Combining Cooperation and Storage for the Integration of Renewable Energy in Smart Grids

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***Abstract*—Two different techniques for the integration of renewable energy in smart grid systems are considered. The first is storage, which smooths the fluctuations of the renewable energy generation across time. The second is the concept of distributed generation combined with cooperation by exchanging energy among the distributed sources. This technique leads to energy aggregation from diverse sources, which averages out the variation in the energy production across space. The trade-off present between the two techniques is analyzed, and the optimal combination of storage and cooperation to achieve a certain grid performance is investigated. The problem is formulated as a stochastic optimization problem with the objective of minimizing the time average cost of energy exchange within the grid. First, an analytical model of the optimal cost is provided by investigating the steady state of the system for some specific scenarios. Then, an algorithm to solve the cost minimization problem using the technique of Lyapunov optimization is developed. The algorithm is implemented on the renewable energy data provided by National Renewable Energy Laboratory (NREL) of the United States. The results show that in the presence of limited storage devices, the grid can benefit greatly from cooperation, whereas in the presence of large storage, cooperation does not yield much benefit.**

I. Introduction

The time varying and intermittent nature of renewable energy causes significant challenges in its integration into the electricity grid. Solution techniques such as the use of fast-ramping fuel-based generators as a backup, and the use of large batteries to store energy are not cost effective with a high penetration of renewable energy.

Another useful technique to average out the fluctuations of renewable energy that has been explored relatively less, is to exploit the diversity in energy harvested across geographically distributed energy sources. Studies have shown that aggregating the diverse renewable resources from geographically distributed areas leads to a substantial reduction in the load variability [1]. In terms of analytical results, the impact of aggregation of wind power has been considered in the framework of coalitional game theory in [2]. Distributed energy production has also been studied within the framework of *micro-grids* (MGs) [3], with focus on distributed storage and decentralized control of MG networks [4],[5]. Energy sharing among MGs has also been studied in [6] by simulations, and shown to reduce energy losses in the network.

In terms of analytical studies, while techniques such as the use of storage and demand-response have been studied extensively, the combination of the dual averaging effect produced by storage and cooperation by energy sharing has not been explored. The objective of this work is to provide an analytical framework for studying the trade-off present between storage and cooperation. We analyze the optimal combination of storage and cooperation to achieve a certain level of grid performance.

We consider a power grid consisting of spatially separated MGs that are powered by harvesting renewable energy and are serving their respective loads, and a macro-grid (typically a utility company that can provide energy in case of shortage). The MGs are equipped with finite capacity storage devices. The energy harvested by the MGs might not always meet the demand. In such a scenario, MGs can cooperate by exchanging the excess harvested energy. Any further deficits can be fulfilled by drawing energy from the macro-grid. Typically, the distance between the MGs is much less than their distance to the macro-grid and hence, the cost of energy exchange between the MGs is lower. Moreover, the MGs can be connected by a short distance DC power line which incurs much less energy loss than the AC power line connection between the MGs and the macro-grid [7]. The objective of our problem is to minimize the time average cost of energy exchange within the entire grid.

We provide a centralized solution to this problem and exhibit the trade-off between storage and cooperation in reducing this cost. We model the battery as a virtual energy queue, and use queuing theoretic techniques to analyze this problem. We first provide an analytical characterization of the optimal cost of energy exchange by examining the steady state behavior of the system for a symmetric system set up consisting of two MGs and a macro-grid. Then, for the general case, we provide an online algorithm to solve the optimization problem using the technique of Lyapunov optimization. The Lyapunov optimization based relaxation technique is mainly motivated by Urgaonkar et al. [8], which studies storage as a means to reduce the cost in an electricity market with time varying price. Unlike that study, we consider the cost minimization with distributed energy sources, and exploit the gains brought about by cooperation in addition to storage.

We validate the results by applying our algorithm on renewable energy data provided by the National Renewable Energy Laboratory (NREL) of the United States. Our result shows that when the storage capacity is low, cooperation among the

978-1-4799-3088-3/14/$31.00

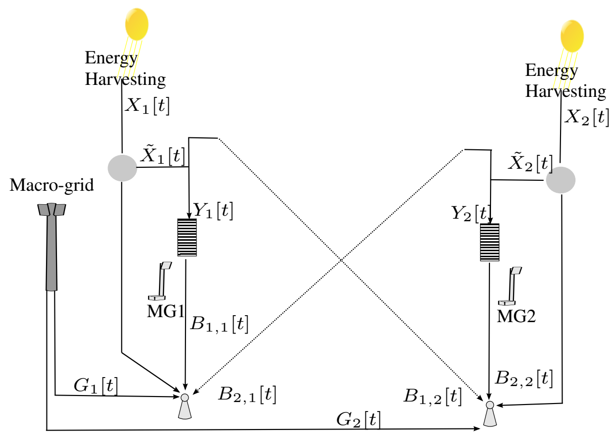

Fig. 1. Power grid consisting of micro-grids and a macro-grid.

