Information Market for TV White Space

Yuan Luo, Lin Gao, and Jianwei Huang

***Abstract*—We propose a novel information market for TV white space networks, where white space databases sell the *information* regarding the TV channel quality to unlicensed white space devices (WSDs). Different from the traditional spectrum market, the information market demonstrates the *positive network externality*, as more WSDs purchasing the information from a database will increase the value of the database's information to each of its buyers. We study an *oligopoly* information market, where two competitive databases compete to sell their information to WSDs, and WSDs decide whether and from which database to purchase the information. Specifically, we first derive the WSD's optimal purchasing behavior under fixed information prices, and show how the market share of each database dynamically evolves over time. We then characterize the *market equilibrium*, where no WSD has an incentive to change its purchasing behavior. Our analysis indicates that given the prices of databases, there may be multiple market equilibria, and which one actually emerges depends on the initial market shares of both databases. We further show that some equilibria are stable, in the sense that a small fluctuation on the equilibrium will drive the market back to the equilibrium, while some equilibria are not.**

I. Introduction

A. Motivations

TV white space network is a novel and promising paradigm of dynamic spectrum sharing, and can effectively alleviate the spectrum scarcity today [1], [2]. In a TV white space network, unlicensed wireless devices (called white space devices, WSDs) opportunistically exploit the under-utilized broadcast television spectrum (called TV white space, TVWS[1]) via a third-party *geo-location* white space database [3]. Specifically, the white space database is required to house a global repository of TV licensees, and update the licensees' channel occupations periodically. Each WSD, before accessing any TV channel, must query a white space database for the available channels at its current location. Figure 1 illustrates such a database-assistant TV white space network with 3 licensed TV stations and 8 unlicensed WSDs, where WSDs 1 and 2 query the available channel information from database 1, WSDs 3 and 4 query the available channel information from database 2, and WSDs 5 to 8 remain inactive. Such a database-assistant TV white space network architecture has been widely supported by spectrum regulators, standards bodies, and industrial organizations [4]–[10].[2] However, the commercial deployment of such a network requires a proper business model, which offers

This work is supported by the General Research Funds (Project Number CUHK 412713 and CUHK 412511) established under the University Grant Committee of the Hong Kong Special Administrative Region, China.

Authors are with the network communications and economics lab (NCEL), Dept. of Information Engineering, The Chinese University of Hong Kong, E-mail: {ly011, lgao, jwhuang}@ie.cuhk.edu.hk

[1]For convenience, we will refer to TV white space as "TV channel".

[2]In January 2011, FCC has conditionally designated 9 companies including Google [4], Spectrum Bridge [5], and LS Telecom to serve as white space database operators in USA, developing trial white space database systems. Based on these certified databases, several TV white space trial networks [9] and commercial networks [10] have been developed.

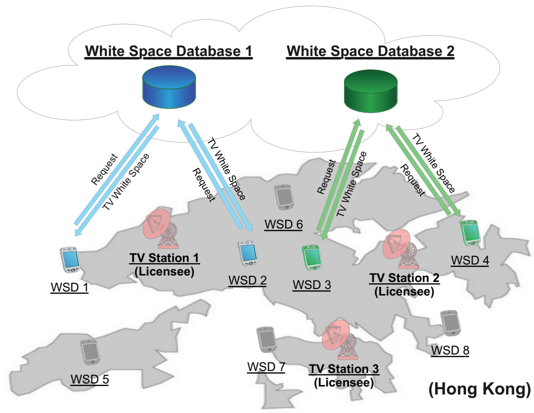

Fig. 1. Illustration of a database-assistant TV white space network. To access a TV channel, each WSD first reports its location to a white space database (request), and then the database returns the available channel list to the WSD.

sufficient economic incentives to the database operators. Such an issue has not been extensively discussed in the existing literature.

The existing business modeling of TV white space network mainly focused on the *spectrum market* [11]–[13], where the database operators, acting as spectrum brokers or agents, sell the TV white spaces to unlicensed WSDs for profit. However, the TV spectrum market model may *not* be suitable in practice due to some regulatory considerations. For example, TV white spaces (especially those not licensed to any licensee) are usually treated as the *public* resources, and designated by regulators for the *public and shared* usage by unlicensed devices. Therefore, TV white spaces may not be freely traded in a spectrum market like other licensed spectrum bands. To this end, a new business model without involving the trading of spectrum is highly desirable.

The world first white space database operator certified by FCC – Spectrum Bridge – proposed an alternative business model called "White Space Plus" [10]. The basic idea is to sell some advanced information (regarding the quality of TV channels) to WSDs, such that the latter can choose and operate on the high quality channels. An example of such information is the degree of interference on every available channel. This essentially leads to an *information market*, where WSDs purchase the information regarding the channel quality from the database, instead of purchasing the channel itself. Clearly, the successful deployment of such an information market requires a deep understanding of the market response and dynamics, which has not been considered in the existing literature. This motivates our study in this paper.

B. Contributions

In this paper, we model and study an *oligopoly* competitive information market with two white space databases. The databases (sellers) compete to sell the following information to WSDs: *the interference level on every TV channel*. The WSDs (buyers) decide whether and from which database to purchase the information. We want to understand the behavior of such an information market, in particular, (i) what is the WSD's optimal purchasing behavior, (ii) how the market share of each database (i.e., the percentage of WSDs purchasing information from the database) dynamically evolves over time, and (iii) what is the stable market shares of both databases (also called *market equilibrium*)? All of these problems are challenging due to the following reasons.

