

Joint Pricing and Proactive Caching for Data Services: Global and User-centric Approaches

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Abstract—In this work, we investigate the profit maximization problem of a network service provider through smart pricing and proactive data services. The demand characteristics of each user are dependent on the price and willingness-to-pay values of each service. By learning these characteristics, the service provider can further improve its profit performance through a *proactive* service of the predictable demand so as to smooth-out its load dynamics over time, and reduce the incurred cost. We formulate the joint *price* and *proactive download* allocation problem and study its impact on the expected user payments and the service provider profit. In particular, we show that proactive downloads can only enhance the expected profit of service provider and at the same time reduce the expected payments by the user, when compared with the no-proactive-service regime. The problem is studied from two perspectives: global optimization, and game theory. From the global optimization perspective, the problem is shown to be non-convex, yet an algorithm that yields a local optimal solution with better profit than the no-proactive-download scenario is developed. From the game theoretical perspective, the problem is posed as a coordination game with the user and the service provider as players. Best response dynamics are shown to converge to a Nash Equilibrium (NE) of the game, which is the local optimal solution achieved by the developed non-convex optimization algorithm.

I. INTRODUCTION

It is well-reported that network service providers have been experiencing a major peak-to-average demand problem over the course of the day. Throughout the daytime, demand levels raise substantially to approach the network capacity and cause excessive service costs and undesirable congestions. On the other hand, at the nighttime, the demand significantly drops to minimum levels rendering the network resources severely underutilized [1], [2]. Such a demand disparity is ultimately tied to the user behavioral pattern which consistently shows that users are very active during the peak hour, and idle at the off-peak time [3]–[5].

Some emerging solutions such as the cognitive radio approach [6] claim to provide an enhanced degree of resource utilization during the off-peak hour. The concept is facilitated by allowing secondary users with cognitive radio capabilities to sense and access underutilized bands with the incentive of less payment as compared to the primary users. However, it is expected that the cognitive radio approach will offer only a partial solution to the problem due to the fact that it conflicts with the natural user activity pattern, that is, the majority of the users will not comply with the pricing incentives that essentially aim to change their activity characteristics.

As the peak-to-average demand problem became more severe, the pricing aspect of the cognitive radio approach has been later generalized to include primary users as well. In fact, some network operators outside the USA (such as Orange, MTN and Uninor) have already started using adaptive pricing schemes to mitigate the excessive cost resulting from high bandwidth consumption [9]–[11]. The main approach is to adjust the service prices depending on the total network load in a way that assigns low prices to off-peak services and higher prices to peak-hour demand. There has been other attempts to formulate and study such a dynamic assignment of service prices in [12], whereby the economic responsiveness of the users has been utilized through discount incentives to defer the peak network load towards the off-peak time.

However, these schemes rely on the assumption that users are willing to change their activity profile based on the network prices, which seems a rather strong assumption since users activities are tied to non-flexible constraints such as work, school, and sleep times. Thus, there is an inherent need for a different paradigm of smart pricing that does not only facilitate the best utilization of the network resources, but reasonably accounts for the user's ability and willingness to comply with certain incentive strategies.

In a recent work [7], we have proposed a different approach for price usage in data networks. In particular, pricing has been concurrently employed with proactive data services [8] to minimize the time average cost incurred by the service providers. The primal use of pricing incentives has been to enhance the certainty about which service that the user will ask for. Hence, pricing has not conflicted with the user activity. The high certainty about the future demand allowed for an efficient employment of *proactive* data services, a scheme through which predictable peak-hour demand is served *ahead of time* (mainly during the off-peak hour) to smooth-out the network loads and minimize the expected service provider costs. In that work, however, the impact of pricing allocation on the expected profit of the service provider has not been studied.

In this paper, we extend our investigations to account for the profit performance of the service provider while joint pricing and proactive services are harnessed to maximize such a profit. Similar to our previous work, prices are not intended to change the activity level of the user throughout any period of the day, instead, they can only change the likelihood of preferring one service over the other. We mathematically show that the employment of proactive services with smart pricing will (1)

strictly increase the expected service provider profit, and (2) strictly decrease the expected payments made by the user, when compared with the no-proactive-service counterpart.