MGs yields a significant reduction in the time averaged cost of energy exchange. Interestingly, most of the gains are obtained by cooperation among only a few neighboring MGs. However, when the MGs have a large storage capacity, cooperation does not yield much benefit. This is because each MG can simply store all its excess harvested energy and use it during the time slots when it is deficient.

II. System Model

We consider an inter-connected power grid consisting of N MGs and a macro-grid as shown in Figure 1. Each MG is capable of harvesting renewable energy (e.g. wind, solar, etc). In addition, the MGs are equipped with batteries in which they can store the harvested energy for future use.

Load Serving: Each MG_i serves a set of users whose aggregate energy[1] demand is denoted by $L_i[t]$ ($\leq L_{\max}$, where $L_{\max}$ is a deterministic bound on the load) units of energy per time slot. This energy demand is met in the following manner.

Firstly, each MG_i harvests an amount of energy denoted by $X_i[t],\ i = 1, \ldots, N$ units per time slot t. We assume that the harvesting process $X_i[t]$ evolves according to an independent and identically distributed (i.i.d.) random process across time. The harvested energy is used to serve the load $L_i[t]$. We consider two cases as follows:

• If $X_i[t] < L_i[t]$, then MG_i uses all the harvested energy to serve its load. The unsatisfied load is denoted by $\tilde{L}_i[t] = (L_i[t] - X_i[t])^+$, and the MG_i does the following to serve the unsatisfied load:

1. Draw energy stored in its own battery: The MG_i uses $B_{i,i}[t]$ units of energy from the energy stored in its own battery.
2. Exchange energy among the MGs: In addition, MG_i can borrow $B_{j,i}[t]$ units of energy from MG_j with $j \neq i$, such that
$$B_{j,i}[t] \leq B^{\text{ex}}_{\max} \qquad \forall j, i, \tag{1}$$
where $B^{\text{ex}}_{\max}$ denotes the maximum amount of energy that can be exchanged between the MGs during a given time slot. Note that $B_{j,i}[t] > 0$ only when $\tilde{X}_j[t] = (X_j[t] - L_j[t])^+ > 0$, i.e., MG_j has excess harvested energy beyond its demand. We also consider that transferring energy from MG_j to MG_i incurs a cost of $p_{j,i}[t]$ units per unit of energy transferred. Also, we assume $p_{i,j}[t] \leq p_{\max}\ \forall i, j, t$ for some finite $p_{\max}$.
3. Buy energy from the macro-grid: In case the energy from the battery and the energy borrowed from neighboring MGs is insufficient to satisfy the demand, MG_i can borrow $G_i[t]$ ($\leq G_{\max}$, for some finite $G_{\max}$) units of energy from the macro-grid. We assume that transferring energy from the macro-grid incurs a cost of $q_i[t]$ units per unit of energy transferred. Also, we assume $q_i[t] \leq q_{\max}\ \forall i, t$ for some finite $q_{\max}$.

The sum of energy drawn from the battery, energy exchange with the neighboring MGs, and the energy borrowed from the macro-grid must satisfy the residual demand, i.e.,
$$B_{i,i}[t] + \sum_{j \neq i} B_{j,i}[t] + G_i[t] = \tilde{L}_i[t],\ \forall t,\ i = 1, \ldots, N. \tag{2}$$

• Now we consider the second case, in which the harvested energy exceeds the energy demand, i.e., if $X_i[t] \geq L_i[t]$, then the MG does the following:

1. As previously mentioned, it can donate an amount $B_{i,j}[t]$ to satisfy the load of MG_j.
2. Store an amount $Y_i[t] \leq Y_{\max}$ (where $Y_{\max} < \infty$) in its own battery to be used at a later time. Accordingly, at each time t, we have
$$Y_i[t] + \sum_{j \neq i} B_{i,j}[t] \leq \tilde{X}_i[t] \qquad \forall t,\ i = 1, \ldots, N. \tag{3}$$

Note that in this work, we assume that the excess renewable energy from MG_i can be exchanged with MG_j only to satisfy the load of MG_j during the same time slot. In other words, MG_j cannot use the excess renewable energy of MG_i to charge its battery. Further, we assume that the energy stored in the battery of MG_i can only be used to serve its own load during a future time slot, and there is no energy exchange possible from the battery. This is done so as to clearly exhibit the trade-off between cooperation (by exchanging energy in the current time slot with neighboring MGs) and storing energy for future use.