First, there is lack of a unified framework to evaluate the value of information to WSDs. In particular, one database's known information may not be the same as the other's, and neither database has the global information. To this end, we propose a general framework to evaluate the value of information for WSDs. The framework considers not only the potential error of the information provided by databases, but also the heterogeneity of WSDs.

Second, the information market has the property of *positive network externality*, i.e., the more WSDs purchasing information from the same database, the higher value of that database's information for each future buyer. This is quite different from traditional spectrum markets which are usually congestion-oriented, i.e., the more users purchasing and using spectrum, the less value of spectrum for each buyer due to the potential co-interference among users. This positive correlation between the information value and market share complicates the market behavior analysis, as a slight change of one WSD's purchasing behavior may affect the information evaluation and purchasing decisions of other WSDs. We will analytically show how the market share of each database dynamically evolves over time, and eventually converges to a market equilibrium.

In summary, the main contributions are as follows.

- *Novelty and Practical Significance.* To the best of our knowledge, this is the first paper proposing and analyzing an oligopoly competitive information market, considering the positive network externality for TV white space networks. Comparing with the traditional spectrum market model, this information market model better fits the regulatory requirements and industry practice.
- *Market Equilibrium Analysis.* We characterize the equilibrium of the proposed information market systematically. Our analysis indicates that given the prices of databases, there may be multiple market equilibria, and which one will actually emerge depends on the initial market shares of both databases. We further show that some equilibria are stable, in the sense that a small fluctuation on the equilibrium will drive the market back to the equilibrium, while some equilibria are not.
- *Performance Analysis.* We quantify the impact of the databases' initial market shares and information prices on the market equilibrium. Our results show that (i) when the prices of two databases are very different, there is a unique stable market equilibrium independent of the databases' initial market shares, where the lower price database achieves most of the market share and the higher price database only achieves a zero market share; and (ii) when the prices of two databases are similar, there are two stable market equilibria depending on the databases' initial market shares, where the database with the higher initial market share achieves most of the market share at the equilibrium, and the database with the lower initial market share achieves a small market share which is close to zero in our simulations.

The rest of the paper is organized as follows. In Section II, we present the system model. In Sections III and IV, we analyze the WSD's purchase dynamics and the market equilibrium, respectively. Finally, we conclude in Section V.

II. System Model

We consider a TV white space network with two white space databases (denoted by s_1 and s_2) and a set $\mathcal{N} = \{1, ..., N\}$ of unlicensed white space devices (WSDs) operating on idle TV channels, as illustrated in Figure 1. Let $\mathcal{K} = \{1, \ldots, K\}$ denote the set of available TV channels in the area of the network. Each WSD queries a database for the available TV channel set, and can only operate on *one* of the available channels at a particular time.

For each WSD $n \in \mathcal{N}$, each channel k is associated with an *interference level*, denoted by $Z_{n,k}$, which reflects the aggregate interference from all other nearby devices (including TV stations and other WSDs) operating on this channel. Due to the fast changing of wireless channels and the uncertainty of WSDs' mobilities and activities, the interference $Z_{n,k}$ is a random variable. For convenience, we assume that

$Z_{n,k}$ is *temporal-independence* and *frequency-independence*.

That is, (i) the interference $Z_{n,k}$ on channel k is independent identically distributed (iid) at different times, and (ii) the interferences on different channels, $Z_{n,k}, k \in \mathcal{K}$, are also iid at the same time.[3] As we are talking about a general WSD n, **we will omit the WSD index n in the notations (e.g., write $Z_{n,k}$ as Z_k), whenever there is no confusion caused.** Let $F_Z(\cdot)$ and $f_Z(\cdot)$ denote the cumulative distribution function (CDF) and probability distribution function (PDF) of $Z_k, \forall k \in \mathcal{K}$.[4]

White Space Database. According to the regulator's ruling (e.g., FCC [1]), a certified white space database provides the following information for WSDs: (i) the list of available TV channels, (ii) the transmission constraints (e.g., maximum transmission power) on each available channel, and (iii) some other optional requirements. This is the basic information (*basic service*) that every database is mandatory to provide for any WSD free of charge.

Beyond the basic service, the white space database can also provide certain advanced information (*advanced service*) to make a profit, under the constraint that it does not conflict with the basic service. Motivated by the practice of Spectrum Bridge [10], we consider such an advanced service, where each database provides the following advanced information to every WSD n subscribing to its advanced service[5]: $\{Z_k\}_{k\in\mathcal{K}}$ (i.e., the interference level on every available channel for

[3] Note that the iid assumption is a reasonable approximation of the practical scenario, where all channel quality distributions are the same but the realized instant qualities of different channels are different (e.g., [15]–[17]).

[4] In this paper, we will conventionally use $F_X(\cdot)$ and $f_X(\cdot)$ to denote the CDF and PDF of a random variable X, respectively.

[5] "Subscribe to a database" means that the WSDpurchases the advanced information from the database.

this particular WSD). With this advanced information, the WSD is able to operate on the best available channel (with the lowest interference level). Accordingly, each database s_i can charge a *subscription fee* (denoted by π_i) to every WSD subscribing to its advanced service. This essentially constitutes an *information market*.

For convenience, we illustrate the basic service and advanced service by the example in the following table. In the basic service, the database provides the available channel set (i.e., {CH1, CH3, CH4, CH6}) to a particular WSD. In the advanced service, the database provides the interference levels on available channels (i.e., $\{Z_1, Z_3, Z_4, Z_6\}$) to the WSD, in addition to the available channel set.