We formulate and study the profit maximization problem from convex optimization and game theoretical perspectives. We show that the problem is non-convex, and develop an algorithm that converges to a local optimal solution to it. The same problem will also be viewed as a coordination game with the service provider and the user being the players. The service provider strategies involve price assignment, and the user strategies are the proactive service requests. We show that repeated best reply dynamics converge to a Nash Equilibrium (NE) of the game which is the local optimal solution obtained through the developed non-convex optimization algorithm.

The rest of this paper is organized as follows. In Section II, we layout the system setup and define the characteristics of its main components. We formulate the problem and study its finite and infinite time horizon duality in Section III. In Section IV, we propose and study a centralized algorithm that tackles the non-convexity of the problem. Subsequently, a user-centric version of that algorithm is presented in Section V, and the expected payment is also studied. Numerical results are provided in Section VI, and the paper is concluded in Section VII.

II. SYSTEM MODEL

We consider a service provider that has a total of M different data items/services that each of its users can request in a random fashion (to be modeled via the *demand profile* of that user). Each data item m is assumed to require $S > 0$ resources to be served. While the service provider responds to requests from all users, we focus our analysis on the single user interactions for the ease of exposition. Nevertheless, our results can directly be generalized to the N user case with potentially coupled contribution to the cost function, as in [7].

In a time-slotted system, the content of data item m is consistently updated every time slot, where such a content could be a movie (as in YouTube and Netflix), a soundtrack (as in Pandora), a social network update (as in Facebook and Twitter), a news update (as in CNN and Fox News), etc. We consider the application-layer timescale in which the duration of a time slot is the time taken for a user to completely consume the requested data item. This covers timescales of minutes or possibly hours. At the beginning of each time slot, the service provider collects the user demand and supplies the requested data item which has been updated over the previous time slot.

User's demand profiles: We assume that the user demand can be tracked, learned, and predicted by the service provider over time. The service provider constructs a *demand profile* for the user at every time slot t , denoted $\mathbf{p}_t = (p_t(m))_{m=1}^M$, where $p_t(m)$ is the probability that the user requests item m in slot t . Then, we model the statistics of the predictable user demands as follows:

- The user demand at slot t is captured by a random variable $\mathbb{I}_t(m)$ where

$$\mathbb{I}_t(m) = \begin{cases} 1, & \text{with probability } p_t(m), \\ 0, & \text{with probability } 1 - p_t(m). \end{cases}$$

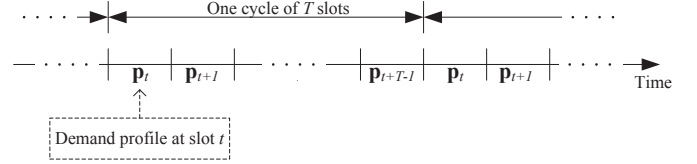


Fig. 1: Cyclostationary demand profiles over time. Every T time slots, the user demand profile is repeated.

- For any t , $\mathbb{I}_t(m)$ is independent of $\mathbb{I}_{t+1}(j)$ for all m, j .
- At slot $t \geq 0$, the user requests at most one data item, i.e., $\sum_{m=1}^M \mathbb{I}_t(m) \leq 1$.
- The probability that the user does not request any data item at slot t is $q_t := 1 - \sum_{m=1}^M p_t(m)$.

Further, the demand profile of the user is assumed to follow a *cyclostationary* pattern that repeats itself consistently in a period of T time slots, as shown in Fig. 1. That is, we can write $\mathbf{p}_{t+l} = \mathbf{p}_{t+kT+l}$ for any non-negative integer k , and $l = 0, \dots, T-1$. As an example, the T -slot period can be interpreted as a single day through which the activity of each user varies each hour, but occurs with the same statistics each day.

Pricing of data items: Define $y_t(m) \in \mathbb{R}_+$ as the price of data item m offered to the user at time slot t . The probability that the user requests item m at time slot t is modeled as $p_t(m) := \phi_{m,t}(y_t(m))$, where $\phi_{m,t} : \mathbb{R}_+ \rightarrow [0, 1]$ is a non-negative function that maps the pricing policy for such a user into a corresponding probability of requesting this item m at time slot t . The function $\phi_{m,t}$ captures the influence of the offered prices on the preferences of the user over the M data items. The assigned price for item m , however, can not be chosen to exceed a predefined quantity $v(m)$ which represents the willingness-to-pay value by the user. That is, $\phi_{m,t}(y) = 0$ for any $y > v(m)$. The willingness-to-pay value is assumed to be known to the service provider for every item m . The user is interested in consuming a specific item if its assigned price is below $v(m)$.

