Smart Data Pricing: To Share or Not to Share?

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***Abstract*—This paper studies a monopoly telecom operator's decision on the adoption of shared data plans. A shared data plan allows sharing data quota among multiple devices or users, while conventional single device data plans only allow the use of a single device. We devise analytical models and compare a simple shared data plan (also called bundling pricing) to single device data plans (also called partitioned pricing). When consumer valuations (utilities) of different devices are independent, we find a threshold on the unit usage cost below which the shared data plan yields more profits than single device data plans. The optimal price for the shared data plan is less than the sum of the single device data plans. If consumers' valuations for different devices have different distributions, this disparity reduces the relative value of the shared data plan against the single device data plans. We also show that shared data plans increases the social welfare and consumer surplus when it yields a higher profit.**

We further extend the analysis to complementary valuations on different devices: a consumer's valuation for using both devices may be higher than the sum of his utilities on the devices when only one device is used. We identify a threshold on the unit usage cost below which the shared data plan yields more profits. The price of the shared data plan is larger than the sum of the single device data plans for very strong complementariness, while it's always less with independent valuations. We also show numerically that a strong complementariness shrinks the range of the unit usage cost where the shared data plan has a higher profit.

I. Introduction

The development and adoption of smart phones and mobile tablet computers have been driving the growth of mobile internet data services in recent years, which also drives mobile network operators to keep innovating their business models and pricing schemes. One of the emerging pricing schemes is a shared data plan. A shared data plan allows the use of multiple devices against a common data quota, which represents a form of bundling pricing. On the other hand, conventional data plans only allow the use of a single device for each subscription, which represents a form of partitioned pricing. Strategy Analytics (2011) shows that 60% of smartphone owners want a single shared data plan that can connect multiple mobile devices. According to Alcatel-Lucent (2013), operators in US, Canada, Sweden, Greece, Austria, UK and Australia are offering shared data plans. Furthermore, for US-based operators, Alcatel-Lucent (2013) shows that within the first 6 months of introduction, 14% to 23% of their own postpaid customers have signed up for shared data plans, and people on shared data plans generally buy bigger data buckets and spend $30 more per month. However, the reasons for the supremacy of the shared data plans over conventional data plans remain misty. In this paper, we seek to unravel these reasons with analytical models. Our goal is to understand what are the key driving forces in these pricing schemes and what conditions make the shared data plans outperform single device data plans. Working towards this goal, we develop analytical models on the operator's pricing decisions on different schemes and derive and compare the solutions.

We follow the marketing literature (e.g., Venkatesh and Kamakura 2003) to develop analytical models. Consumer utilities (valuations) of different devices, which are assumed to be equal to their usage, follow independent uniform distributions. We use uniform distributions because of its wide use in analytical models on bundling and its tractability. The cost of each subscription is a linear function of the his/her total data usage. To gain sharper insight, we first consider two independent devices for which the utility of using two devices is the sum of the utilities of two individual devices. We analyze optimal partitioned pricing and bundling pricing respectively and then compare the profitability, consumer surpluses and social welfare under the two pricing schemes. We find a threshold on the unit usage cost below which the shared data plan yields more profits than single device data plans. The optimal price for the shared data plan is less than the sum of the single device data plans. If consumers' valuations for different devices have different distributions, this disparity reduces the relative value of the shared data plan against the single device data plans. We also show that shared data plans increases the social welfare and consumer surplus when it yields a higher profit.

We further generalize our analysis to the setting where the two devices are economic complements, that is, the utility of using two device is greater than the sum of the utilities on the devices when only one device is used. We identify a threshold on the unit usage cost below which the shared data plan yields more profits. The price of the shared data plan is larger than the sum of the single device data plans for very strong complementariness, while it's always less with independent valuations. We also show numerically that a strong complementariness shrinks the range of the unit usage cost where the shared data plan has a higher profit.

The remainder of the paper is organized as the following:

Section II reviews related literature; Section III presents the model and results on the problem with independent utilities; Section IV presents those on the problem with complementary utilities; Section V provides a summary of the paper and some potential extensions.

II. Related Literature

Our work is closely related to two streams of literature: data plan pricing and bundling pricing.

