

# Scheduling long term energy storage

Rozanna N. Jesudasan and Lachlan L. H. Andrew  
Monash University, Australia

**Abstract**—Eventually, society will need to be powered almost entirely by renewable energy sources. This will require very large scale storage for long time periods, such as between seasons or even between years. On these timescales, issues such as gradual leakage of energy and price rises must be considered. This paper considers optimal storage management schedules on these long time scales, from the point of view of minimizing the purchase price of electricity while meeting an inelastic demand. We show that the two factors (leakage and price rises) have complementary effects on the optimal storage level, which implies that peak generation is non-monotonic when either is varied by itself.

## I. INTRODUCTION

Building a sustainable and non-polluting global energy infrastructure is a huge task. It is not likely to be achieved in the short term, but it must eventually be accomplished. In the long term, society must be powered entirely by sustainable energy. The leading renewable sources are dependent on the weather, be it wind, sunshine or rainfall. The resulting intermittency is a major challenge. Much work is going into developing storage systems to provide energy during the sunless nights and occasional windless days. However, both short- and long-term storage is needed for a system powered entirely by renewables [1]. The availability of and demand for energy also vary on much longer timescales, both seasonally within a year [2], and between years. Examples include a lack of rainfall affecting hydroelectric generation in Portugal [3], Brazil [4] and New Zealand [5].

Currently, fossil fuels are used as backup. For example, Portugal's coal consumption rose 56% year-on-year in June 2012, due to the 63% year-on-year drop in its hydroelectric production. However, as argued in [6], if society is eventually to be powered entirely by renewable energy then the only alternative is to store energy for periods of many months. Such large-scale storage can be achieved by technology such as compressed air energy storage (CAES) [7] or the underground storage of hydrogen generated by the electrolysis of water [1].

In this paper, we consider the optimal management of such a long term storage system. We investigate the structural properties of the optimal charging schedule subject to the influence of two effects that are insignificant for short-term storage but have a measurable influence when storing energy on a long-term basis.

The first issue is energy leakage. Most forms of storage leak energy on a timescale varying from hours for flywheels [8] or months for batteries through to years for compressed hydrogen. This form of loss is very different from the “round-trip” loss incurred by the charging and discharging process, and must be managed differently. Although the latter is considered in many studies of storage management (e.g., [9]), the former is largely ignored.

The second issue is the fact that energy prices will rise in real terms. Market prices are currently dominated by fluctuations in demand due to economic cycles and fluctuations in supply due to individual energy extraction projects. However, with the advent of peak oil, there will be an inevitable upward trend in prices driven by a fundamentally diminishing supply, combined with the need to avoid polluting fuels.

There has been considerable work on managing energy storage. Managing ideal storage is a “warehouse problem”, which has been studied since the 1940s [10]. When storing natural gas, it is common to apply receding horizon control to a finite horizon dynamic program, known in the economics literature as the rolling intrinsic method [11]. Such models often consider the charge/discharge inefficiency [9], [12], but do not consider leakage. They also often treat price as a martingale, with no upward trend. The understanding of the implications of leakage and rising prices provided by the present paper will assist such management policies to incorporate these important factors.

Another factor often ignored [9], [12] is the nonlinearity in the price of generation. Wholesale electricity markets around the world are typically structured so that all electricity purchased at a particular time is bought at the system marginal price (SMP), which is the cost of the most expensive generator called upon at that time. This causes the price per unit of electricity to increase with the demand. Following [13], we incorporate this important feature.

In this paper, we consider storage owned and managed by a load serving entity (LSE), who must serve an inelastic demand and purchase electricity from a separate entity or market. We assume that the primary objective of the LSE is to minimize the cost of purchasing electricity. However, the generation and transmission companies are interested in minimizing the peak generation and transmission required, and so we evaluate the resulting charging schedules in terms of the peak generation requirements, and also the total energy consumption.

In order to study long-term storage management, we develop a new approach to studying infinite horizon problems in which the average cost is increasing. Although good approximations could be obtained by truncating the optimizations to a suitably large finite horizon, to be sustainable we must make decisions that are optimal not only for the next year, next decade or even next century. We must consider the implications on the timescale of civilizations. As suggested by David McKay [14], a timescale of 1000 years is more suitable. This calls for us to increase our understanding of infinite horizon models.

After introducing the model in Section II, we consider the implications of self-discharge in Section III and rising prices in Section IV.