We will focus our analysis on the demand profile function $\phi_{m,t}(y)$ that *linearly* decreases in the price y . While the linearity assumption will enable a convex constraint set, it also agrees with the proposed linear model in [12] which has captured the user responsiveness to the pricing discounts. Hence, we formally consider

$$\phi_{m,t}(y_t(m)) := (1 - q_t) \frac{v(m) - y_t(m)}{D_t}, \quad \forall m, t, \quad (1)$$

where D_t is a normalizing constant.

Incurred cost: To supply the requested data items, the service provider incurs a certain cost due to the resources consumed at each time slot. Letting L denote the amount of resources consumed at a given slot, the aggregate cost incurred by the service provider is given by $C(L)$, where $C : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a *smooth, strictly convex, and monotonically increasing* cost function with $C(0) = 0$. We assume also that the total load at a given time slot is *not observable* by the service provider, an assumption that is justified by practical data delivery scenarios whereby the user requests arrive to a widespread content delivery network associated with the service provider,

rendering the observability of the total load centrally at the outset of a slot impractical.

With the introduced notation, the total load encountered at slot t for a *non-proactive* network is given by

$$L_t := S \sum_{m=1}^M \mathbb{I}_t(m), \quad t \geq 0. \quad (2)$$

Next, we formulate the time-average profit maximization problem for the service provider that utilizes proactive data download.

III. PROBLEM FORMULATION AND BASIC OBSERVATIONS

In the proactive design, the service provider can send potential user demand ahead of time in order to minimize its expected incurred cost, but at the risk of possibly not receiving the predicted request from the user. Further, it can use new pricing policies to slightly change the demand profiles and render them more deterministic, and hence enhance the efficiency of the proactive downloads. Yet, its ultimate objective is to maximize its expected profit. These motivate us to formulate the *joint pricing and proactive download problem* next.

In our view of a *proactive network*, we denote by $x_{t+1}(m)$ the portion of data item m to be consumed at time $t+1$ that is sent ahead to the user at time slot t , $m = 1, \dots, M$, $t = 0, 1, \dots$.

Moreover, the service provider is allowed to *slightly* modify the demand profile of each user at every time slot, so as to strike a balance between enhancing the *predictability* of future demands, and maximizing its expected profit.

The employed pricing strategies are considered to affect only the user preferences over the data items based on the willingness-to-pay values \mathbf{v} , and do not affect the user activity. That is, the modified demand profile under any pricing strategy always satisfies $q_t = 1 - \sum_{m=1}^M p_t(m)$, as q_t captures the user inactivity over time.

The **joint pricing and proactive download profit maximization problem** is defined as

$$\begin{aligned} \pi^* := & \underset{\{\mathbf{y}_t\}_t, \{\mathbf{x}_t\}_t}{\text{maximize}} \quad \liminf_{\tau \rightarrow \infty} \frac{1}{\tau} \sum_{t=0}^{\tau-1} \pi(\mathbf{x}_t, \mathbf{y}_t) \\ \text{subject to} \quad & 0 \leq x_t(m) \leq S, \quad \forall m, t, \\ & 0 \leq y_t(m) \leq v(m), \quad \forall m, t \\ & \phi_{m,t}(y_t(m)) = \phi_{m,kT+t}(y_{kT+t}(m)), \quad \forall m, k, t \\ & \sum_{m=1}^M \phi_{m,t}(y_t(m)) = 1 - q_t, \quad t = 0, \dots, \end{aligned} \quad (3)$$

where the optimization is jointly performed over the prices, and proactive downloads. The profit function π captures the difference between the expected payment made by the user, and the expected incurred cost in a given time slot. Exact expression for π is given in (4).

The first constraint set captures the fact that proactive downloads can not be negative or exceed the size of the whole data item. The second set of constraints implies that the assigned prices do not exceed the willingness-to-pay values, since otherwise users will not request the data item. The third and fourth constraints sets ensure that the user will maintain

his cyclostationary demand pattern with unaffected activity performance (measured by q_t) under the new pricing scheme.