Data Plan Pricing: The literature on data plan pricing, as part of the telecom pricing research, is growing rapidly due to the fast growth of smart mobile device markets; see Sen et al. (2013) for an excellent review on past, current and potential future developments on data pricing. Many studies focus on the comparison of time-based and usage-based pricing schemes (see, e.g., Chen and Huang 2013 and the references therein), or static and dynamic pricing strategies (see, e.g., Ha et al. 2012). Sen et al. (2013) highlight "shared data plan" as an emerging pricing option that US telecom operators are offering. Sen et al. (2012) discuss the economics of shared data plan. They propose a consumer utility model of price and usage cap and then conduct a numerical study. However, whether the telecom operators should adopt the shared data plan and what the key driving factors are for this decision remain unanswered. Different from Sen et al. (2012), we seek to answer these questions in this work. We follow the economics and marketing literature on product bundling, treating shared data plan as a special bundled product, to develop an analytical model which enables us to derive the closed-form solution to compare the shared data plan and single device data plans.

Bundling: There is an extensive literature in marketing and economics on bundling; see Venkatesh and Mahajan (2009) for a comprehensive review on the recent developments. Rooted from the seminal paper of Adams and Yellen (1976), the early studies treat bundling as a price discrimination device and typically assume that the consumer reservation price for the bundle is equal to the sum of the separate reservation prices for the component products. Products that conform to this assumption are referred to as independently valued products. Mainly through the use of numerical examples, Adams and Yellen (1976) examine the possible implications of choices among partitioned, bundling and mixture strategies for seller profit and net welfare (profit plus consumer surplus). Assuming bivariate normal distribution of valuation (reservation prices), Schmalensee (1984) numerically demonstrates that bundling reduces buyer diversity, which allows the seller to capture more consumer surplus. Hence, bundling could make the seller more profitable while generally making consumers worse off. These studies, though insightful, are restricted to the numerical examples.

In contrast to those early studies, Venkatesh and Kamakura (2003) point out that firms often seek to use bundling strategies to manage interrelated products. For complement products (e.g., iPad and its smart cover), consumers' reservation price for the bundle is superadditive in those for the component products (e.g., Coke and Pepsi), whereas for substitute products, consumers' reservation price for the bundle is subadditive in those for the component products. They propose an analytical model to analyze the optimal bundling and pricing strategies for a monopoly in the presence of these contingent valuations. They provide closed-form solution to the optimal pricing decisions under both partitioned (pure component) and bundling strategies, and show that *when the variable cost is zero* bundling is weakly more profitable than partitioned pricing. To gain sharper insights analytically, we adopt Venkatesh and Kamakura's (2003) approach to model the data plan pricing. Different from these marketing and economics models in which the unit cost of the component product is fixed and is independent of the consumer valuation, the valuation of data plans and the corresponding cost are contingent on data usage in our work. Such a feature allows us to build a different model and to analytically compare the partitioned and bundling strategies in terms of operator's profits, consumer surplus, and social welfare.

III. The Model with Independent Utilities

We analyze the pricing strategy for a monopoly telecom operator who is providing mobile data services in a market. The market size is normalized to one. Assume that each subscriber may subscribe two mobile devices (e.g., two smart phones or a smart phone and a tablet). The usages of the devices are exogenous and heterogeneous among consumers. We denote the usage by u_i, $i = 1, 2$, and its cumulative and density distribution function by Φ_i and ϕ_i respectively. We assume u_i follows a uniform distribution between $[0, b_i]$, and $\phi_i(u_i) = \frac{1}{b_i}$. The utility of a device for consumers is equal to its usage. The utility is independent of each other between the two devices. The operator's cost for providing the service is linear in the total usage. We use $C(x) = cx$ $(0 < c \leq 1)$ to denote the cost where c is the unit cost over the usage. For simplicity, our analysis is restricted to "unlimited" data plans at fixed fare rates (fixed prices per time period).

Two pricing schemes are considered: partitioned pricing (single device data plan) and bundling pricing (shared data plan). In the partitioned pricing, the operator provides single device data plans for each device, while in the bundling pricing pricing the data plan is shared by the two devices. Its objective is to maximize the total expected profit.