Modeling the Battery: Next, we consider the energy model of the battery at each MG. At MG_i, the battery evolves according to the following rule:
$$E_i[t+1] = E_i[t] - B_{i,i}[t] + Y_i[t]\ \forall t,\ i = 1, \ldots, N, \tag{4}$$
where the energy availability constrains the battery at each MG_i to satisfy $B_{i,i}[t] \leq E_i[t]\ \forall t,\ i = 1, \ldots, N$. The energy availability constraint combined with the battery discharge constraint yields
$$B_{i,i}[t] \leq \min(E_i[t], B^s_{\max})\ \ \forall t,\ i = 1, 2, \ldots, N, \tag{5}$$
where $B^s_{\max}$ is the maximum energy discharge from the battery during every time slot. Similarly the battery input energy constraint $Y_i[t] \leq Y_{\max}$ can be combined with the battery capacity constraint $E_i[t] \leq E_{\max}$ to yield
$$Y_i[t] \leq \min(E_{\max} - E_i[t], Y_{\max})\ \ \forall t,\ i = 1, 2, \ldots, N. \tag{6}$$

[1]With slight abuse of terminology, we use the terms power and energy interchangeably.

Additionally, we make the following practical assumption on the battery capacity:

$$E_{\max} > Y_{\max} + B^s_{\max}, \tag{7}$$

i.e., the battery size is at least larger than the sum of one charge and one discharge capacity.

Cost Minimization: The total cost incurred for energy transfer to MG_i is given by

$$\text{Cost}_i[t] = q_i[t]G_i[t] + \sum_{j\neq i} p_{j,i}[t]B_{j,i}[t],\ i = 1,\dots,N. \tag{8}$$

The objective of the controller is to design the system parameters in order to minimize the time average cost of energy transfer across the grid, as follows:

$$\begin{aligned} \min \quad & \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\Big[\sum_{i=1}^{N} \text{Cost}_i[t]\Big] \\ s.t. \quad & (1), (2), (3), (4), (5), (6) \end{aligned} \tag{9}$$

where during each time slot t, the decision variables are $Y_i[t], B_{i,i}[t], B_{i,j}[t] (\forall j \neq i), G_i[t] \ \forall i$. We denote the optimal value of the cost function $\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\Big[\sum_{i=1}^{N} \text{Cost}_i[t]\Big]$ over all possible control actions by f^*_N. Note that we have explicitly mentioned the subscript N to denote the optimal solution when N MGs cooperate.

One possible way to interpret the prices of energy exchange is to view them as the price paid for the losses incurred during the energy exchange. However, in this work, we do not consider any explicit models for the prices.

III. An analytical characterization of the optimal cost

We first provide an analytical characterization of the optimal cost for a symmetric scenario consisting of two MGs. We consider that the cost of energy exchanges $p_{1,2}[t] = p_{2,1}[t] = p_{\max}\ \forall t$ and $q_1[t] = q_2[t] = q_{\max}\ \forall t$. Each MG has an excess energy arrival process $\tilde{X}_i[t] = X_i[t] - L_i[t]$, assumed to be i.i.d. across time slots, whose probability mass function (p.m.f.) is given by

$$f_{\tilde{X}_i}(x_i) = \begin{cases} -1 & \text{w.p. } d \\ 0 & \text{w.p. } 1-a-d \\ 1 & \text{w.p. } a. \end{cases} \tag{10}$$

Further we also assume that $\tilde{X}_1[t]$ and $\tilde{X}_2[t]$ are independent of each other. In order to model the energy transfer within the grid, we consider the following policy. Whenever, MG_1 produces excess energy and MG_2 has an energy deficit, MG_1 transfers its excess energy to MG_2 with probability $\alpha \in [0, 1]$. Otherwise, MG_1 stores the excess energy in its battery with a probability $1 - \alpha$. Since the system is perfectly symmetric, MG_2 does the same in the case when it overproduces and MG_1 has an energy deficit. In all other cases, there is no requirement to exchange energy. Let the random variables $\tilde{Z}_1$ and $\tilde{Z}_2$ denote the effective excess energy arrivals. They are related to $\tilde{X}_1$ and $\tilde{X}_2$ as follows:

- If $\tilde{X}_1 = 1$ and $\tilde{X}_2 = -1$ (which happens with probability ad) then,

$$\tilde{Z}_1 = \begin{cases} 1 & \text{w.p. } (1-\alpha)ad \\ 0 & \text{w.p. } \alpha ad. \end{cases} \quad \tilde{Z}_2 = \begin{cases} -1 & \text{w.p. } (1-\alpha)ad \\ 0 & \text{w.p. } \alpha ad. \end{cases} \tag{11}$$

- If $\tilde{X}_1 = -1$ and $\tilde{X}_2 = 1$ (which happens with probability ad) then,

$$\tilde{Z}_1 = \begin{cases} -1 & \text{w.p. } (1-\alpha)ad \\ 0 & \text{w.p. } \alpha ad. \end{cases} \quad \tilde{Z}_2 = \begin{cases} 1 & \text{w.p. } (1-\alpha)ad \\ 0 & \text{w.p. } \alpha ad. \end{cases} \tag{12}$$

- In all other cases, $\tilde{Z}_1 = \tilde{X}_1$ and $\tilde{Z}_2 = \tilde{X}_2$.