## II. MODEL

The notation in this paper will typically use upper case letters for fixed parameters of the problem and lower case letters for variables controllable in real time.

The grid must balance supply and demand; without energy storage, the draw from the grid,  $g(t)$ , should be able to satisfy the demand (load)  $D(t) \geq 0$  at all time  $t$ , i.e.,  $g(t) = D(t)$ . We consider demand to be inelastic (independent of changes in price). Following common practice [15], [16], [17], [18], we consider the hypothetical case that  $D(t)$  is known in advance. Consider an energy storage system with capacity  $B > 0$  is installed between the generator and the user (load), allowing the user to store energy  $b(t)$ , by drawing extra power at a rate of  $c(t)$  or consuming the stored energy at a rate of  $d(t)$ . The maximum charge and discharge rates are  $C_{\max}, D_{\max}$  respectively. Further the storage has a charging efficiency of  $\eta \leq 1$  and leaks energy such that, if no charging occurs, then  $b(t) = Sb(t-1)$  for some  $S \in (0, 1]$ .

The cost of drawing power  $g(t)$  from the grid (“generation”) is assumed to have the form  $P(t)N(g(t))$  for  $P(\cdot) > 0$  and nonlinearity  $N(\cdot)$ , which is strictly convex increasing, to model the fact that peak grid power increases the cost for the utility and increases the strain on the grid.

The user then adjusts its charging pattern to minimise the generation cost, assuming that the cost savings by the utility will be passed on to the user. In particular, we would like to know the optimal long term charging and grid use schedule under arbitrary prices and arbitrary demands. Specifically, for a long horizon  $T$ , we want to solve

$$\arg \min_{g,b,c,d} \sum_{t=1}^T P(t)N(g(t)) \quad (1)$$

subject to,

$$b(t) - Sb(t-1) - \eta c(t) + d(t) = 0 \quad (2a)$$

$$D(t) + c(t) - d(t) - g(t) = 0 \quad (2b)$$

$$g(t) \geq 0 \quad (2c)$$

$$B - b(t) \geq 0 \quad b(t) \geq 0 \quad [\bar{\beta}(t), \underline{\beta}(t)] \quad (2d)$$

$$C_{\max} - c(t) \geq 0 \quad c(t) \geq 0 \quad (2e)$$

$$D_{\max} - d(t) \geq 0 \quad d(t) \geq 0 \quad (2f)$$

with  $b(0) = 0$ , where the variables in square brackets are the Lagrange dual variables corresponding to the constraints. Note that capital letters denote parameters of the problem instance and lower case letters denote decision variables.

If  $\eta = 1$  the solutions for  $c$  and  $d$  need not be unique. We consider only the solution in which  $\min(c(t), d(t)) = 0$  for all  $t$ , which corresponds to the storage never charging and discharging simultaneously.

The structure of the optimal solution to this problem was presented in [19], and is repeated here for completeness. The superscript  $*$  denotes the optimal solution.

**Theorem 1.** Consider an interval  $[t_1, t_2]$  on which  $b^*(t) \in (0, B)$ . There exists a constant  $\xi^*$  and a constant

$$R = \xi^* + \sum_{\tau=t_1}^{T-1} S^\tau (\underline{\beta}^*(\tau) - \bar{\beta}^*(\tau)) \quad (3)$$

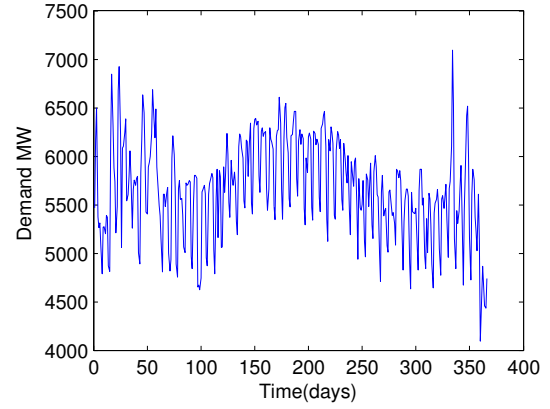


Fig. 1: Annual daily demand for Victoria in 2012, from 1 January to 31 December. Note the weekly cycle, the trend for more demand in winter (July) and isolated peaks of demand in summer (December/January).

such that, when the storage is neither charging at maximum rate nor discharging at maximum rate (i.e.,  $d < D_{\max}$  and  $c < C_{\max}$ ) then

$$P(t)N'(g^*(t))S^t \in [\eta R^+, R^+] \quad (4)$$

where  $R^+ = \max(R, 0)$ . The left hand side is at the lower limit of this interval when the storage is charging,  $c(t) > 0$ , and the upper limit when the storage is discharging,  $d(t) > 0$ .