A. Partitioned Pricing

In the partitioned pricing, the operator provides single device data plans for each device. Since consumers' utilities of the two devices are independent, the operator can decide the prices for the two devices separately. We denote the fixed price per time period by p_i, $i = 1, 2$. A consumer's utility surplus for device i is

$$U_i = u_i - p_i$$

A consumer subscribes to the device if and only if $U_i \geq 0$. The sales for device i is

$$\alpha_i = \Pr(U_i \geq 0) = \Pr(u_i \geq p_i)$$

The total expected usage for device i is

$$\alpha_i \cdot E[u_i | u_i \geq p_i] = \int_{p_i}^{b_i} u_i \phi_i(u_i) du_i$$

The pricing problem for device i is

$$\max_{p_i} \quad \Pi_{P,i} = p_i \int_{p_i}^{b_i} \phi_i(u_i) du_i - c \int_{p_i}^{b_i} u_i \phi_i(u_i) du_i$$

Derived from the first order conditions, the optimal prices in the partitioned pricing are $p_i = \frac{b_i}{2-c}$. The corresponding maximum profit is

$$\Pi_P = \frac{(1-c)^2}{2-c} \frac{b_1 + b_2}{2} \tag{1}$$

B. Bundling Pricing

In the bundling pricing, the operator provides a data plan that is shared by the two devices. We denote the fixed price per time period by p. A customer's utility surplus is

$$U = u_1 + u_2 - p$$

A consumer subscribes to the device iff $U \geq 0$. Let Φ, ϕ be the cumulative and density distribution functions of $u = u_1 + u_2$.

Without loss of generality, we assume $b_2 = tb_1$ with $t \geq 1$ for the uniform distributions of u_i. The distribution of $u = u_1 + u_2$ is then

$$\phi(u) = \begin{cases} \frac{u}{b_1 b_2}, & \text{if } 0 \leq u \leq b_1 \\ \frac{1}{b_2}, & \text{if } b_1 < u \leq b_2 \\ \frac{b_1 + b_2 - u}{b_1 b_2}, & \text{if } b_2 < u \leq b_1 + b_2 \end{cases}$$

Similar to the partitioned pricing problem, the pricing problem for the bundle is

$$\max_{p} \quad \Pi_B = p \int_{p}^{b_1+b_2} \phi(u) du - c \int_{p}^{b_1+b_2} u \phi(u) du$$

The first order derivative of Π_B over p is continuous in p. It first decreases then increases in p. It remains negative in the interval it increases. Thus the first order condition has a unique solution. We find the candidate optimal solution in each of the intervals $[0, b_1]$, $(b_1, b_2]$ and $(b_2, b_1 + b_2]$ and identify the conditions when the optimal solution falls in each interval. The optimal price in the bundling pricing and its corresponding maximum profit are

- If $c \leq \frac{1}{2}$ and $t \leq \frac{3}{2} - c$, $p = \sqrt{\frac{b_1 b_2}{\frac{3}{2} - c}}$,

$$\Pi_B = b_1 \left[\frac{2}{3} \sqrt{\frac{t}{\frac{3}{2} - c}} - c \frac{1+t}{2} \right] \tag{2}$$

- If $c \leq \frac{1}{2}$ and $t > \frac{3}{2} - c$, or if $c > \frac{1}{2}$ and $t > \frac{1}{2(1-c)}$, $p = \frac{\frac{b_1}{2} + b_2}{2-c}$,

$$\Pi_B = b_1 \frac{1+t}{2} \cdot \frac{(1-c)^2}{2-c} + \frac{b_1}{2t} \left(\frac{1}{4(2-c)} - \frac{c}{3} \right) \tag{3}$$

- If $c > \frac{1}{2}$ and $t \leq \frac{1}{2(1-c)}$, $p = \frac{b_1 + b_2}{2\left(\frac{3}{2} - c\right)}$,

$$\Pi_B = b_1 \frac{(1+t)^3}{4t} \cdot \frac{2(1-c)^3}{3\left(\frac{3}{2} - c\right)^2} \tag{4}$$

C. Comparing Partitioned Pricing and Bundling Pricing

We next compare the optimal pricing strategies under the two schemes. We find that the unit usage cost c determines whether the shared data plans outperform the single device data plans and the disparity t between the distributions affects the profit differences. Thus c is a more defining factor than t.