Channel ID	**CH1**	**CH2**	**CH3**	**CH4**	**CH5**	**CH6**
Availability	Yes	No	Yes	Yes	No	Yes
Interference Level	Z_1	-	Z_3	Z_4	-	Z_6

White Space Devices (WSDs). After obtaining the available channel list through the free basic service, each WSD has 3 choices (denoted by l) in terms of channel selection:

(i) $l \in \{1, 2\}$: subscribing to the database s_l's advanced service, and picking the channel with the lowest interference indicated by database s_l;

(ii) $l = 0$: choosing an available channel randomly;

The *payoff* of WSD equals (i) the benefit (*utility*) from the achieved data rate on the selected channel minus (ii) the *subscription fee* if subscribing to an advanced service. Formally, the payoff of WSD can be defined as:

$$\Pi^{\text{WSD}} = \begin{cases} \theta \cdot U(R_l) - \pi_l, & \text{if } l \in \{1, 2\}, \\ \theta \cdot U(R_0), & \text{if } l = 0, \end{cases} \tag{1}$$

where R_l is the expected data rate when the WSD chooses a strategy $l \in \{0, 1, 2\}$, $U(\cdot)$ is the utility function (concave and increasing) of the WSD, and θ is the WSD's evaluation for the achieved utility. Note that different WSDs may have the different utility evaluation factor θ (e.g., for different applications), that is, WSDs are heterogeneous in term of θ. For the analytical convenience, we assume that θ is uniformly distributed in $[0, 1]$ for all WSDs.

Next we compute the WSD's expected data rate R_l under different strategies $l \in \{0, 1, 2\}$. Specifically, when $l = 0$ (choosing channel randomly), the WSD's expected data rate is

$$R_0 = \mathrm{E}_Z[\mathcal{R}(Z)] = \int_z \mathcal{R}(z) \mathrm{d}F_Z(z), \tag{2}$$

where $\mathcal{R}(\cdot)$ is the transmission rate function (e.g., the Shannon capacity) under any given interference.

It is notable that under the strategy $l \in \{1, 2\}$ (subscribing to the database s_l's advanced service), the WSD's expected data rate R_l cannot be directly computed, as it depends on the accuracy of the database s_l's information. We will provide more details regarding the computation of R_l in (6).

Interference Level (Information). For a particular WSD, its experienced interference Z_k on a channel k is the aggregate interference from all other (nearby) devices operating on channel k, and usually consists of the following three components:

1) L_k: the interference from licensed TV stations;
2) $W_{k,m}$: the interference from another WSD m operating on the same channel k;
3) I_k: any other interference from outside systems.

The total interference on channel k is $Z_k = L_k + W_k + I_k$, where $W_k \triangleq \sum_{m \in \mathcal{N}_k} W_{k,m}$ is the total interference from all other WSDs operating on channel k (denoted by $\mathcal{N}_k$). Similar to Z_k, we assume that $L_k, W_k, W_{k,m}$, and I_k are random variables with *temporal-independence* (i.e., iid across time) and *frequency-independence* (i.e., iid across frequency). We further assume that $W_{k,m}$ is *user-independence*, i.e., $W_{k,m}, m \in \mathcal{N}_k$, are iid. It is important to note that **different WSDs may experience different interferences L_k (from TV stations), $W_{k,m}$ (from another WSD), and I_k (from outside systems) on a channel k, as we have omitted the WSD index n for all these notations for clarity.**

Next we discuss these interferences in more details.

- Each database is able to compute the interference L_k from TV stations to every WSD (on channel k), as it knows the locations and channel occupancies of all TV stations.
- Each database cannot compute the interference I_k from outside systems, due to the lack of outside interference source information. Thus, the interference I_k will *not* be included in a database's information sold to WSDs.
- Each database may or may not be able to compute the interference $W_{k,m}$ from another WSD m, depending on whether WSD m subscribes to the database's advanced service. Specifically, if WSD m subscribes to the advanced service, the database can predict its channel selection (since the WSD is fully rational and will always choose the channel with the lowest interference level indicated by the database at the time of subscription), and thus can compute its interference to any other WSD. However, if WSD m does not subscribe to the advanced service, the database cannot predict its channel selection, and thus cannot compute its interference to other WSDs.

For convenience, we denote $\mathcal{N}_k^{[l]}, l \in \{1, 2\}$, as the set of WSDs operating on channel k and subscribing to the database s_l's advance service (i.e., those choosing the strategy $l \in \{1, 2\}$), and $\mathcal{N}_k^{[0]}$ as the set of WSDs operating on channel k and *not* subscribing to any advance service (i.e., those choosing the strategy $l = 0$). That is, $\mathcal{N}_k^{[1]} \bigcup \mathcal{N}_k^{[2]} \bigcup \mathcal{N}_k^{[0]} = \mathcal{N}_k$. Then, for a particular WSD, its experienced interference (on channel k) **known by database** s_l is

$$Z_k^{[l]} \triangleq L_k + \sum_{m \in \mathcal{N}_k^{[l]}} W_{k,m}, \tag{3}$$

which contains the interference from TV licensees and all WSDs (operating on channel k) subscribing to the database s_l's advanced service. The WSD's experienced interference (on channel k) ***not* known by database** s_l is

$$\widetilde{Z}_k^{[l]} \triangleq I_k + \sum_{m \in \mathcal{N}_k^{[0]}} W_{k,m} + \sum_{m \in \mathcal{N}_k^{[i]}, i \neq l} W_{k,m}, \tag{4}$$

which contains the interference from outside systems and all WSDs (operating on channel k) not subscribing to the database s_l's advanced service. Obviously, both $\widetilde{Z}_k^{[l]}$ and $Z_k^{[l]}$ are also random variables with temporal- and frequency-independence. Accordingly, the total interference on channel k for a WSD can be written as $Z_k = Z_k^{[l]} + \widetilde{Z}_k^{[l]}$.