The unconditional p.m.f. of $\tilde{Z}_1$ and $\tilde{Z}_2$ can be easily computed by summing across all the possible cases. It is given by

$$f_{\tilde{Z}_1}(z) = \begin{cases} -1 & \text{w.p. } d(1-\alpha a) \\ 0 & \text{w.p. } 2\alpha ad + (1-a-d) \\ 1 & \text{w.p. } a(1-\alpha d). \end{cases} \tag{13}$$

With this formulation, the system can be viewed as two independent MGs with effective energy arrival processes given by $\tilde{Z}_i\ i = 1, 2$. The evolution of the battery at each MG_i can now be modeled as a random walk that evolves as follows: $E_i[t+1] = \min(\max(E_i[t] + \tilde{Z}_i[t], 0), E_{\max})$. Since the $\tilde{Z}_i[t]$ are i.i.d. across time, it can be verified that the random process $(E_i[t], t \geq 0)$ is Markovian. The steady state distribution of the Markov chain corresponding to this random walk $\lim_{t\to\infty} \Pr(E_i[t] = j) \triangleq \pi(j)$ is given by

$$\pi(j) = r^j \left(\frac{1-r}{1 - r^{(E_{\max}+1)}} \right),\ i = 1, 2, \tag{14}$$

where $r = \frac{a(1-\alpha d)}{d(1-\alpha a)}$. Note that $\lim_{t\to\infty} \Pr(E_1[t] = j) = \lim_{t\to\infty} \Pr(E_2[t] = j)$ due to symmetry. Now, the cost of energy exchange within the grid can be computed based on the steady state of the system. It can be written as the sum of two components: the cost of energy exchanged between the MGs which is equal to $2\alpha ad p_{\max}$. Next, we model the cost incurred by borrowing energy from the macro-grid. This occurs in the case when $\tilde{X}_i = -1,\ i = 1, 2$, and if the corresponding battery is in the 0 energy state. This cost associated with this event is equal to $2d(1-\alpha a)\pi(0)q_{\max}$. Therefore, the total cost incurred in the steady state can be written as the sum of the two components:

$$\text{Cost}(\alpha) = 2\alpha ad p_{\max} + 2d(1-\alpha a)\pi(0)q_{\max}. \tag{15}$$

The minimum cost is then obtained by optimizing over the choice of α. Let us define $\alpha^* = \arg\min_\alpha \text{Cost}(\alpha)$, and hence, the minimum cost of energy exchange is given by $\text{Cost}(\alpha^*)$. From (15), we can analyze two extreme cases, namely, the case with no storage and the case with infinite storage.

- $E_{\max} = 0$ - No storage

In this case $\pi(0) = 1$. Therefore,

$$\overline{\text{Cost}} = 2dq_{\max} + ad\alpha(-q_{\max} + p_{\max}), \tag{16}$$

and the cost is minimized when $\alpha = 1$ if $p_{\max} < q_{\max}$. Thus, in the absence of storage, the MGs must always share the excess available energy as long as $p_{\max} < q_{\max}$. This result is quite intuitive since in the absence of storage, one can always reduce the cost by exchanging energy locally between the MGs.

• $E_{\max} = \infty$ - Infinite storage

In this case, $\pi(0) = \frac{d-a}{d(1-\alpha a)}$. Therefore,

$$\overline{\text{Cost}} = 2d(d-a)q_{\max} + 2ad\alpha. \tag{17}$$

Thus, the cost is minimized when $\alpha = 0$. This implies that in the presence of infinite storage, any excess energy must always be stored rather than exchanging.

In what follows, we provide a practical algorithm to solve the time average cost minimization problem under a general system model.

IV. Algorithm Design based on Lyapunov Optimization

We use the idea of Lyapunov optimization to solve this problem. The Lyapunov optimization method provides simple online solutions based only on the current knowledge of the system state, as opposed to approaches such as dynamic programming which suffer from very high complexity.