Theorem 1: When the usages of the devices follow uniform distributions between $[0, b_i]$, $i = 1, 2$,

- the maximum profit Π_B in the bundling pricing is greater than that in the partitioned pricing Π_P if and only if $c < \frac{1}{2}$.
- the optimal price p in the bundling pricing is smaller than the sum of optimal prices in the partitioned pricing $p_1 + p_2$.

The bundling pricing has a lower price. This increases the revenue and the total usage of the consumers simultaneously. If the unit usage cost is relatively low, the increase in the revenue outweighs the increase in the usage cost. Thus the bundling pricing generates a higher profit. On the other hand, when the unit cost is high, the increase in the revenue is not sufficient for compensating the increase in the usage cost. The bundling pricing then has a lower profit.

We are also interested in understanding whether the improved profit of the bundling pricing comes at the cost of a smaller consumer surplus or social welfare.

The consumer surplus in the partitioned pricing is

$$S_P = \frac{b_1(1+t)}{2} \frac{(1-c)^2}{(2-c)^2} \tag{5}$$

The consumer surplus in the bundling pricing is

- If $c \leq \frac{1}{2}$ and $t \leq \frac{3}{2} - c$, $p = \sqrt{\frac{b_1 b_2}{\frac{3}{2} - c}}$,

$$S_B = b_1 \frac{1+t}{2} - b_1 \sqrt{\frac{t}{\frac{3}{2} - c}} \frac{(4 - 3c)}{3\left(\frac{3}{2} - c\right)} \tag{6}$$

- If $c \leq \frac{1}{2}$ and $t > \frac{3}{2} - c$, or if $c > \frac{1}{2}$ and $t > \frac{1}{2(1-c)}$, $p = \frac{\frac{b_1}{2} + b_2}{2-c}$,

$$S_B = b_1 \frac{1+t}{2} \cdot \frac{(1-c)^2}{(2-c)^2} + \frac{b_1}{2t} \left(-\frac{3-2c}{4(2-c)^2} + \frac{1}{3} \right) \tag{7}$$

- If $c > \frac{1}{2}$ and $t \leq \frac{1}{2(1-c)}$, $p = \frac{b_1 + b_2}{2\left(\frac{3}{2} - c\right)}$,

$$S_B = \frac{(1+t)^3}{4t} \cdot \frac{2b_1(1-c)^3}{3\left(\frac{3}{2} - c\right)^3} \tag{8}$$

The social welfare is the sum of the consumer surplus and the operator profit. It turns out the bundling pricing yields higher social welfare and consumer surplus most of the time.

Theorem 2: Suppose the usages of the devices follow uniform distributions between $[0, b_i]$, $i = 1, 2$.

If $c \leq 1/2$, the social welfare and consumer surplus is higher in the bundling pricing at the optimal price that those in the partitioned pricing.

If $c > 1/2$,

- the social welfare is higher in the bundling pricing if and only if $\frac{(1+t)^2}{4t} > \frac{3(3/2-c)^3}{4(2-c)^2(1-c)} \cdot \frac{3-c}{5/2-c}$,
- the consumer surplus is higher in the bundling pricing if and only if $\frac{(1+t)^2}{4t} > \frac{3(3/2-c)^3}{4(2-c)^2(1-c)}$

When $c \leq 1/2$, it's optimal for the operator to offer the bundling pricing. The total usage is increased in this situation. This leads to an increased consumer surplus and social welfare. In other words, the bundling pricing is a "win-win" solution for the operator and the consumers. For $c > 1/2$, the bundling pricing brings a greater social welfare and consumer surplus if the disparity between the distributions is sufficiently large, since $\frac{(1+t)^2}{4t}$ increases in t.

However, a large disparity between the distributions reduces the profit difference between the two pricing scheme.