Now, we turn to the question of the existence of a *one-cycle* steady-state solution. The cyclostationary nature of the demand profiles and the unobservability of the instantaneous load suggest the existence of a steady-state solution to (3) which is summarized in the following theorem.

Theorem 1: The joint pricing and proactive download problem can be optimally solved through a one cycle optimization of (3) with the following additional constraints:

$$\begin{aligned} \mathbf{x}_t &= \mathbf{x}_{kT+t}, \quad \forall k, t \geq 0, \\ \mathbf{y}_t &= \mathbf{y}_{kT+t}, \quad \forall k, t \geq 0, \end{aligned}$$

Proof. The proof follows by Jensen's inequality and the strict convexity of the cost function C . First, it shows a globally optimal solution $(\mathbf{x}_t^*, \mathbf{y}_t^*)_t$ exists, and $\mathbf{x}_t^* = \mathbf{x}_{t+kT}^*$. Then, the best pricing response to it is also periodic. Full proof is detailed in [17]. ■

As such, only one cycle of the time average profit maximization is tantamount to the optimal infinite-horizon time average. In the following section we investigate the convexity of the one-cycle optimization problem and compare the optimal expected profit with that obtained under no proactive downloads.

IV. CENTRALIZED UTILITY MAXIMIZATION

In this section, we address the centralized optimization of joint pricing and proactive downloads from a convex optimization perspective, where the service provider is to make a centralized optimization decision. The one-cycle profit maximization problem takes on the following form.

$$\pi^* := \underset{\{\mathbf{x}_t, \mathbf{y}_t\}_{t=0}^{T-1}}{\text{maximize}} \quad \frac{1}{T} \sum_{t=0}^{T-1} \pi(\mathbf{x}_t, \mathbf{y}_t) \quad (5)$$

$$\begin{aligned} \text{subject to} \quad & 0 \leq x_t(m) \leq S, \quad \forall m, t, \\ & 0 \leq y_t(m) \leq v(m), \quad \forall m, t \end{aligned} \quad (6)$$

$$\phi_{m,T}(y_t(m)) = \phi_{m,0}(y_0(m)), \quad \forall m, k, t$$

$$\sum_{m=1}^M \phi_{m,t}(y_t(m)) = 1 - q_t, \quad t = 0, \dots, T-1$$

With the linear dependence of $\phi_{m,t}$ on y , the last constraint in (5) reduces to $\frac{1}{M} \sum_{m=1}^M y_t(m) = \bar{y}_t$, which is an average per-item price constraint with $\bar{y}_t := \sum_m v(m) - D_t$. Thus, the resulting constraint set will become convex since it involves only affine functions of the optimization variables $(\mathbf{x}_t, \mathbf{y}_t)_t$.

A. Non-convexity of the Problem

By considering the objective function $f_0(\mathbf{x}, \mathbf{y}) := \frac{1}{T} \sum_{t=0}^{T-1} \pi(\mathbf{x}_t, \mathbf{y}_t)$, with the profit function π defined in (4), it follows that the objective function is non-concave. The parameters \mathbf{x} and \mathbf{y} are vectors of all the assigned proactive services and prices, respectively. The non-concavity of objective function is due to the products of functions of the

$$\pi(\mathbf{x}_t, \mathbf{y}_t) = \sum_m y_t(m) \phi_{m,t}(y_t(m)) - \mathbb{E} \left[C \left((S - x_t(m)) \mathbb{I}_t(m) + \sum_j x_{t+1}(j) \right) \right]. \quad (4)$$

optimization variables \mathbf{x} and \mathbf{y} that result from the expected cost expression [13], which is given by

$$\eta(\mathbf{x}, \mathbf{y}) := \frac{1}{T} \sum_{t=0}^{T-1} q_t C \left(\sum_{j=1}^M x_{t+1}(j) \right) + \sum_{m=1}^M \left((S - x_t(m)) + \sum_{j=1}^M x_{t+1}(j) \right) \phi_{m,t}(y_t(m)). \quad (7)$$

With the optimization (5) having a non-concave objective function, obtaining an optimal solution to it is a computationally intractable problem. However, it can be noted that for a fixed value of \mathbf{x} , the objective function f_0 is concave¹ in \mathbf{y} , and for a fixed \mathbf{y} , the objective function is concave² in \mathbf{x} . Accordingly, if we consider the price allocation under *no-proactive downloads*, i.e., $\mathbf{x} = \mathbf{0}$, as a **baseline** for our smart-pricing technique, we will develop an iterative algorithm that always yields a strictly larger profit than the *baseline*'s.