Before doing so, first note that the Lyapunov optimization method is not directly applicable to our problem due to the presence of the battery constraints (5) and (6), which couples the decisions across time slots. To overcome this, we use an approach similar to [8], and first consider an approximate version of the optimization problem (9) in which we relax all the constraints associated with the battery, given as follows:

$$\begin{aligned}
\min \quad & \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\Big[\sum_{i=1}^{N} \text{Cost}_i[t]\Big] \qquad (18)\\
s.t. \quad & \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}[Y_i[t]] \le \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}[B_{i,i}[t]]\\
& (1), (2), (3), B_{i,i}[t] \le B^s_{\max}, Y_i[t] \le Y_{\max},\ \forall i, t.
\end{aligned}$$

Note that in the relaxed problem, the dependence of control actions on the battery size $E[t]$ is removed. Let us denote the optimal cost over all possible control actions for the relaxed problem by g_N^*. First, it is easy to see that $g_N^* \le f_N^*$, i.e. the solution to the relaxed problem acts as a lower bound on the original problem (since the relaxed problem has fewer constraints compared to the original problem and any feasible solution of the original problem is feasible for the relaxed problem as well). We now apply Lyapunov optimization to solve (18). Further, we will show that the solution developed for (18) by our method also satisfies all the constraints associated with battery, hence making it applicable for solving the original problem (9).

We consider the Lyapunov function associated with the virtual energy queues (batteries) defined as follows:

$$\Psi[t] = \frac{1}{2}\sum_i (E_i[t] - \theta)^2 \tag{19}$$

where $\theta > 0$ is a perturbation parameter, given by $\theta = B^s_{\max} + Vq_{max}$. The choice of the value of θ will become clear once we examine the structure of our solution. We now examine the Lyapunov drift which represents the expected change in the Lyapunov function from one time slot to the next, which is defined as $\Delta[t] = \mathbb{E}\left[\Psi[t+1] - \Psi[t] \middle| \mathbf{E}[t]\right]$, where the expectation is with respect to the random processes associated with the system, given the energy queue-length values $\mathbf{E}[t] = [E_1[t], \ldots, E_N[t]]$. Using the equation for evolution of the virtual queue associated with the battery in (4), and some standard manipulations, it can be shown that the Lyapunov drift can be bounded as

$$\Delta[t] \le C - \mathbb{E}\Big[\sum_i (E_i[t] - \theta)(B_{i,i}[t] - Y_i[t]) \Big| \mathbf{E}[t]\Big] \tag{20}$$

where $C < \infty$ is a constant. We will henceforth denote $\tilde{E}_i[t] = E_i[t] - \theta$. Adding the performance metric $V\mathbb{E}\left[(\sum_{i,j} p_{i,j}B_{i,j}[t] + \sum_i q_i[t]G_i[t])|\mathbf{E}[t]\right]$ (where V is another control parameter which will be specified later) to both the sides of (20), and using the fact that $G_i[t] = \tilde{L}_i[t] - B_{i,i}[t] - \sum_{j\neq i} B_{j,i}[t]$, we obtain

$$\begin{aligned}
\Delta_V[t] \le\ & C + V\sum_i q_i[t]\tilde{L}_i[t] + \sum_i \tilde{E}_i[t]Y_i[t] \qquad (21)\\
& + \sum_{i,j\neq i} VB_{i,j}[t](p_{i,j}[t] - q_j[t])] - \sum_i B_{i,i}[t](\tilde{E}_i[t] + Vq_i[t])
\end{aligned}$$

where we have denoted $\Delta_V[t] = \Delta[t] + V\mathbb{E}\big[\sum_{i,j} p_{i,j}B_{i,j}[t] + \sum_i q_i[t]G_i[t]|\mathbf{E}[t]\big]$, and rearranged the terms in the equation.

According to the theory of Lyapunov optimization [9], the control decisions are chosen to minimize the bound on the modified Lyapunov drift obtained in (21). Before we proceed, for the sake of completeness, we provide the main intuition behind this approach. Note that the problem (18) can viewed as minimizing the time average cost of energy exchange in the grid while maintaining the stability of the virtual energy queue (battery). The modified Lyapunov drift has two components, the Lyapunov drift term $\Delta[t]$, and the $V \times \text{Cost}[t]$ term. Intuitively, minimizing the Lyapunov drift term alone pushes the queue-length of the virtual energy queue to a lower value. The second metric $V \times \text{Cost}[t]$ can be viewed as a penalty term, with the parameter V representing the trade-off between minimizing the queue-length drift and minimizing the penalty function. A larger value of V represents a greater priority to minimizing the cost metric at the expense of a greater size of the virtual energy queue and vice versa. Therefore, we minimize the modified Lyapunov drift $\Delta_V[t]$, and subsequently analyze the performance of this algorithm.