Theorem 3: Suppose the usages of the devices follow uniform distributions between $[0, b_i]$, $i = 1, 2$. The absolute value of the profit difference $|\Pi_B - \Pi_P|$ first increases, then decreases in t. The absolute value of the relative profit difference $\frac{|\Pi_B - \Pi_P|}{\Pi_P}$ decreases in t.

Figure 1 shows the behavior of the absolute and relative profit differences. As the disparity t increases, the influence of the distribution of u_1 is increasingly marginal. In the partitioned pricing, the revenue and usage cost from device 1 becomes dominated by those from device 2. In the bundling pricing, the distribution of u becomes closer to that of u_2. Thus the bundling pricing problem and partitioned pricing problem becomes much similar to each other. This leads to a reduced profit difference.

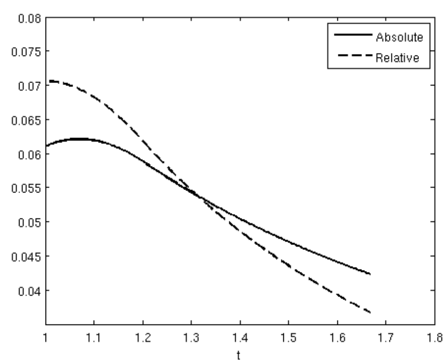


Fig. 1. Absolution and Relative Profit Difference ($b_1 = 3$, $c = 0.3$)

IV. The Model with Complementary Utilities

The preceding analysis assumes that the utilities of different devices are independent of each other. While this is true in some situations, there are many situations where the utilities are complementary: a consumer's utility for using both devices may be higher than the sum of his utilities on the devices when only one device is used. For instance, some computer applications have mobile versions; an increased use of these applications on the computer may lead to an increased use of their mobile versions, and vice verse. Thus in this section, we consider the case of complementary utilities. Following the treatment of Venkatesh and Kamakura (2003), we assume that when both devices are used, the usage and consequently the utility increases by a factor θ to $(1+\theta)(u_1+u_2)$ with $0 < \theta < 1$. The total usage cost then increases accordingly.

A. Partitioned Pricing

Since the utilities are no longer independent, the operator has to decide the prices jointly even for the partitioned pricing. When only 1 device is used, a consumer's utility surplus is still $U_i = u_i - p_i$. However, when both devices are used, his utility surplus is

$$U_P = (1+\theta)(u_1+u_2) - (p_1+p_2)$$

A consumer is going to maximize his utility surplus by deciding the device(s) to use.

- If $U_i > 0$ and $U_i = \max\{U_1, U_2, U_P\}$, he uses device i.
- If $U_P > 0$ and $U_P = \max\{U_1, U_2, U_P\}$, he uses both devices.

Let Ω_i (Ω_P) denote the set of consumers who use device i (both devices). The sales for device i is

$$\alpha_i = \Pr((u_1,u_2) \in \Omega_i) + \Pr((u_1,u_2) \in \Omega_P)$$

The expected usage from device i is

$$\begin{aligned} s_i &= E[u_i|(u_1,u_2) \in \Omega_i] \cdot \Pr((u_1,u_2) \in \Omega_i) \\ &\quad + E[(1+\theta)u_i|(u_1,u_2) \in \Omega_P] \cdot \Pr((u_1,u_2) \in \Omega_P) \end{aligned}$$

The operator's pricing problem is

$$\max_{p_1,p_2} \quad \Pi_P = \sum_i [p_i\alpha_i - cs_i]$$

B. Bundling Pricing

A consumer's utility surplus is now

$$U = (1+\theta)(u_1+u_2) - p$$

The optimal bundling pricing problem is

$$\max_p \ \Pi_B = p\int_{\frac{p}{1+\theta}}^{+\infty} \phi(u)du - c\int_{\frac{p}{1+\theta}}^{+\infty} (1+\theta)u\phi(u)du$$

Define $p_\theta = \frac{p}{1+\theta}$. The optimal bundling pricing problem becomes

$$\max_{p_\theta} \ (1+\theta)\left[p_\theta\int_{p_\theta}^{+\infty} \phi(u)du - c\int_{p_\theta}^{+\infty} u\phi(u)du\right]$$

This problem is just $(1+\theta)$ times the bundling pricing problem with independent utilities. Thus we just need to convert the solutions from that problem properly.