Since the database s_l knows only $Z_k^{[l]}$, it will provide this information (instead of the total interference Z_k) as

the advanced service to a subscribing WSD. It is easy to see that the more WSDs subscribing to the database s_l's advanced service, the more information the database s_l knows, and the more accurate the database s_l's information will be.

Next we can characterize the accuracy of a database's information explicitly. Due to the frequency independence assumption, it is reasonable to assume that each channel $k \in \mathcal{K}$ will be occupied by an average of $\frac{N}{K}$ WSDs. Let η_l denote the percentage of WSDs subscribing to the advanced service of database s_l (called the *market share* of database s_l). Then, among all $\frac{N}{K}$ WSDs operating on channel k, there are, *on average*, $\frac{N}{K}\cdot\eta_l$ WSDs subscribing to the database s_l's advanced service, and $\frac{N}{K}\cdot(1-\eta_1-\eta_2)$ WSDs not subscribing to any advanced service. That is, $|\mathcal{N}_k| = \frac{N}{K}$, $|\mathcal{N}_k^{[l]}| = \frac{N}{K}\cdot\eta_l, \forall l \in \{1,2\}$, and $|\mathcal{N}_k^{[0]}| = \frac{N}{K}\cdot(1-\eta_1-\eta_2)$.[6] Finally, by the user-independence of $W_{k,m}$, we can immediately calculate the distributions of $Z_k^{[l]}$ and $\widetilde{Z}_k^{[l]}$ under any given market share η_l via (3) and (4).

Information Value. Now we evaluate the value of database s_l's information to WSDs, which is reflected by the WSD's benefit (utility) that can be achieved from this information.

We first consider the expected utility of a WSD when not subscribing to any advanced service (i.e., $l=0$). In this case, the WSD will randomly select a channel from the available channel list, and its expected utility is

$$B \triangleq U(R_0) = U\left(\textstyle\int_z \mathcal{R}(z)\mathrm{d}F_Z(z)\right), \tag{5}$$

where R_0 is the expected data rate given in (2). Obviously, B depends only on the distribution of the total interference Z_k, while not on the specific distributions of $Z_k^{[l]}$ and $\widetilde{Z}_k^{[l]}$. This implies that the accuracy of the database s_l's information does not affect the utilities of those WSDs not subscribing to its advanced service.

Then we consider the expected utility of a WSD when subscribing to the database s_l's advance service (i.e., $l \in \{1,2\}$). In this case, the database s_l returns the interference $\{Z_k^{[l]}\}_{k\in\mathcal{K}}$ to the WSD, together with the basic information such as the available channel list. For a rational WSD, it will always choose the channel with the minimum $Z_k^{[l]}$ (since $\{\widetilde{Z}_k^{[l]}\}_{k\in\mathcal{K}}$ are iid). Let $Z_{\text{MIN}}^{[l]} = \min\{Z_1^{[l]},\ldots,Z_K^{[l]}\}$ denote the minimum interference indicated by the database s_l's information. Then, the actual interference experienced by a WSD (subscribing to the database s_l's advanced service) can be formulated as the sum of two random variables, denoted by $Z^{[l]} = Z_{\text{MIN}}^{[l]} + \widetilde{Z}$. Accordingly, the WSD's expected data rate and expected utility under the strategy $l \in \{1,2\}$ can be computed by

$$\begin{aligned} R_l &= \mathrm{E}_{Z^{[l]}}\left[\mathcal{R}\left(Z^{[l]}\right)\right] = \textstyle\int_z \mathcal{R}(z)\mathrm{d}F_{Z^{[l]}}(z), \\ A_l &\triangleq U\left(R_l\right) = U\left(\textstyle\int_z \mathcal{R}(z)\mathrm{d}F_{Z^{[l]}}(z)\right), \end{aligned} \tag{6}$$

where $F_{Z^{[l]}}(z)$ is the CDF of $Z^{[l]}$. It is easy to see that both R_l and A_l depend on the distributions of $Z_k^{[l]}$ and $\widetilde{Z}_k^{[l]}$, and thus depend on the market share η_l. Thus, we will write A_l as $A_l(\eta_l)$. We can further check that $A_l(\eta_l)$ increases with η_l.

Problem Formulation. Suppose that each WSD is rational, and will always choose the strategy that maximizes its payoff.

[6]Note that the above discussion is from the aspect of expectation, and in a particular time period, the realized numbers of WSDs in different channels may be different.

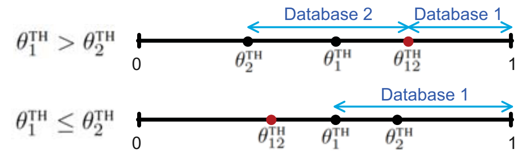

Fig. 2. Illustration of θ_1^{TH}, θ_2^{TH}, and θ_{12}^{TH} when $\eta_1^0 > \eta_2^0$.

We are interested in the following problems: given the information prices $\{\pi_1,\pi_2\}$ and initial market shares $\{\eta_1^0,\eta_2^0\}$ of databases, how these market shares dynamically evolve and what is the market equilibrium?

III. WSD Subscription Dynamics

In this section, we will study the WSD subscription dynamics in an information market. Specifically, we will first show what is the WSD's optimal subscription choice given the initial market shares of databases. Then we will show how the databases' market shares dynamically evolve, and eventually converge to a market equilibrium.