B. Iterative Successive Approximation

We denote by $\tilde{\mathbf{y}}$ the optimal pricing solution to the baseline problem with no proactive downloads. Since the point $(\mathbf{0}, \tilde{\mathbf{y}})$ does not necessarily satisfy the KKT conditions for (5) [13], then a point $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ which satisfies $f_0(\hat{\mathbf{x}}, \hat{\mathbf{y}}) > f_0(\mathbf{0}, \tilde{\mathbf{y}})$, as well as the KKT conditions of (5), can be obtained through the following iterative procedure to approximate it with convex problems.

Lemma 1: Let \hat{f}^k be a concave function in (\mathbf{x}, \mathbf{y}) that replaces the objective function f_0 of (5) at iteration k . Denote by $(\mathbf{x}^{k-1}, \mathbf{y}^{k-1})$ the optimal solution to the resulting convex optimization problem at the $(k-1)^{th}$ iteration, $k = 1, 2, \dots$. If

- 1) $\hat{f}^k(\mathbf{x}, \mathbf{y}) \leq f_0(\mathbf{x}, \mathbf{y})$ for all feasible (\mathbf{x}, \mathbf{y}) ,
- 2) $\nabla \hat{f}^k(\mathbf{x}^{k-1}, \mathbf{y}^{k-1}) = \nabla f_0(\mathbf{x}^{k-1}, \mathbf{y}^{k-1})$,
- 3) $\hat{f}^k(\mathbf{x}^{k-1}, \mathbf{y}^{k-1}) = f_0(\mathbf{x}^{k-1}, \mathbf{y}^{k-1})$,

then $f_0(\mathbf{x}^{k-1}, \mathbf{y}^{k-1}) < f_0(\mathbf{x}^k, \mathbf{y}^k)$, $\forall k$, and the sequence $\{(\mathbf{x}^k, \mathbf{y}^k)\}_k$ converges to a point $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ which is a locally optimal solution to (5).

The above lemma is a special case of Theorem 1 in [14] which aims at providing local optimal solutions to non-convex optimization problems.

Corollary 1: Starting from the baseline initial point $(\mathbf{x}^0, \mathbf{y}^0) = (\mathbf{0}, \tilde{\mathbf{y}})$, a sequence of approximate functions $\{\hat{f}^k\}$ generated as in Lemma 1 and resulting in a KKT-satisfying point $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ leads to $f_0(\mathbf{0}, \tilde{\mathbf{y}}) < f_0(\hat{\mathbf{x}}, \hat{\mathbf{y}})$.

In the following theorem, we use this general construction to provide a particular approximation to f_0 of (5) at each new iteration k that satisfies the requirements and thus converges to a locally optimal solution.

Theorem 2: For f_0 being the objective function of (5), the approximate function

$$\hat{f}^k(\mathbf{x}, \mathbf{y}) = f_0(\mathbf{x}^{k-1}, \mathbf{y}) + f_0(\mathbf{x}, \mathbf{y}^{k-1}) \quad (8)$$

at iteration $k \geq 1$ is concave in (\mathbf{x}, \mathbf{y}) . Further, the sequence of solutions to the problem resulting from replacing f_0 with $\{\hat{f}^k\}_k$ converges to a locally optimal solution of (5).

Proof. Omitted for brevity, yet we apply the above three conditions to the proposed approximate function. ■

The approximate function (8), can now be used to replace f_0 of (5) at iteration $k \geq 1$. Starting with the baseline no-proactive downloads strategy, $(\mathbf{x}^0, \mathbf{y}^0) = (\mathbf{0}, \tilde{\mathbf{y}})$, the successive solutions to approximate optimization problems with f_0 being replaced by \hat{f}^k of (8) converges to a point $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ with $f_0(\hat{\mathbf{x}}, \hat{\mathbf{y}}) > f_0(\mathbf{0}, \tilde{\mathbf{y}})$.