Moreover, when the storage is discharging at maximum rate,  $P(t)N'(g^*(t))S^t \geq R$  and when it is charging at maximum rate,  $P(t)N'(g^*(t))S^t \leq \eta R^+$ . When  $P(t)N'(g^*(t))S^t$  is in the interior of the interval, the storage is neither charging nor discharging.

Note that the lower limit for the sum in the definition of  $R$  could be any  $t \in [t_1, t_2]$ , since  $\bar{\beta}(t) = \underline{\beta}(t) = 0$  for  $t \in [t_1, t_2]$ .

## III. IMPACT OF SELF-DISCHARGE

On long timescales, the leakage of energy and increases in electricity price affect the optimal charging schedule. This section explores the impact of leakage with constant prices, and the following considers rising prices. Leakage is important even on a short timescale for some storage technologies such as flywheels, which lose up to 50% of their stored energy in 24 hours [8].

The structure will be illustrated using total daily demand from the Australian state of Victoria throughout the year 2012 [20] (see Fig. 1). This demand exhibits both weekly fluctuations and a seasonal trend. The average load is higher in winter, but the load is more variable in summer with isolated hot days causing spikes in cooling-related demand.

Since the only way leakage  $S$  appears in Theorem 1 is as a multiplicative factor of  $S^t$ , it is tempting to assume that the optimal charging schedule for  $S < 1$  might differ from that for  $S = 1$  only in that the marginal price increases exponentially for  $S < 1$  where it is flat for  $S = 1$ . Figure 2 shows this is not the case. Increased leakage causes the storage to empty and fill more frequently. As a result, intervals over which the marginal cost is constant with  $S = 1$  are split into multiple

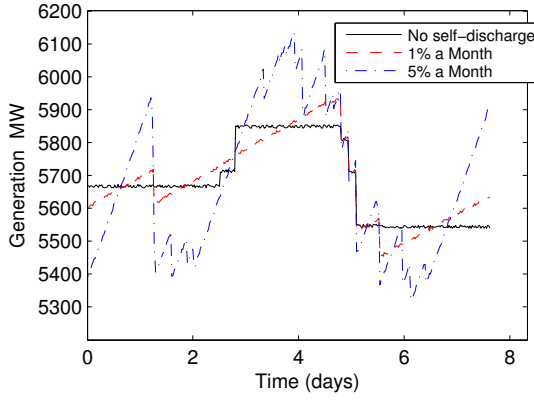


Fig. 2: Optimal generation,  $g$ , with  $B = 8000$  MWh leaky storage. Generation is piecewise increasing.

intervals of increasing marginal cost, separated by downward jumps.

The figure suggests that whenever the price decreases for a low rate of leakage, the price also decreases for a higher rate of leakage. Conversely, whenever the price jumps *up* for a *high* rate of leakage, it also jumps *up* for a *lower* rate. This is because these decreases occur when the storage is empty, and the optimal storage level  $b(t)$  is typically smaller for storages with more leakage (smaller  $S$ ). However, this is not always the case. The following is the main technical result of this paper:

**Theorem 2.** *The energy level  $b^T(t)$  can decrease with  $S$ .*

*Proof:* The proof will be constructive. To simplify notation, the superscript  $T$  will be omitted. Consider the function  $N(g) = g^2 a/2$ , and a short sequence of demands  $D(t-1)$ ,  $D(t)$ ,  $D(t+1)$ , with  $D(t-1)$ ,  $D(t+1)$  sufficiently large that  $b(t-1) = b(t+1) = 0$ , and  $0 < D(t) \ll D(t+1)$ , and a capacity  $B$  large enough not to limit  $b(t)$ . Further assume that  $P(t) = P(t+1)$ . Then  $b(t)$  satisfies

$$\begin{aligned} g(t+1) &= b(t+1) + D(t+1) - b(t)S \\ g(t) &= b(t) + D(t) - b(t-1)S \end{aligned}$$