During each time slot t, the control decisions are chosen as solutions to the following linear programming problem (obtained by minimizing the right hand side of (21)):

$$\begin{aligned}
\min_{\substack{Y_i\\ B_{i,i}, B_{i,j}}} \quad & \sum_i \tilde{E}_i[t]Y_i + V\sum_{j\neq i} B_{i,j}(p_{i,j}[t] - q_j[t])\\
& - \sum_i B_{i,i}(\tilde{E}_i[t] + Vq_i[t]) \qquad (22)\\
s.t. \quad & Y_i + \sum_{j\neq i} B_{i,j} \le \tilde{X}_i[t],\ B_{i,i} + \sum_{j\neq i} B_{j,i} \le \tilde{L}_i[t]\\
& 0 \le Y_i \le Y_{\max},\ 0 \le B_{i,i} \le B^{\text{s}}_{\max}\ \forall i,\\
& 0 \le B_{i,j} \le B^{\text{ex}}_{\max}\ \forall j \neq i, \forall i.
\end{aligned}$$

Let us denote the solution corresponding to (22) as $Y_i^*[t], B_{i,i}^*[t]$ and $B_{i,j}^*[t]$. The value of $G_i^*[t]$ is then given by $G_i^*[t] = L_i[t] - B_{i,i}^*[t] - \sum_{j\neq i} B_{j,i}^*[t]$.

A. Algorithm Performance Analysis

We will now analyze the performance of the algorithm.

Lemma 1. *By choosing the parameters V and θ as*

$$0 < V \leq \frac{E_{\max} - (Y_{\max} + B^s_{\max})}{q_{\max}} \tag{23}$$

$$\theta = B^s_{\max} + V q_{\max} \tag{24}$$

the following hold true:

1. *If $E_i[t] > E_{\max} - Y_{\max}$, then $Y_i^*[t] = 0$.*
2. *If $E_i[t] < B^s_{\max}$, then $B_{i,i}^*[t] = 0$.*

We omit the proof of this lemma due to a lack of space. From the result of Lemma 1, it can be seen that the battery charging decisions are non-zero only when $E_i[t] < E_{\max} - Y_{\max}$ and the discharge decisions are non-zero only when $E_i[t] > B^s_{\max}$. As a consequence of this lemma, it can be noticed that the algorithm developed by the Lyapunov optimization method applied to the relaxed problem naturally satisfies the battery constraints (5) and (6). Therefore, this algorithm is feasible for the original problem (9) as well. We will now provide the main result of our algorithm.

Theorem 1. *For the algorithm developed in the previous subsection, the virtual energy queue-lengths can be bounded as follows:*

$$0 \leq E_i[t] \leq E_{\max} \qquad \forall t, i = 1, \ldots, N, \tag{25}$$

and the time average cost function achieved by this algorithm satisfies

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\Big[\sum_{i=1}^{N} Cost_i[t]\Big] \leq g_N^* + \frac{\tilde{B}}{V} \tag{26}$$

where $\tilde{B} < \infty$ is a constant.

We omit the proof of (25) but prove the result of (26) in Appendix A. The theorem implies that the performance of our algorithm can be made arbitrarily close to the optimal value by increasing the value of parameter V. However, this comes at the cost of increasing the battery capacity $E_{\max}$ (due to the bound on the value of V in (23), given $E_{\max}$). Also, note that since $g_N^* \leq f_N^*$, the bounds of our algorithm hold with respect to f_N^* as well (and are tighter).

V. Numerical Results

In this section, we present some numerical results to study the normalized time average cost of energy exchange within the grid. The parameters for the simulations are chosen as follows: For the renewable energy production, we obtain data from the NREL of the United States (website [10]). We utilize the 2006 annual recorded wind power from 5 different geographically distributed wind turbines with the following ID and location:

ID	Position	Mean Power Output ($\bar{X}_i$)
13925	41.06N,105.76W	13.186 MW
15017	41.39N,107.11W	12.879 MW
16209	41.61N,106.07W	12.454 MW
21102	42.44N,107.72W	11.924 MW
21633	42.51N,106.59W	12.3423 MW

We consider the data corresponding to an interval of one year. The data is sampled once every hour. Therefore, we have $T = 8760$ samples. The mean power output $\bar{X}_i$ is calculated by averaging the renewable energy production over these 8760 samples. For simplicity, we consider that the load on each MG_i is $L_i[t] = \bar{X}_i\ \forall t$. This is done in order to ensure that the aggregate energy production of each turbine matches the aggregate load, and the only reason to store/exchange energy is to compensate for the fluctuations in renewable energy (thus capturing the most essential aspects of the problem considered in this work). We consider 5 different values of the storage capacities, 2, 10, 50, 100 and 200 MWh, at each MG. In each of these cases, we choose $B^s_{\max}$ and $Y_{\max}$ to satisfy the constraint (7). Specifically, we choose $(B^s_{\max}, Y_{\max}) = (0.5, 0.5), (2, 2), (10, 10), (20, 20), (50, 50)$ MW in the five cases of storage capacity respectively. We set $B^{\text{ex}}_{\max} = 20$ MW.