It is worth noting that the above algorithm can start from other initial points than the baseline non-proactive solution, and will always yield a better profit performance than that it started from. For instance, service providers that apply flat prices with proactive downloads can apply the algorithm and strictly enhance their profit with optimized smart prices and corresponding proactive services.

In the following section, we consider a decentralized approach to the problem whereby the user can decide on the proactive download values in response to the pricing decisions made by the service provider.

V. USER-CENTRIC APPROACH

In this section, we investigate the scenario when the proactive download decisions are carried out by the user so as to minimize the expected cost it creates at the service provider end. While this cooperation apparently helps improve the service provider's profit, we will show that it will nevertheless *reduce* the expected payments made by the user. To that end, we view the profit maximization problem as a two-player game with the user and the service provider as its players. The strategy set for the user is \mathcal{X} which contains all proactive download vectors \mathbf{x} that satisfy the first set of constraints in (5). On the other hand, the strategy set for the service provider is \mathcal{Y} and consists of all pricing vectors \mathbf{y} satisfying the rest of constraints.

Suppose that the user device can track and learn its own demand profile $\mathbf{p} = \{\mathbf{p}_t\}_t$, yet it does not necessarily know the dependence of \mathbf{p} on the pricing strategy \mathbf{y} (i.e., $\phi_{m,t}$ need not be known at the user). However, the user is interested in minimizing the expected cost it creates at the service provider for the sake of reduced expected payments. Hence, we adopt the negative of the expected service provider cost, $-\eta(\mathbf{x}, \mathbf{y})$ (defined in (7)), as the profit function for the user for a fixed \mathbf{y} . The service provider, however, is interested in maximizing its expected profit $f_0(\mathbf{x}, \mathbf{y}) = \mu(\mathbf{y}) - \eta(\mathbf{x}, \mathbf{y})$ for a fixed \mathbf{x} ,

¹Quadratic with a negative definite Hessian.

²This follows by the convexity of the cost function C .

where

$$\mu(\mathbf{y}) := \frac{1}{T} \sum_{t=0}^{T-1} \sum_{m=1}^M y_t(m) \frac{(v(m) - y_t(m))(1 - q_t)}{D_t} \quad (9)$$

is the average expected payment by the user. Next, we study the existence of a NE to the game and investigate its achievability.

A. Iterative Best-response Dynamics

Since the expected payment μ does not explicitly depend on the proactive download \mathbf{x} , we can extend the utility function for the user to $f_0(\mathbf{x}, \mathbf{y})$ without affecting its unilateral decisions \mathbf{x} in response to the offered prices \mathbf{y} . With this update, we can now see that both players have the same utility functions, yet with different strategy spaces. Hence the game reduces to a *coordination game*, with at least one NE point in pure strategies (c.f. Theorem 17 [15]).

In addition, the structure of the objective function implies that the game is convex, as the utility function for each player is concave in its strategy space, given the strategy played by the other. Such convexity suggests that *best response* dynamics converge to a local optimal point of the utility function f_0 , and this point is also a NE of the game (c.f. Theorem 20 in [15]). The best response dynamics involve a repeated version of the game where players take turns responding to each other's decisions. Thus, at iteration $k \geq 0$ if the service provider is to determine the best pricing policy to the user's strategy in the previous iteration, \mathbf{x}^{k-1} , it should solve $\max_{\mathbf{y} \in \mathcal{Y}} f_0(\mathbf{x}^{k-1}, \mathbf{y})$. In the next iteration, $k + 1$, the user optimizes its proactive download requests given \mathbf{y}^k , that is $\max_{\mathbf{x} \in \mathcal{X}} f_0(\mathbf{x}, \mathbf{y}^k)$.

The resulting sequence of objective functions satisfy $f_0(\mathbf{x}^{k-1}, \mathbf{y}^k) < f_0(\mathbf{x}^{k+1}, \mathbf{y}^k) < f_0(\mathbf{x}^{k+1}, \mathbf{y}^{k+2}), \dots$, until convergence to a local optimal point of f_0 .

Lemma 2: Starting from $(\mathbf{x}^0, \mathbf{y}^1) = (\mathbf{0}, \tilde{\mathbf{y}})$, the best response dynamics will converge to $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$, the optimal solution achieved through the successive approximation algorithm developed in the previous section.