A. WSD's Best Subscription Choice

We first consider the optimal choice of a WSD given the information prices $\{\pi_1,\pi_2\}$ and the initial market shares $\{\eta_1^0,\eta_2^0\}$ where $\eta_1^0+\eta_2^0 \le 1$. Notice that each WSD is rational and will always choose the strategy that maximizes its payoff. Hence, for a type-θ WSD, it will (i) subscribe to the database s_1's advanced service if and only if (iff)[7]

$$\theta\cdot A_1 - \pi_1 > \max\{\theta\cdot A_2 - \pi_2,\ \theta\cdot B\},$$

(ii) subscribe to the database s_2's advanced service iff

$$\theta\cdot A_2 - \pi_2 > \max\{\theta\cdot A_1 - \pi_1,\ \theta\cdot B\},$$

and (iii) not subscribe to any database's advanced service iff

$$\theta\cdot B > \max\{\theta\cdot A_1 - \pi_1,\ \theta\cdot A_2 - \pi_2\}.$$

where $A_1 = A_1(\eta_1^0)$ and $A_2 = A_2(\eta_2^0)$ defined in (6).

Based on the WSDs' best choices, we can compute the new derived market shares η_1 and η_2 of databases. For convenience, we introduce the following notaitons:

$$\theta_i^{\text{TH}} = \tfrac{\pi_i}{A_i - B}, \quad \text{and} \quad \theta_{ij}^{\text{TH}} = \tfrac{\pi_i-\pi_j}{A_i-A_j}, \quad i,j\in\{1,2\}.$$

Intuitively, θ_i^{TH} represents the smallest θ such that a type-θ WSD prefers the advanced service of database s_i than the basic service, and θ_{ij}^{TH} represents the smallest θ such that a type-θ WSD prefers the advanced service of database s_i than the advanced service of database s_j.

Suppose $\eta_1^0 > \eta_2^0$ (and thus $A_1 > A_2$). For clarity, we illustrate the values of θ_1^{TH}, θ_2^{TH}, and θ_{12}^{TH} in Figure 2. Specifically, when $\theta_1^{\text{TH}} > \theta_2^{\text{TH}}$, we can easily check that $\theta_{12}^{\text{TH}} > \theta_1^{\text{TH}}$ as shown in the upper subfigure. Then, the WSDs with $\theta\in(0,\theta_2^{\text{TH}})$ will not subscribe to any database, the WSDs with $\theta\in(\theta_2^{\text{TH}},\theta_{12}^{\text{TH}})$ will subscribe to database s_2, and the WSDs with $\theta\in(\theta_{12}^{\text{TH}},1)$ will subscribe to database s_1. Similarly, when $\theta_1^{\text{TH}} \le \theta_2^{\text{TH}}$, we can check that $\theta_{12}^{\text{TH}} \le \theta_1^{\text{TH}}$ as shown in the lower subfigure. Then, the WSDs with $\theta\in(0,\theta_1^{\text{TH}})$ will not subscribe to

[7]Note that we omit the case of $\theta\cdot A_1 - \pi_1 = \max\{\theta\cdot A_2 - \pi_2,\ \theta\cdot B\}$, which is negligible (i.e., occurring with zero probability) due to the continuous distribution of θ. The same is applicable to the following two conditions.

any database, the WSDs with $\theta \in (\theta_1^{\mathrm{TH}}, 1)$ will subscribe to database s_1. Accordingly, the new derived market shares $\{\eta_1, \eta_2\}$ of two databases are

- If $\theta_1^{\mathrm{TH}} > \theta_2^{\mathrm{TH}}$, then $\eta_1 = 1 - \theta_{12}^{\mathrm{TH}}$ and $\eta_2 = \theta_{12}^{\mathrm{TH}} - \theta_2^{\mathrm{TH}}$;
- If $\theta_1^{\mathrm{TH}} \le \theta_2^{\mathrm{TH}}$, then $\eta_1 = 1 - \theta_1^{\mathrm{TH}}$ and $\eta_2 = 0$.

The case of $\eta_1^0 < \eta_2^0$ (hence $A_1 < A_2$) is symmetric to the above case and we omit the analysis. Formally, we summarize the above derived market shares in the following lemma.

Lemma 1: If $\eta_1^0 > \eta_2^0$, the derived market shares are

$$\begin{aligned} \eta_1 &= \max\left\{1 - \max\{\theta_{12}^{\mathrm{TH}}, \theta_1^{\mathrm{TH}}\},\ 0\right\}, \\ \eta_2 &= \max\left\{\min\{\theta_{12}^{\mathrm{TH}}, 1\} - \theta_2^{\mathrm{TH}},\ 0\right\}. \end{aligned} \tag{7}$$

If $\eta_1^0 < \eta_2^0$, the derived market shares are

$$\begin{aligned} \eta_1 &= \max\left\{\min\{\theta_{21}^{\mathrm{TH}}, 1\} - \theta_1^{\mathrm{TH}},\ 0\right\}, \\ \eta_2 &= \max\left\{1 - \max\{\theta_{21}^{\mathrm{TH}}, \theta_2^{\mathrm{TH}}\},\ 0\right\}. \end{aligned} \tag{8}$$

The results in Lemma 1 assume that all WSDs change their choices simultaneously. Note that both θ_i^{TH} and $\theta_{ij}^{\mathrm{TH}}$ are functions of the initial market shares η_1^0 and η_2^0 (as A_1 and A_2 are functions of η_1^0 and η_2^0). Thus, the derived market shares η_1 and η_2 are also functions of η_1^0 and η_2^0, and can be written as $\eta_1(\eta_1^0, \eta_2^0)$ and $\eta_2(\eta_1^0, \eta_2^0)$.