We assume that the cost of transporting energy among the different elements of the grid is directly proportional to the distance between them. Therefore, $p_{i,j}[t] = \beta_1 d_{i,j},\ \forall t$, units (where $d_{i,j}$ is the distance between MG_i and MG_j in km) and $q_i[t] = \beta_2 D_i,\ \forall t$, (where D_i is the distance between MG_i and the macro-grid in km). The distances are calculated based on the coordinates provided in the table. β_1 and β_2 are the proportionality constants. We have chosen different constants for the connection between the MGs and the connection between the MG and macro-grid to indicate that they might be connected by different power lines (e.g. DC power lines between the MGs and AC power lines to the macro-grid). Let us consider the time average cost incurred, $\bar{C} = \frac{1}{T}\sum_{t=0}^{T-1}\sum_{i=1}^{N} C_i[t] = \frac{1}{T}\sum_{i=1}^{N}\big(\sum_{j\neq i}\beta_1 d_{i,j}\sum_{t=0}^{T-1} B_{i,j}[t] + \beta_2 D_i \sum_{t=0}^{T-1} G_i[t]\big)$, in transferring energy across the grid, where N is the number of cooperating MGs. We plot the normalized cost incurred $\frac{\bar{C}}{N}$, as a function of N, for different values of the storage capacity in Figure 2. The following observations can be made.

- For a given storage capacity, the cost of energy exchange decreases with an increase in the number of cooperating MGs. This is due to the fact that a greater number of cooperating MGs leads to a greater diversity in energy production and hence greater possibility of sharing energy among the MGs (reducing the need for borrowing energy from the macro-grid).
- The decrease in the normalized cost (as a function of the number of cooperating MGs) is greater for lower values of storage capacity. For higher values of storage capacity, the normalized cost does not reduce with increasing N. This is due to the fact that with greater storage capacity, the MGs are able to store all the excess harvested energy, and use it during time slots when the harvested energy is deficient, thereby, eliminating the need for energy

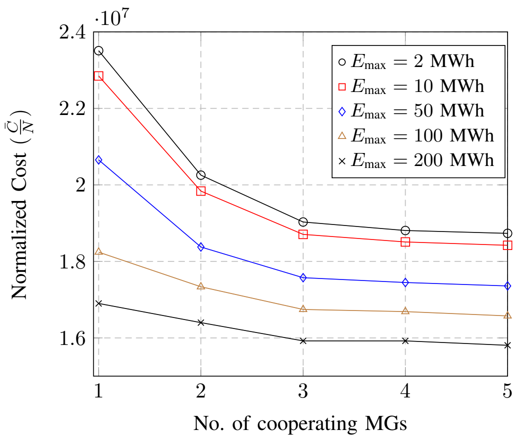

Fig. 2. Normalized cost versus the number of cooperating MGs for different values of storage capacity.

cooperation.

VI. Conclusions

In this work, we have explored the combined benefits of energy storage and cooperation among interconnected MGs as a means to combat the intermittent nature of the renewable energy. Our results show that in the presence of limited storage devices, the grid can benefit greatly by cooperating, even among only a few distributed sources. However, in the presence of large storage capacity, cooperation does not yield much benefit. Our solution can be useful for power grid designers in terms of choosing the optimal combination of storage size and infrastructure for cooperation in order to meet a specific cost criterion. The framework presented in this work can be extended in multiple ways, so as to include demand response and load shedding.

Appendix A: Proof of Theorem 1

Consider the bound on the Lyapunov drift function of (21). It is clear that the control action chosen according to the solution of (22) minimizes the bound on the Lyapunov function over all possible control actions. To compare this control action with an alternate control policy called the stationary and randomized policy that takes control actions purely based on the system state $\tilde{X}, p_{i,j}, q_i$ and independent of the virtual queue-length (we denote this policy with the superscript α), we can obtain the following:

$$\begin{aligned}\Delta_V[t] \le C + \mathbb{E}\Big[V \sum_i q_i[t]\tilde{L}_i[t] + \sum_i \tilde{E}_i[t] Y_i^*[t] \\ + \sum_{i,j\neq i} V B_{i,j}^*[t](p_{i,j}[t] - q_j[t]) \\ - \sum_i B_{i,i}^*[t](\tilde{E}_i[t] + V q_i[t]) \big| \mathbf{E}[t] \Big] \quad (27) \\ \le C + \mathbb{E}\Big[V \sum_i q_i[t]\tilde{L}_i[t] + \sum_i \tilde{E}_i[t] Y_i^\alpha[t] \\ + \sum_{i,j\neq i} V B_{i,j}^\alpha[t](p_{i,j}[t] - q_j[t]) \\ - \sum_i B_{i,i}^\alpha[t](\tilde{E}_i[t] + V q_i[t]) \big| \mathbf{E}[t] \Big]. \quad (28)\end{aligned}$$