Proof. Follows by noting that the approximate function \hat{f}^k in (8) yields the best response decisions for the preceding iteration's optimal solution. ■

This result, therefore, shows that the user device can monitor and learn its own demand profile corresponding to a certain pricing strategy, and accordingly adjust the proactive download decisions, and report them to the service provider. The service provider in turn will supply these proactive downloads and update a new pricing policy which maximizes its profit. The iterations will proceed until the two parties converge to an equilibrium with an enhanced utility function for the service provider.

Next, we show that the user also benefits from the profit maximization process as its expected payments will eventually decrease below the no-proactive-download baseline.

B. Reduced Expected Payment

In this subsection we study the effect of proactive downloads on the expected payment, μ . We first establish the statement

that proactive downloads can not yield a larger expected payment than the no-proactive download scenario. Subsequently, we characterize the necessary and sufficient conditions for the expected payment to be strictly less than the that of the no-proactive downloads. To that end, we maintain the same notation that $\tilde{\mathbf{y}}$ is optimal pricing under no proactive downloads, and $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ is locally optimal pricing and proactive download solution to the profit maximization problem.

Theorem 3: The expected payment by the user under the best response dynamics satisfies $\mu(\hat{\mathbf{y}}) \leq \mu(\tilde{\mathbf{y}})$.

Proof. Since $f_0(\mathbf{0}, \tilde{\mathbf{y}}) = \max_{\mathbf{y} \in \mathcal{Y}} \mu(\mathbf{y}) - \eta(\mathbf{0}, \mathbf{y})$ with $\eta(\mathbf{0}, \mathbf{y}) = \sum_{t,m} C(S) \phi_{m,t}(y_t(m))$, the constraint that $\sum_m \phi_{m,t}(y_t(m)) = 1 - q_t$ implies $\eta(\mathbf{0}, \mathbf{y}) = C(S) \sum_t (1 - q_t)$ is independent of \mathbf{y} . In this case, profit maximization will reduce to maximizing the expected payment. Therefore, $\mu(\tilde{\mathbf{y}})$ is the maximum expected payment by the user. ■

The user will be charged the maximum possible payment when the service provider can not modify its load dynamics over time. With the proactive downloads, however, the potentially smoothed-out load characteristics will never yield extra payments on the user. The following theorem characterizes an even stronger result.

Theorem 4: Suppose that $\max_m v(m) > \min_m v(m)$, then the expected payment satisfies $\mu(\hat{\mathbf{y}}) < \mu(\tilde{\mathbf{y}})$ if and only if

$$\phi_{m_0, t_0}(\tilde{y}_{t_0}(m_0)) > 1 - q_{t_0-1}, \quad (10)$$

for some time slot t_0 , and data item m_0 .

Proof. (Sketch) The proof relies on the mean value theorem to compare the expected cost under proactive downloads with that of $\mathbf{x} = \mathbf{0}$. It shows that (10) is necessary and sufficient to have $\hat{\mathbf{x}} > \mathbf{0}$, and that $\hat{x}_{t_0}(m_0) \neq \hat{x}_{t_0}(m)$ for at least one item m . Full proof will be provided in [16]. ■

Remark 1: In Theorem 4, suppose that $\min_m v(m) \geq |2\bar{y}_{t_0} - \bar{v}|$, where $\bar{v} = \frac{1}{M} \sum_m v(m)$. Then Condition (10) reduces to

$$\frac{1}{M} + \frac{1}{2} \cdot \frac{\max_m v(m) - \bar{v}}{D_{t_0}} > \frac{1 - q_{t_0-1}}{1 - q_{t_0}}. \quad (11)$$

The requirement on $\min_m v(m)$ facilitates a full characterization of $\tilde{\mathbf{y}}$ as a function of $(v(m))_m$ and D_t .

The left hand side of (11) is the probability of requesting the service with the maximum willingness-to-pay value, given that the user is going to request a service at time t_0 . Thus, it is required to have this probability greater than the ratio between its activity at the previous slot $t_0 - 1$ over the activity at t_0 .

VI. NUMERICAL RESULTS

We consider a simple setup of $T = 5$ time slots with the user inactivity captured by $\mathbf{q} = (q_t)_t = (0.03, 0.9, 0.02, 0.01, 0.9)$. The number of data items is $M = 3$, the data item size is $S = 100$, the willingness-to-pay values are $(2.5, 2.33, 2.25)$ for the three items, and the average price per item $\bar{y}_t = 2$ for all slots. We consider a quadratic cost function $C(L) = BL^2$ with $B = 2 \times 10^{-4}$.