B. WSD Subscription Dynamics

Notice that when the databases' market shares change according to Lemma 1, the information structure and interference distribution will also change according to (3) and (4). This will affect the values of A_1, A_2, and B, and hence affect the WSDs' evaluations for the databases' information. As a result, WSDs may have incentives to change their subscription choices again based on these new values. Therefore, the market share will dynamically evolve, until it reaches a stable market share (called *market equilibrium*), where no WSD has the incentive to change its subscription choice (and thus the market share no longer changes). In what follows, we will study such a WSD's subscription dynamics *given the information prices*.

For convenience, we introduce a virtual time-discrete system with slots $t = 1, 2, ..., T$, where WSDs change their subscription decisions at the beginning of every slot, based on the derived market share in the previous slot. Consider a particular slot t. The market shares $(\eta_1^{t-1}, \eta_2^{t-1})$ achieved in the previous slot $t-1$ serve as the initial market shares, and the derived market share (η_1^t, η_2^t) in the current slot t is given by Lemma 1. Let $\triangle\eta_1$ and $\triangle\eta_2$ denote the changes (dynamics) of market shares between two successive time slots t and $t-1$, i.e.,

$$\triangle\eta_1 = \eta_1^t - \eta_1^{t-1}, \quad \triangle\eta_2 \ = \eta_2^t - \eta_2^{t-1}. \tag{9}$$

A positive (negative) $\triangle\eta_i$ implies that the market share η_i will increase (decrease) along the dynamics. Note that $\triangle\eta_i$ is a function of η_1^{t-1} and η_2^{t-1}. Hence, we can write $\triangle\eta_i$ as $\triangle\eta_i(\eta_1^{t-1}, \eta_2^{t-1}), \forall i \in \{1, 2\}$.

A *market equilibrium* is defined as a fixed point of the market shares. In other words, if an equilibrium market share is achieved, it will not change any more. Formally,

Lemma 2 (Market Equilibrium): A pair of market shares $\boldsymbol{\eta}^* = \{\eta_1^*, \eta_2^*\}$ is a market equilibrium, if and only if

$$\triangle\eta_1(\eta_1^*, \eta_2^*) = 0, \quad \text{and} \quad \triangle\eta_2(\eta_1^*, \eta_2^*) = 0. \tag{10}$$

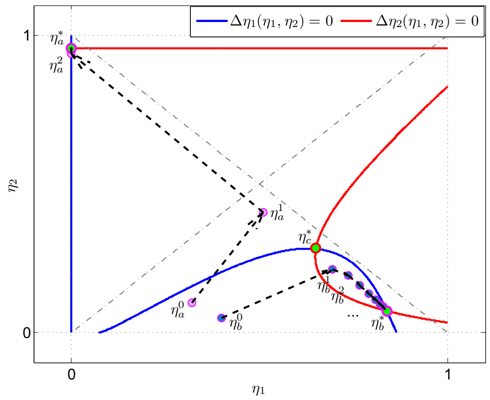

Fig. 3. Dynamics of the market shares η_1 and η_2. From the initial market shares $\boldsymbol{\eta}_a^0 = \{0.3, 0.1\}$, the market shares will gradually evolve to $\boldsymbol{\eta}_a^1$, $\boldsymbol{\eta}_a^2$, ... , and eventually achieves the equilibrium $\boldsymbol{\eta}_a^*$ located on the top-left corner. From the initial market shares $\boldsymbol{\eta}_b^0 = \{0.4, 0.05\}$, the market shares will evolve to the equilibrium $\boldsymbol{\eta}_b^*$ located on the bottom-right corner.

We illustrate the dynamics of market shares in Figure 3. The blue curve denotes the isoline of $\triangle\eta_1 = 0$, and the red curve denotes the isoline of $\triangle\eta_2 = 0$. By Lemma 2, the intersections between the blue curve and red curve are the market equilibria. In this example, there are three equilibrium points $\boldsymbol{\eta}_a^*$, $\boldsymbol{\eta}_b^*$, and $\boldsymbol{\eta}_c^*$. From this figure, we can see that given the information prices of databases, there may be multiple equilibria, and which will eventually emerge depends on the initial market shares of databases. For example, from the initial market shares $\boldsymbol{\eta}_a^0 = \{0.3, 0.1\}$, the market shares will change following the route $\boldsymbol{\eta}_a^0 \to \boldsymbol{\eta}_a^1 \to \boldsymbol{\eta}_a^2 \to \ldots \to \boldsymbol{\eta}_a^*$ as shown by the dash arrow. From the initial market shares $\boldsymbol{\eta}_b^0 = \{0.4, 0.05\}$, the market shares will change following $\boldsymbol{\eta}_b^0 \to \boldsymbol{\eta}_b^1 \to \boldsymbol{\eta}_b^2 \to \ldots \to \boldsymbol{\eta}_b^*$. Note that no initial market shares other than $\boldsymbol{\eta}_c^*$ will converge to the market equilibrium $\boldsymbol{\eta}_c^*$. In fact, the equilibria $\boldsymbol{\eta}_a^*$ and $\boldsymbol{\eta}_b^*$ are stable, in the sense that a small fluctuation on the equilibrium will drive the market back to the equilibrium. The equilibrium $\boldsymbol{\eta}_c^*$ is not.

IV. Market Equilibrium

In the previous section, we have shown how the market shares dynamically evolve to a market equilibrium given the information prices. In this section, we will further characterize the market equilibrium under different information prices.

