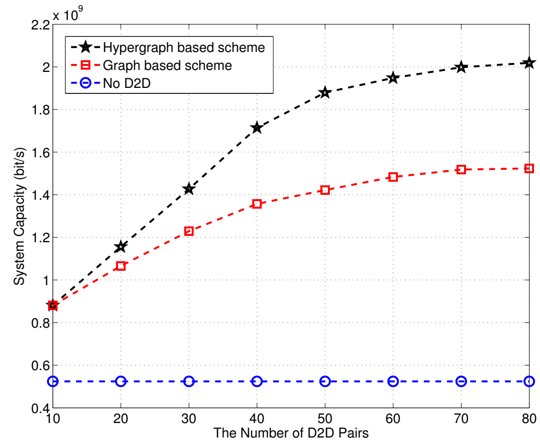

Fig. 2. System capacity with the number of D2D pairs M for $K = 30$, $N = 20$.

In Fig. 2, it shows the system capacity as a function of M with $N = 20$, $K = 30$. We can see that the system capacity using both our proposed hypergraph based scheme and graph based scheme increases with the growing number of D2D pairs M. In addition, it shows that when M grows, the rate of the increase in system capacity slows down, which indicates that when the number of D2D pairs become sufficiently large, the system capacity will be limited by the number of K. Comparing the system capacity using hypergraph method with that using graph method, due to the cumulative interference modeling, the interference from the D2D pairs is well controlled, and thus the system capacity using hypergraph method is always higher than that using the graph method.

V. Conclusions

In this paper, we have realized channel allocation using hypergraph theory to effectively increase the total capacity of the system network. The hypergraph is constructed according to the location for the mutual interference coordination. We formulate this channel sharing problem as a hypergraph coloring problem to maximize capacity of the system, and also propose a greedy coloring algorithm. Simulation results indicate that the proposed hypergraph based channel sharing scheme improves the system capacity significantly compared with that using the traditional graph method.

References

[1] K. Doppler, M. Rinne, C. Wijting, C. Ribeiro, and K. Hugl, "Device-to-Device Communication as an Underlay to LTE-Advanced Networks," *IEEE Communications Magazine*, vol. 7, no. 12, pp. 42-49, Dec. 2009.

[2] Q. Li, G. Kim, and R. Negi, "Maximal Scheduling in A Hypergraph Model for Wireless Networks," in *2008 IEEE International Conference on Communication (ICC)*, Beijing, China, May. 2008.

[3] V. I. Voloshin, *Introduction to Graph and Hypergraph Theory*. Nova Science Publishers, New York, USA, Inc, 2008.