We run the best response solution starting from the no proactive downloads initial point $(\mathbf{0}, \tilde{\mathbf{y}})$. Convergence results of the expected profit as a function of the players strategies are plotted versus the iteration number.

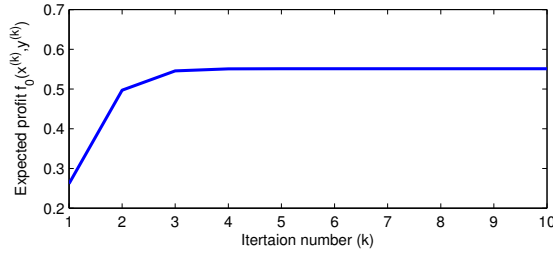


Fig. 2: Evolution of expected profit of best response dynamics.

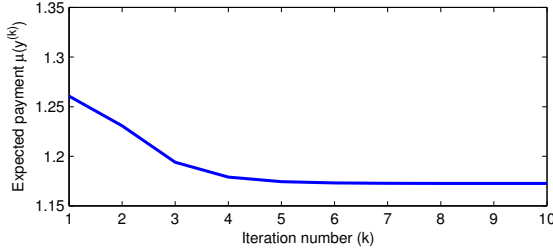


Fig. 3: Decreasing payments under proactive downloads.

Fig. 2 depicts the evolution of the expected profit with the iteration number. Clearly, the sequence of resulting best response strategies yields an increasing profit until convergence at about twice the profit of no-proactive downloads baseline.

In Fig. 3, we plot the convergence of the expected user payment. The result confirms Theorem 4, since the NE payment is strictly less than the initial baseline. Moreover, the figure reveals a monotonically decreasing trend for the expected payment. This fact reveals that the payments are implicitly utilized to improve the proactive download performance, that is, reduced prices for some items enhance the certainty of the user demand at the peak-hour slots, hence proactive downloads can substantially mitigate the excessive peak costs.

To further elaborate on the effect of the pricing strategy on the certainty of the user demand, we evaluate the entropy of the normalized profile $\psi_{m,t} = \phi_{m,t}/(1 - q_t)$ and plot it versus the iteration number. The value of $\psi_{m,t}$ captures the probability of requesting item m at time t conditioned on the event that the user requests at least one item at time t . Fig. 4 plots the entropy of the normalized demand profile for all slots $t = 0, \dots, 4$ with the iteration number. The peak-hour slots 0 and 2 witness reduced entropy values to improve the proactive download performance. Note that, no proactive downloads are

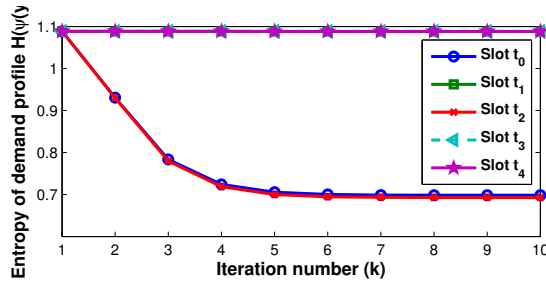


Fig. 4: Entropy of the normalized demand profiles.

applied for slot 3, since it is preceded by a higher activity slot. The off-peak slots 1 and 4 are not assigned proactive download values too.

VII. CONCLUSION

In this work we have addressed the optimal allocation of data prices and proactive downloads for users with predictable demand characteristics. We have formulated the problem of profit maximization from the service provider's perspective, with the objective function involves the expected data delivery costs subtracted from the expected revenue. We have assumed that the user activity at any period of the day is preserved under the pricing strategy used, yet pricing different data items based on their willingness-to-pay values can change the distribution of the item to be requested. Under this setup, we have developed centralized and user centric algorithms that achieve a local optimal value of the profit function that is strictly larger than the no-proactive download scenario. Further, we have proven that the resulting expected payment by the user will be less than the no-proactive case. This step is believed to highlight the significance of proactive data services for predictable demand to smooth-out the network traffic, without changing the user activity pattern over the course of the day.

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