We illustrate the market evolution under different information prices $\{\pi_1, \pi_2\}$ in Figure 4. For a better illustration, we fix the database s_1's price π_1 in all three subfigures, while increase the database s_2's price π_2 from 0 to π_1 in the three subfigures.[8] Similar to Figure 3, the blue curve denotes the isoline of $\triangle\eta_1 = 0$, the red curve denotes the isoline of $\triangle\eta_2 = 0$, and the intersection between the blue curve and red curve denote the market equilibria. The gray arrows denote the market share changing directions at any given market share.[9] It is easy to see that with the increase of π_2, the blue arc and red arc in the bottom-right area become closer, and finally intersect

[8]The case of $\pi_2 > \pi_1$ is totally symmetric to the case of $\pi_1 > \pi_2$. Thus, we skip the analysis for the case of $\pi_2 > \pi_1$ due to the space limit.

[9]Note that a feasible market share pair $\{\eta_1, \eta_2\}$ implies that $\eta_1 + \eta_2 \le 1$. Thus, in Figure 4, only the evolutions from the points below $\eta_1 + \eta_2 = 1$ are meaningful, while the evolutions from the points above $\eta_1 + \eta_2 = 1$ are meaningless. Nevertheless, we draw all evolutions for a clear illustration.

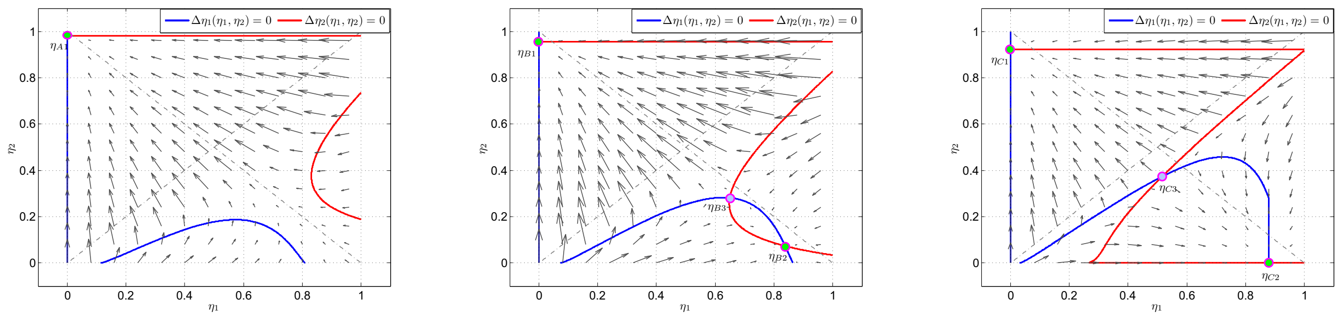

Fig. 4. Market evolution under (a) $\pi_2 \in [0, \pi_{\rm A}]$, (b) $\pi_2 \in (\pi_{\rm A}, \pi_{\rm B}]$, and (c) $\pi_2 \in (\pi_{\rm B}, \pi_1]$, where $\pi_{\rm A} < \pi_{\rm B} < \pi_1$.

(in the middle and right two subfigure). For convenience, let $\pi_{\rm A}$ denote the database s_2's price such that the red arc is just tangent to the blue arc, and $\pi_{\rm B}$ denote the database s_2's price such that the red arc just intersects with the line of $\eta_2 = 0$.

(a) $\pi_2 \in [0, \pi_{\rm A}]$. In this case, there is a unique market equilibrium, denoted by $\boldsymbol{\eta}_{A1}$. The gray arrows (directions) show that any initial market share will evolve to the market equilibrium $\boldsymbol{\eta}_{A1}$. This implies that a small fluctuation on the equilibrium $\boldsymbol{\eta}_{A1}$ will drive the market back to the equilibrium $\boldsymbol{\eta}_{A1}$. Thus, $\boldsymbol{\eta}_{A1}$ is a stable equilibrium.

(b) $\pi_2 \in (\pi_{\rm A}, \pi_{\rm B}]$. In this case, there are three market equilibria, denoted by $\boldsymbol{\eta}_{B1}$, $\boldsymbol{\eta}_{B2}$, and $\boldsymbol{\eta}_{B3}$. The gray arrows show that an initial point with a larger initial market share for database s_1 (or s_2) will more likely evolve to the equilibrium $\boldsymbol{\eta}_{B1}$ (or $\boldsymbol{\eta}_{B2}$). Moreover, a small fluctuation on the equilibrium $\boldsymbol{\eta}_{B1}$ (or $\boldsymbol{\eta}_{B2}$) will drive the market back to the equilibrium $\boldsymbol{\eta}_{B1}$ (or $\boldsymbol{\eta}_{B2}$). Thus, $\boldsymbol{\eta}_{B1}$ and $\boldsymbol{\eta}_{B2}$ are stable market equilibria. This subfigure further show the equilibrium $\boldsymbol{\eta}_{B3}$ is unstable, as a slight fluctuation on $\boldsymbol{\eta}_{B3}$ will drive the market to the equilibrium $\boldsymbol{\eta}_{B1}$ or $\boldsymbol{\eta}_{B2}$.

(c) $\pi_2 \in (\pi_{\rm B}, \pi_1]$. In this case, there are three market equilibria, denoted by $\boldsymbol{\eta}_{C1}$, $\boldsymbol{\eta}_{C2}$, and $\boldsymbol{\eta}_{C3}$. Similarly, $\boldsymbol{\eta}_{C1}$ and $\boldsymbol{\eta}_{C2}$ are stable, while $\boldsymbol{\eta}_{C3}$ is unstable. The difference between (c) and (b) is as follows. In (b), the database s_2 will achieve a small positive market share at the equilibrium $\boldsymbol{\eta}_{B2}$, while in (c), the database s_2 will achieve a zero market share at equilibrium $\boldsymbol{\eta}_{C2}$.