C. Comparing Partitioned Pricing and Bundling Pricing

As the complementarity introduces more complexity, we focus on the case of symmetric distributions to keep the problem tractable. We assume that u_i follows a uniform distribution between $[0, b]$. The distribution of $u = u_1 + u_2$ becomes

$$\phi(u) = \begin{cases} \frac{u}{b^2}, & \text{if } 0 \le u \le b \\ \frac{2b-u}{b^2}, & \text{if } b < u \le 2b \end{cases}$$

When the prices in the partitioned pricing are greater than b, consumers won't choose to use a single device since that gives them a negative utility surplus. If consumers use devices, they always use both. In other words, the partitioned pricing effectively becomes a bundling pricing. Thus we restrict our attention to the optimal price below b in the partitioned pricing. The partitioned pricing yields a greater profit if and only if it does so with a price below b.

Derived from the first order conditions, the optimal prices in the partitioned pricing and the corresponding maximum profits are

- If $c \le \frac{3}{2} - \frac{(1+\theta)^2}{\theta \cdot l_\theta}$, where $l_\theta = ((1-\theta)(1+\theta)^2 + 4\theta^2)$

$$p_i = (1+\theta)b\sqrt{\frac{\theta}{l_\theta} \cdot \frac{2}{3-2c}} \tag{9}$$

$$\Pi_P = (1+\theta)b\left(\frac{4}{3}\sqrt{\frac{\theta}{l_\theta} \cdot \frac{2}{3-2c}} - c\right) \tag{10}$$

- If $c > \frac{3}{2} - \frac{(1+\theta)^2}{\theta \cdot l_\theta}$,

$$p_i = b\left(\sqrt{\left(\frac{2-c}{3-2c} \cdot \frac{1+\theta}{\theta(3-\theta)}\right)^2 + \frac{(2+3\theta)(1+\theta)}{\theta(3-\theta)(3-2c)}} - \frac{2-c}{3-2c} \cdot \frac{1+\theta}{\theta(3-\theta)}\right) \tag{11}$$

  The profit can be obtained by inserting the optimal price to the objective funciton.

For the bundling pricing, the optimal price and the maximum profit are just $1 + \theta$ times those with independent utilities.

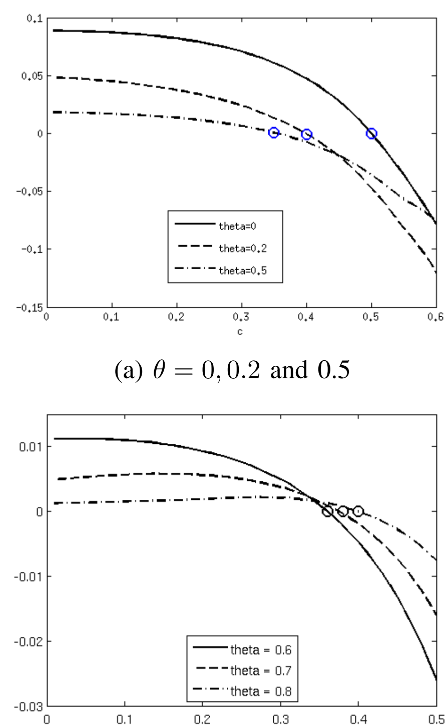
In the comparison of the two pricing schemes, we find that the unit usage cost c alone can no longer determine whether the shared data plans outperform the single device data plans. It's a relationship between c and the scaling factor θ determines that.

Theorem 4: When the usage of the devices follow uniform distribution between $[0, b]$, if $c < \frac{3}{2} - \frac{(1+\theta)^2}{\theta((1-\theta)(1+\theta)^2+4\theta^2)}$

- the maximum profit Π_B in the bundling pricing is greater than that in the partitioned pricing Π_P for the complementary utilities.
- the optimal price p in the bundling pricing is greater than the sum of optimal prices in the partitioned pricing $p_1 + p_2$.