and so the cost of these two steps is

$$N(b(t) + D(t) - b(t-1)S) + N(b(t+1) + D(t+1) - b(t)S).$$

This is minimized either for the  $b(t)$  that satisfies

$$N'(b(t) + D(t) - b(t-1)S) - SN'(b(t+1) + D(t+1) - b(t)S) = 0 \quad (5)$$

or, if that is negative, for  $b(t) = 0$ . For  $N(g) = g^2 a/2$ , (5) is solved for

$$b(t) = \frac{S(b(t-1) + b(t+1) + D(t+1)) - D(t)}{1 + S^2}$$

Since, by construction,  $b(t-1) = b(t+1) = 0$ , we have

$$b(t) = \frac{SD(t+1) - D(t)}{1 + S^2}.$$

If  $D(t) > SD(t+1)$  then this solution is negative and so  $b(t) = 0$ . Otherwise,

$$\frac{db(t)}{dS} = \frac{D(t+1) + 2D(t)S - D(t+1)S^2}{(1 + S^2)^2}$$

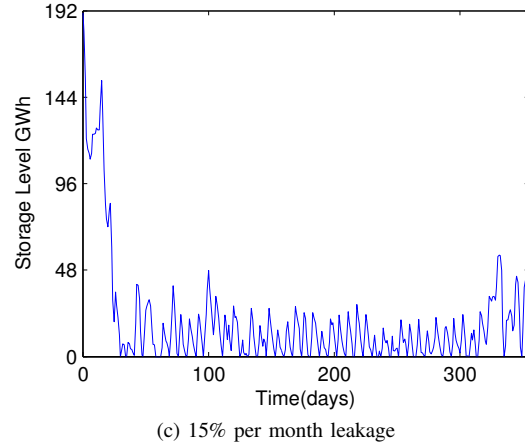
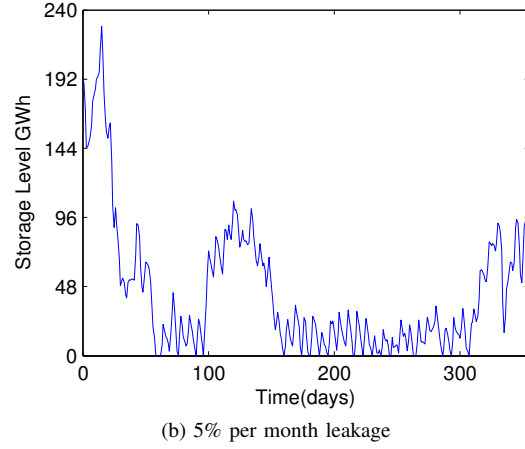
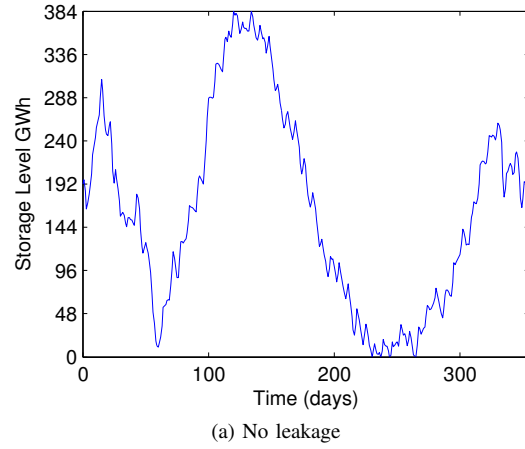


Fig. 3: Optimal storage level as a function of time, for three leakage rates and fixed prices. The optimal level typically decreases as the leakage rate increases. Storage size is  $B = 384$  GWh. Note the different vertical scales.

which is negative for

$$S \in \left( \frac{\sqrt{D^2(t) + D^2(t+1)} - D(t)}{D(t+1)}, 1 \right].$$

This interval is non-empty for  $D(t) > 0$ . ■

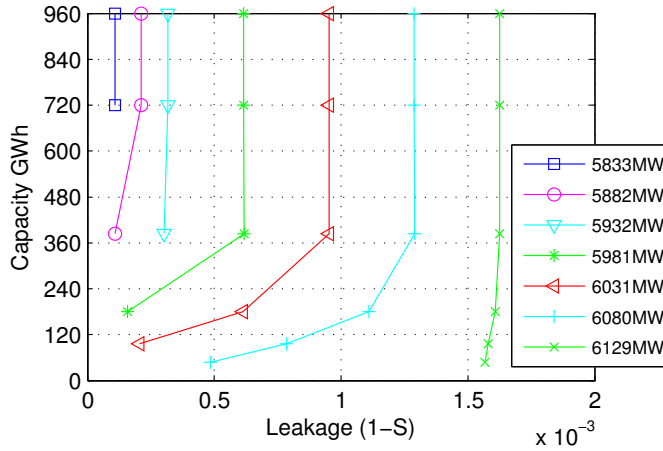


Fig. 4: Capacity vs Leakage for a given peak generation. (Note:  $S$  is leakage per day.)