Rearranging, we have

$$\begin{aligned}\Delta_V[t] \le C + \mathbb{E}\Big[V \sum_i q_i[t](\tilde{L}_i[t] - B_{i,i}^\alpha[t] + \sum_{j\neq i} B_{j,i}^\alpha[t]) \\ + \sum_i \tilde{E}_i[t](Y_i^\alpha[t] - B_{i,i}^\alpha[t]) + \sum_{i,j\neq i} V p_{i,j}[t] B_{i,j}^\alpha[t] \big| \mathbf{E}[t] \Big]. \quad (29)\end{aligned}$$

In particular, we consider the stationary and randomized policy that satisfies

$$\mathbb{E}[Y_i^\alpha[t]] \le \mathbb{E}[B_{i,i}^\alpha[t]] \quad (30)$$
$$\mathbb{E}[B_{i,i}^\alpha[t] + \sum_{j\neq i} B_{j,i}^\alpha[t] + G_i^\alpha[t]] = \mathbb{E}[\tilde{L}_i[t]] \quad (31)$$
$$\mathbb{E}[Y_i^\alpha[t] + \sum_{j\neq i} B_{i,j}^\alpha[t]] = \mathbb{E}[\tilde{X}_i[t]] \quad (32)$$
$$\mathbb{E}\Big[\sum_i \sum_{j\neq i} p_{i,j}[t] B_{i,j}^\alpha[t] + \sum_i q_i[t] G_i^\alpha[t]\Big] = g_N^*. \quad (33)$$

The existence of such a policy for the relaxed problem in (18) can be proven by using Caratheodory's theorem similar to that in [9] and is omitted here for brevity. Using this policy in (29), we obtain

$$\begin{aligned}\Delta_V[t] \le C + \mathbb{E}\Big[V \sum_i q_i[t] G_i^\alpha[t] + + \sum_{i,j\neq i} V p_{i,j}[t] B_{i,j}^\alpha[t] \\ + \sum_i \tilde{E}_i[t](Y_i^\alpha[t] - B_{i,i}^\alpha[t]) \big| \mathbf{E}[t] \Big] \quad (34) \\ = C + V g_N^*. \quad (35)\end{aligned}$$

Taking the expectation on both sides, summing from $t = 0, \ldots, T-1$, normalizing by T and taking the limit, we obtain the result of (26).

References

[1] "Western Wind and Solar Integration Study," May 2009. [Online]. Available: http://www.nrel.gov/docs/fy10osti/47434.pdf

[2] E. Baeyens, E. Bitar, P. Khargonekar, and K. Poolla, "Wind energy aggregation: A coalitional game approach," in *Proc. IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, 2011, pp. 3000–3007.

[3] N. Hatziargyriou, H. Sano, R. Iravani, and C. Marnay, "Microgrids: an overview of ongoing research development and demonstration projects," *IEEE Power and Energy Magazine*, vol. 27, pp. 78–94, Aug. 2007.

[4] S. Nykamp, M. Bosman, A. Molderink, J. Hurink, and G. Smit, "Value of storage in distribution grids - competition or cooperation of stakeholders?" *IEEE Transactions on Smart Grid*, vol. 4, no. 3, pp. 1361–1370, Sep. 2013.

[5] J. Guerrero, M. Chandorkar, T. Lee, and P. Loh, "Advanced control architectures for intelligent microgrids - part i: Decentralized and hierarchical control," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 4, pp. 1254–1262, Apr. 2013.

[6] T. Zhu, Z. Huang, A. Sharma, J. Su, D. Irwin, A. Mishra, D. Menasche, and P. Shenoy, "Sharing renewable energy in smart microgrids," in *Proc. ACM/IEEE International Conference on Cyber-Physical Systems (ICCPS), 2013*, Apr. 2013, pp. 219–228.

[7] D. M. Larruskain, I. Zamora, A. J. Mazn, O. Abarrategui, and J. Monasterio, "Transmission and distribution networks: AC versus DC," in *Proc. 9th Spanish-Portuguese Congress on Electrical Engineering*, 2005.

[8] R. Urgaonkar, B. Urgaonkar, M. J. Neely, and A. Sivasubramaniam, "Optimal power cost management using stored energy in data centers," in *Proc. ACM SIGMETRICS*, 2011, pp. 221–232.

[9] L. Georgiadis, M. J. Neely, and L. Tassiulas, *Resource Allocation and Cross-layer Control in Wireless Networks*. Foundations and Trends in Networking, Now Publishers, vol. 1, no. 1, pp. 1-144, 2006.

[10] Western Wind Resources Dataset. [Online]. Available: http://wind.nrel.gov/Web_nrel/