This is a sufficient condition on when the bundling pricing yields a higher profit. The bundling pricing may continue to yield a higher profit when $c \ge \frac{3}{2} - \frac{(1+\theta)^2}{\theta((1-\theta)(1+\theta)^2+4\theta^2)}$. Under this sufficient condition, the optimal price in the bundling pricing is actually greater than the sum of the optimal prices in the partitioned pricing. This is different from the result in the case with independent utilities where the optimal price in the bundling pricing is always lower. This sufficient condition contains a non-empty set of c when θ is large enough. For instance, this sufficient condition is $c < 0.23$ for $\theta = 0.8$. In other words, when the complementary effect is very strong, the bundling pricing can afford to have a high price and still yields a higher profit.

As the complementary effect becomes stronger (θ increases from 0 to 1), intuitively one would also expect that the profit difference between the two pricing schemes would increase, and the threshold on c under which bundling pricing yields a higher profit would become higher. However, our numerical study paints a different picture. Figure 2 shows how the relative profit difference $\frac{\Pi_B - \Pi_P}{\Pi_P}$ changes over c for different values of θ. The small circles indicate where the curves intersect with the X axes. As θ increases from 0 to 0.5, the intersection point moves towards 0 in Figure 2(a). That is, the threshold on the unit usage cost c decreases in θ. The opposite is true when θ increases from 0.6 to 0.8 in Figure 2(b). The relative profit difference tends to decrease in θ in both cases.



(a) $\theta = 0, 0.2$ and 0.5

(b) $\theta = 0.6, 0.7$ and 0.8

Fig. 2. Relative Profit Difference $\frac{\Pi_B - \Pi_P}{\Pi_P}$ ($b = 3$)

As θ increases, consumers becomes increasingly inclined to use both devices in the partitioned pricing. This makes the partitioned pricing problem approach the bundling pricing problem. Thus the relative profit difference decreases. This effect also depresses the threshold on c until the complementary effect is very strong.

V. Conclusion

In this work, we study a monopoly telecom operator's decision on whether to adopt the pricing scheme of shared data plans. When consumers have heterogeneous utilities on each of the two devices they hold and the utilities are independent between the two devices, we identify the unit usage cost as the defining factor and find a threshold on it below which the shared data plan yields more profits than single device data plans. In the shared data plan, the price is less than the sum of the prices of the single device data plans, and the total usage is generally increased. When the usage doesn't cost the operator too much, an increased usage increases the operator's profit since the increased revenue coming with the increased usage outweighs the increase in the usage cost. In this situation, the consumer surplus is also increased, which means the shared data plan is a "win-win" solution for both consumers and the operator. If consumers have rather different utilities on the two devices, an increase in this disparity reduces the profit difference between the shared data plan and single device data plans.

We further study the problem when the utilities on the devices are complementary to each other. The unit usage cost alone can no longer determine whether the shared data plan yields more profits. It's a relationship between the unit usage cost and the scaling factor determines that. We identify a sufficient condition, in the form of a threshold on the unit usage cost as a function of the scaling factor, for the shared data plan to yield more profits. Under this sufficient condition, the price in the shared data plan is larger than the sum of the prices of the single device data plans. This is different from the case with independent utilities where the price in the shared data plan is always lower. Our numerical study shows that an increase in the complementary effect actually makes the single device data plans closer to the shared data plan, since more consumers use both device with the single device data plans.

The devices can also be substitutes for each other instead of being complementary. In this case, a consumer's utility for using both devices is lower than the sum of his utilities on the devices when only one device is used. As one extension to our work, it's interesting to see whether the substitution between the devices has the opposite effect as the complementariness. On one hand, the substitution makes more consumers to use a single device in the single device plans and thus diverges the single device data plans further from the shared data plan. On the other hand, the substitution pushes down the utilities when both devices are used and thus makes the shared data plan less attractive to the operator. The exact outcome of the substitution is then worth investigating.

Our work studies the decisions of a monopoly operator. As future work, the problem with competing operators warrants consideration. Competition among operators brings more dynamics into the problem. A shared data plan helps to recruit and retain consumers and reduce the churn rate. This should lead to an increase in the long-term values of consumers for the operators. Modeling of this effect is critical for studying the problem with competing operators.

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