#### A. Implications

As seen in Figure 2, the piecewise increase of generation tends to increase peak power demand, which reduces the ability of the storage to shave peaks of demand. Hence, to achieve a given level of peak shaving, a more leaky storage facility must have a higher capacity. See Fig. 4. As a result, selecting the appropriate storage technology involves the expected tradeoff<sup>1</sup> between capacity and leakage.

However, this expected conclusion only applies when the self discharge rate is relatively small. Remarkably, the reverse is true when the discharge rate increases. This is illustrated in Figure 5, which shows the peak generation required as a function of the leakage for several storage capacities. For low leakage, the larger storage capacities provide more smoothing and so reduce the peak demand. However, for high leakage, having storage larger than a threshold does not provide further saving. Moreover, that threshold of storage capacity is *smaller* as the leakage increases.

To understand why, consider lossless storage smoothing a given demand. Once the capacity is large enough to supply all the peaks without fully discharging, increasing the capacity does not provide further smoothing. For leaky storage, it is not optimal to charge the storage too much, as shown in Figure 3, because this increases the rate of energy loss. Thus the size of storage required to accommodate the maximum range of states of charge decreases as the leakage increases. Results for round-trip charge/discharge inefficiency are analogous [19].

Leakage also wastes energy. Figure 6 shows the total energy generated as a function of the leakage for the Victorian load. As expected, the total generation (and hence loss) initially increases. However, as the leakage becomes very large, the total generation begins to reduce. That is again because the optimal storage level reduces as the leakage rate increases; since the rate of losing energy is proportional to the state of charge of the storage, the reduction in storage level reduces the

<sup>1</sup>These storages are vastly larger than today's storages, and are extremely expensive. This huge expense does not affect the optimal management of the storage, but does affect these tradeoffs in the choice of storage.

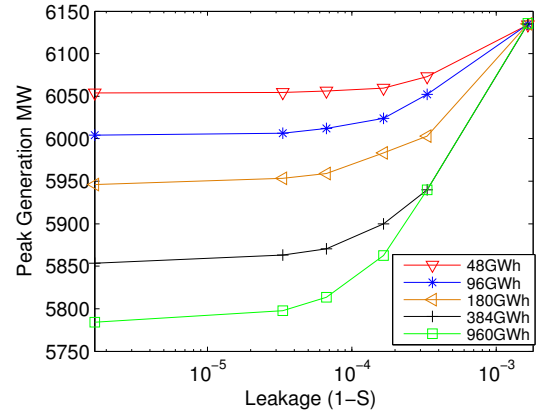


Fig. 5: Maximum generation vs leakage with fixed prices. (Note:  $S$  is leakage per day.)

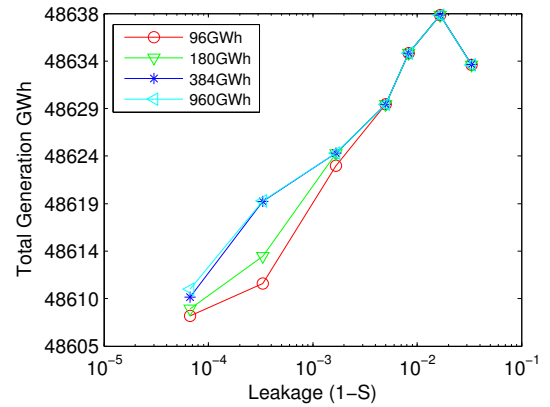


Fig. 6: Total generation vs leakage with fixed prices.

total loss. As a result, the maximum total energy loss due to leakage with these parameters is less than 0.1% of generation. This is almost an order of magnitude lower than the loss due to round-trip inefficiency reported in [19].

#### IV. IMPACT OF RISING PRICES

Let us now turn to the other factor that comes into play on long timescales: the increase of prices in real terms. There are many reasons to believe that the trend will be for energy prices to grow faster than inflation for many decades to come.