In summary, we have the following theorem for the stable market equilibrium.

Theorem 1: The stable market equilibrium is given by

(a) If $\pi_2 \in [0, \pi_{\rm A}]$, there is a unique stable equilibrium $\boldsymbol{\eta}_{A1} = \{\eta_1^*, \eta_2^*\}$, where $\eta_1^* = 0$, and η_2^* satisfies

$$1 - \theta_2^{\rm TH}(\eta_2^*) - \eta_2^* = 0; \tag{11}$$

(b) If $\pi_2 \in (\pi_{\rm A}, \pi_{\rm B}]$, there exist two stable market equilibria $\boldsymbol{\eta}_{B1} = \{\eta_1^*, \eta_2^*\}$ and $\boldsymbol{\eta}_{B2} = \{\eta_1^\dagger, \eta_2^\dagger\}$, where the equilibrium $\boldsymbol{\eta}_{B1}$ is: $\eta_1^* = 0$ and η_2^* satisfies

$$1 - \theta_2^{\rm TH}(\eta_2^*) - \eta_2^* = 0, \tag{12}$$

and the equilibrium $\boldsymbol{\eta}_{B2}$ satisfies

$$\begin{cases} 1 - \theta_{12}^{\rm TH}(\eta_1^\dagger, \eta_2^\dagger) - \eta_1^\dagger = 0, \\ \theta_{12}^{\rm TH}(\eta_1^\dagger, \eta_2^\dagger) - \theta_2^{\rm TH}(\eta_2^\dagger) - \eta_2^\dagger = 0; \end{cases} \tag{13}$$

(c) If $\pi_2 \in (\pi_{\rm B}, \pi_1]$, there exist two stable equilibria $\boldsymbol{\eta}_{C1} = \{\eta_1^*, \eta_2^*\}$ and $\boldsymbol{\eta}_{C2} = \{\eta_1^\dagger, \eta_2^\dagger\}$, where the equilibrium $\boldsymbol{\eta}_{C1}$ is: $\eta_1^* = 0$ and η_2^* satisfies

$$1 - \theta_2^{\rm TH}(\eta_2^*) - \eta_2^* = 0, \tag{14}$$

and the equilibrium $\boldsymbol{\eta}_{C2}$ is: $\eta_2^\dagger = 0$, and $\eta_1^\dagger$ satisfies

$$1 - \theta_1^{\rm TH}(\eta_1^\dagger) - \eta_1^\dagger = 0. \tag{15}$$

V. Conclusion

In this paper, we study an oligopoly competitive information market for TV white space networks, where white space database operators sell the interference information to WSDs. We analyze the WSDs' subscription dynamics and the market equilibrium systematically. One future direction is to study the databases' optimal pricing decisions and analyze the price competition game between databases. Our market equilibrium analysis in this paper can serve as the first step of analyzing the databases' price competition game.

References

[1] FCC 10-174, *Second Memorandum Opinion and Order*, 2010.
[2] OFCOM, *TV White Spaces - A Consultation on White Space Device Requirements*, Nov. 2012.
[3] Ofcom, *Implementing Geolocation*, Nov. 2010.
[4] Google Spectrum Database, http://www.google.org/spectrum/whitespace/
[5] Spectrum Bridge TV White Space, [Online] http://www.spectrumbridge.com/ProductsServices/WhiteSpacesSolutions/WhiteSpaceOverview.aspx
[6] IEEE 802.22 WRAN, [Online] http://www.ieee802.org/22/
[7] Whitespace Alliance, [Online] www.whitespacealliance.org
[8] Microsoft Reserach WhiteFiServiice, http://whitespaces.msresearch.us/
[9] Microsoft Report, *Cambridge TV White Space Trial: A Summary of the Technical Findings*, April. 2012.
[10] Spectrum Bridge White Space Plus, http://www.spectrumbridge.com/ProductsServices/WhiteSpacesSolutions/OperatorsUsers.aspx
[11] Y. Luo, L. Gao, and J. Huang, "Spectrum Broker by Geo-location Database", *IEEE GLOBECOM*, 2012.
[12] X. Feng, J. Zhang, and J. Zhang, "Hybrid Pricing for TV White Space Database," *IEEE INFOCOM*, 2013.
[13] Y. Luo, L. Gao, and J. Huang, "White Space Ecosystem: A Secondary Network Operator's Perspective", *IEEE GLOBECOM*, 2013.
[14] M. Gladwell, *The Tipping Point: How Little Things Can Make a Big Difference*, Back Bay Books, Jan. 2002.
[15] N. B. Chang, and M. Liu, "Optimal Channel Probing and Transmission Scheduling for Opportunistic Spectrum Access", *IEEE/ACM Transactions on Networking*, vol. 17, no. 6, pp. 1805-1818, Dec. 2009.
[16] H. Jiang, L. Lai, R. Fan, and H. V. Poor, "Optimal selection of channel sensing order in cognitive radio", *IEEE Transactions on Wirieless Communications*, vol. 8, no. 1, pp. 297-307, Jan. 2009.
[17] H. Zhou, P. Fan, D. Guo, "Joint Channel Probing and Proportional Fair Scheduling in Wireless Networks", *IEEE Transactions on Wirieless Communications*, vol. 10, no. 10, pp. 3496-3505, Oct. 2011.