In the short term, this may be driven by the need for further investment in aging electricity generation and transmission infrastructure. It is not clear how long this process will take, or when it will need to be repeated.

Another natural reason to expect prices to increase is the transition away from carbon-intensive energy sources towards either renewable resources or nuclear. Where this is done by putting a price on greenhouse gas pollution, it is clear that it causes an increase in price, but such increases also occur where it is done by explicit regulation.

Even without the concern over pollution, fossil fuel energy sources are set to become more expensive as they become harder to extract. The recent development of “unconventional” extraction methods for oil and gas is providing a short-term

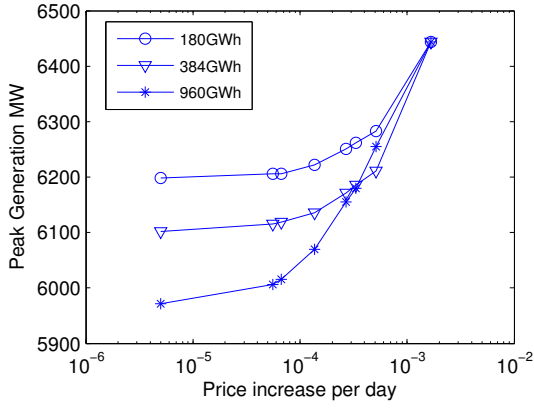


Fig. 7: Peak generation with rising prices and no leakage.

fluctuation in this trend, but paradoxically, the current rapid extraction of these unconventional reserves will increase the rate at which the price rises when these reserves are also nearing exhaustion.

For these reasons, it is safe to assume that prices will rise over the very long term. However, this leaves open the question of whether these price rises will be rapid enough to affect storage management. There are two cases in which this can occur. The first is if energy storage by individual customers allows “panic buying” in response to short-term price fluctuations [21]. The second, the focus of this paper, is if storage is used to smooth seasonal fluctuations in the availability of renewable energy. This smoothing will occur over the timescale of months or years, during which the underlying price can rise by a non-negligible amount.

Let us first look at the impact of rising prices on the management of ideal storage. We will take the standard economic model that the relative rate of change of price is constant (i.e., that price grows exponentially), and take  $P(t) = (1 + Q)^t$ . Figure 7 shows a trend complementary to that of Fig. 5, with the horizontal axis reversed; that is, the “ideal” case (no price increases) is on the left this time rather than the right. Once again, the less ideal the case is (the faster the price rise), the less peak shaving occurs, and the less benefit there is to having more storage.

This suggests that the use of leaky storage when prices are rising would be doubly bad. However, as Figure 8 shows, this is not the case. In fact, there is a dip in peak generation, which becomes more pronounced as the storage capacity increases.

A hint as to the reason can be seen by considering the actual generation when the price is increasing but there is no loss. Figure 9 shows that the generation *decreases* between discontinuities when the price is increasing, in contrast to the increase seen in Figure 2 for the case of leakage with constant prices. Intuitively, it is sensible to charge the storage early when energy is cheaper. This suggests that the dip may occur when the two effects “cancel out” in the sense of having a flat generation profile between discontinuities. Since  $P(t) = (1 + Q)^t$ , (4) shows that this occurs when  $(1 + Q)^t S^t$  is constant, that is,  $(1 + Q)S = 1$ . Moreover, the form in (3) suggests that  $g^*$  is the same for any  $Q$  and  $S$  such that  $(1 + Q)S = 1$ . However, note that  $\underline{\beta}^*$  and  $\bar{\beta}^*$  in (3) are Lagrange multipliers,

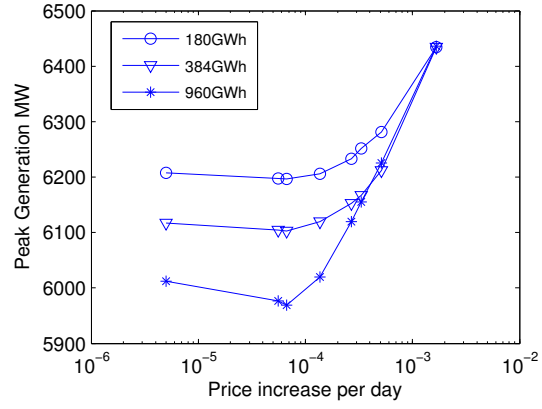


Fig. 8: Peak generation with both leakage and rising prices. Leakage 0.2% per month ( $\approx 6 \times 10^{-4}$  per day). Note the minimum peak generation occurs when the price increase roughly matches the leakage rate.

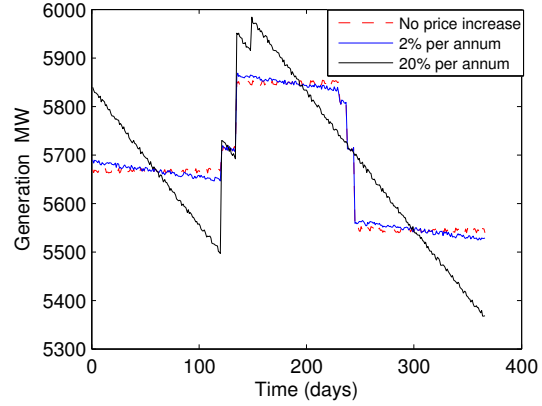


Fig. 9: Optimal generation with rising prices, without leakage. Generation decreases between steps. Price increase 0%, 12% and 20% per annum.

and depend on  $Q$  and  $S$ . Hence  $R$  and  $g^*$  may also depend on  $Q$  and  $S$  even subject to the constraint  $(1 + Q)S = 1$ . This is investigated in Figure 10, which shows that increasing self discharge increases the amount of peak generation required. That is, although the fluctuations of Figures 2 and 9 have been flattened out, they are flattened out at a higher level for large  $Q$  and  $1/S$ . This is not surprising, since the leakage increases the total generation required.

Note also that minimizing the peak generation is one of many objectives. In an energy constrained future, the most important goal is to minimize the total energy consumption used to meet a given demand. Even though having rising prices can cancel the effect that leakage has on the peak generation, it cannot avoid the fact that leakage still wastes energy. In fact, having rising prices actually *increases* the amount of energy lost. This can be seen in Figure 11, which plots the total power generation against the loss rate for different rates of price increase. Recall that the reason for the reduction in generation for very high loss was that it becomes optimal to eschew use of storage altogether. However, when prices are rising, there is an increased financial benefit in using storage,

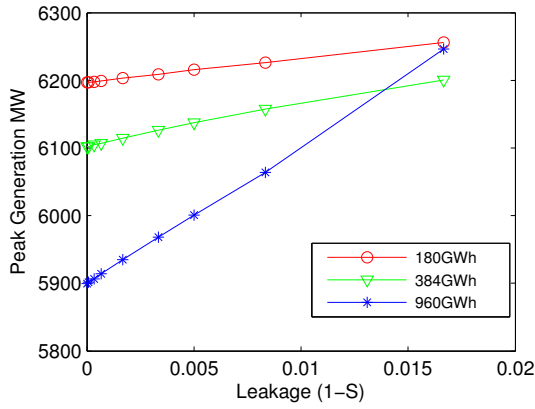


Fig. 10: Peak generation as a function of leakage, subject to the constraint  $(1 + Q)S = 1$ , i.e., generation is piecewise constant.

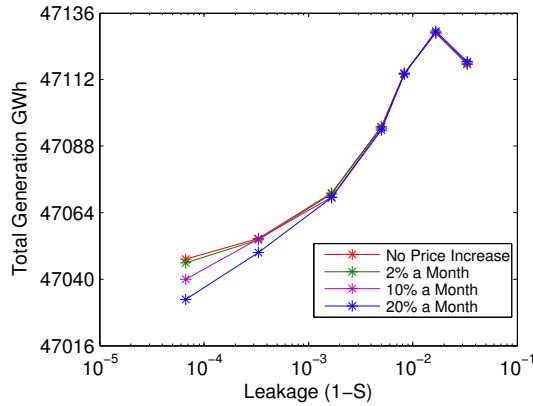


Fig. 11: Total generation vs loss with increasing prices. Note the conflict between obtaining minimum peak generation minimum total generation.  $B = 960$  GWh

and so it remains in use even when it incurs higher losses.

## V. CONCLUSION

In contrast to most work, this paper has considered storage for periods of months or years rather than hours. It studied the impact of energy storage loss and rising prices on the optimal management of energy storage. Both factors are important when energy is stored for long periods, as required in renewable-only systems.

Each factor reduces the ability of storage to reduce the peak demand, but using opposite mechanisms: energy leakage causes the optimal generation to be piecewise increasing, whereas increasing prices cause it to be piecewise decreasing. Hence peak shaving is greatest when the rate of price increase approximately matches the rate of energy leakage such that, in the absence of charging or discharging, the total market value of the stored energy remains constant. This causes a local minimum in the peak generation as a function of leakage rate.

Conversely, there is a local maximum in the total generation; this is because the total generation matches the total demand if either there is no leakage or there is so much leakage that the storage is hardly used.

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## REFERENCES

- [1] K. Agbossou, M. Kolhe, J. Hamelin, and T. Bose, "Performance of a stand-alone renewable energy system based on energy storage as hydrogen," *IEEE Trans. Energy Conversion*, vol. 19, no. 3, pp. 633–640, 2004.
- [2] E. Akpinar and S. Akpinar, "An assessment on seasonal analysis of wind energy characteristics and wind turbine characteristics," *Energy Conversion and Management*, vol. 46, no. 11, pp. 1848–1867, 2005.
- [3] "Energy: Coal consumption peaks as hydro plummets," *Portugal Economy Probe*, Aug. 2012.
- [4] J. Leahy, "Fears grow of brazil power shortages," *Financial Times*, *FT.com*, Jan. 2013.
- [5] A. Rudd, "Hydro lake levels on alert 'cusp'," *Otago Daily Times*, May 2012.
- [6] A. O. Converse, "Seasonal energy storage in a renewable energy system," *Proceedings of the IEEE*, vol. 100, no. 2, pp. 401–409, 2012.
- [7] S. Schoenung, "Characteristics and technologies for long-vs. short-term energy storage," *Sandia Report SAND2001-0765*, 2001.
- [8] EPRI, *Handbook of Energy Storage for Transmission or Distribution Applications*, 2002.
- [9] O. Wu, D. Wang, and Z. Qin, "Seasonal energy storage operations with limited flexibility: The price-adjusted rolling intrinsic policy," *Manufacturing & Service Operations Management*, vol. 14, no. 3, pp. 455–471, 2012.
- [10] A. Cahn, "The warehouse problem," *Bull. Amer. Math. Soc.*, vol. 54, p. 1073, 1948.
- [11] G. Lai, F. Margot, and N. Secomandi, "An approximate dynamic programming approach to benchmark practice-based heuristics for natural gas storage valuation," *Oper. Res.*, vol. 58, no. 3, pp. 564–582, 2010.
- [12] Y. Zhou, A. Scheller-Wolf, N. Secomandi, and S. Smith, "Is it more valuable to store or destroy electricity surpluses?," *Available at Social Sciences Research Network (SSRN)*, 2012.
- [13] K. Chandy, S. Low, U. Topcu, and H. Xu, "A simple optimal power flow model with energy storage," in *Proc. IEEE Conference on Decision and Control (CDC)*, pp. 1051–1057, Dec. 2010.
- [14] D. MacKay, *Sustainable Energy — without the hot air*. UIT Cambridge, 2008.
- [15] F. Figueiredo, P. Flynn, and E. Cabral, "The economics of energy storage in 14 deregulated power markets," *Energy Studies Review*, vol. 14, no. 2, p. 2, 2006.
- [16] F. Graves, T. Jenkin, and D. Murphy, "Opportunities for electricity storage in deregulating markets," *The Electricity Journal*, vol. 12, no. 8, pp. 46–56, 1999.
- [17] R. Sioshansi, P. Denholm, T. Jenkin, and J. Weiss, "Estimating the value of electricity storage in pjm: Arbitrage and some welfare effects," *Energy Economics*, vol. 31, no. 2, pp. 269–277, 2009.
- [18] R. Walawalkar, J. Apt, and R. Mancini, "Economics of electric energy storage for energy arbitrage and regulation in new york," *Energy Policy*, vol. 35, no. 4, pp. 2558–2568, 2007.
- [19] R. N. Jesudasan, L. L. H. Andrew, and H. L. Vu, "Managing inefficient storage," in *IEEE Power and Electrical Society General Meeting*, 2013.
- [20] Australian Energy Market Operator. <http://www.aemo.com.au/Electricity/Data/Price-and-Demand/Aggregated-Price-and-Demand-Data-Files/Aggregated-Price-and-Demand-2011-to-2015>.
- [21] R. N. Jesudasan, L. L. H. Andrew, and H. L. Vu, "User-optimal storage with rising energy prices," Tech. Rep. CAIA Technical report 120906A, Swinburne University of Technology, 2012.