Unidirectional Probabilistic Direct Control for Deferrable Loads

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***Abstract*—The idea of harnessing the inherent flexibility in demand of many types of electric loads has been largely discussed in the last years for coping with the need to maintain the energy demand-supply balance. In particular, the fine tuning of the operation conditions of different thermostatic loads (such as air-conditioning, refrigerators, etc.) has appeared as the most natural solution for load control with minimal user discomfort. In this paper we focus on an alternative approach: deploying simple open-loop control strategies for deferrable loads with minimal communication overhead. The idea is to send a multicast control message to a group of users, on the basis of the expected and desired load profiles, for probabilistically enabling or deferring the activation requests of a specific load type. The control law and the most important performance metrics can be easily derived analytically. Despite the simplicity of the approach, which requires minimal or null investments, our results show that significant load shifts can be achieved.**

I. Introduction

Load control in modern power grids is becoming more and more important for maintaining a balance between energy supply and demand. Traditionally, the demand was much more variable and less controllable than supply, so that the energy balance was achieved by adapting dynamically generation levels to match the consumption. The increasing penetration of renewable energies has radically changed the scenario, due to their lower predictability. The possibility to control the load demand is then becoming more appealing for several actors, such as the energy utilities (that can better plan the production as well as control the grid reliability) and the end customers (that can actively participate to the energy market).

However, despite the many proposals in the literature [1] discussing different demand response programs, load control for residential users (which significantly affect the overall energy load variability [2]) is still limited to pilot projects [3], [4] with little penetration perspective in the near future. The reason is that the implementation of these mechanisms requires investments (for updating user appliances and communication infrastructures) which are not clearly justified for the end users.

In this paper we focus on a low-cost deployment of direct load control for deferrable appliances in a large scale power grid, with very limited infrastructure investments for communication and appliances' control. Our mechanism only requires the controller to periodically send a control message, specifying the *control policy* for a given type of appliances to a group of residential users. The control policy is expressed in terms of the probability that an appliance activation request originated at a given time of the day may be satisfied. The idea is to implement an open-loop probabilistic control on the expected aggregated load generated by groups of residential users without requiring bidirectional control messages or high-frequency metering readings (which may be critical for privacy issues and communication overhead). Indeed, the activation probability function can be loaded once a day on the smart meters (by using the metering communication network), or on a PC acting as a home energy controller (by using an independent data network such as the user internet access network). In turn, the appliances to be controlled need to be equipped with a simple actuator able to send the activation requests to the smart meter or home controller and to respond to the enabling (disenabling) signals by connecting (disconnecting) the appliance to the electric socket. The cost of similar actuators is then relatively small (e.g. not exceeding 40$ for WiFi, 4G or power line network interfaces).

Moreover, the actuator does not require any specific knowledge of the appliance it is controlling, so it can be simply implemented as a small device to be interposed between the socket and the appliance itself. Low-cost and appliance agnosticism are two important features that make our solution attractive for a large scale deployment of direct load control also to older "non-smart" appliances and then to target the residential customers. This would allow the electric utility to control a significant fraction of the power demand, thus enabling economies of scale and further reducing the costs.

The counterpart of the absence of a feedback from the appliances is that our control cannot provide deterministic guarantees on the total power consumption, but only probabilistic ones. In particular the control signal will be determined in order to assure a maximum probability to exceed a given constant bound on the power consumption. As it will become clearer from the rest of the paper, the better the a priori knowledge of the system (in terms for example of the prediction of users' power consumption) the less stringent the control signal to be implemented, i.e. the smaller the delay the users will experience.

II. Related Work

Direct load control is a mechanism that allows electric utilities to turn specific users' appliances on and off during peak demand periods and critical events. The problem has been largely studied in literature with several proposals formulating the control mechanism under different optimization objectives, related to the power grid reliability or operation savings. The usual approach is based on a *central controller*, working on the basis of dynamic programming optimization [5], [6], fuzzy logic-based decisions [7], or other profit maximization schemes [8]. An admission control mechanism based on the exact knowledge of the total load generated by the controlled users has also been proposed in [9].

Recently, some utilities have been involved in pilot projects enrolling real users in direct load control programs. The appliances considered in these programs are usually limited to air conditioners, water heaters and pool pumps [3], [4]. In [3] the control signal is based on the traditional monitoring of the power-grid voltage and frequency signals, in order to detect critical load conditions in terms of variations of the expected signals. Users appliances were modified to respond to the underfrequency signals by reducing their energy demand. Conversely, in [4], an energy management device, connected to the utility controller, has been provided to the users for switching on and off traditional (unmodified) appliances. In both cases, investments have been required for modifying the appliances or deploying the control network.

The bandwidth requirements of the control network, as well as the privacy concerns arising in case of continuous monitoring of users' loads, have been addressed in some recent work proposing some simplifications of the optimal control schemes or *distributed controllers*. For example, in [10], the tradeoff between the importance of exact load characterization (exploration) and control (exploitation) has been analyzed in a restless bandit framework, according to which loads are ranked for their relevance to demand-response actions. In [11] a distributed controller for a pool pump is designed on the basis of a Markovian Decision Process model with randomized decisions for avoiding synchronization of pools in the grid. An aggregated model for a large collection of loads operating under the same controller is then broadcast by the utility to all the users for driving the decision process. A similar approach based on a generalized input signal for a group of users and distributed control actions is pursued in this paper, but is applied to deferrable uninterruptible loads.

III. Model and Control

In our scenario the energy utility would like to enforce a given power consumption level P_g during a time interval $[T_s, T_e]$ for the set of appliances under control. Our solution provides probabilistic guarantees: the instantaneous power consumption ($P_c(t)$) can exceed P_g with probability at most ϵ, i.e., $\text{Prob}\{P_c(t) > P_g\} \leq \epsilon$.

A. Appliance Model

Our methodology applies to deferrable appliances, whose activation time can be postponed, such as washing machines or laundry machines. We can easily take into account different types of appliances, but, we only consider a single class in this paper to keep the exposition simple. We assume that every appliance in this class has the same deterministic operation time D and its instantaneous power consumption is a random variable $X(t)$ with known time-invariant probability density function $f_X(x)$. This probabilistic description can easily incorporate the incertitude about the characteristic of the appliance.

Some statistical studies [12], [13] have characterized the percentage of users activating a specific residential appliance along different intervals of the day. In these studies, the day is divided into equal size intervals and the percentage of active users is averaged in each interval. Table I has been obtained from the data in [12] and shows the percentage of dishwashers/laundry machines active during 2-hour time intervals. Assuming that the user population U is large enough and considering an observation time of one day, we can model the activation instants of a given appliance as a non-homogeneous Poisson process with arrival rate $\lambda(t)$. For our numerical experiments we used the empirical arrival rate for the laundry machines with 30 minute granularity.

Appliance	6	8	10	12	14	16	18	20	22	24
Dishwasher.	3	9	9	3	13	0	16	38	13	3
Laundry m.	16	28	38	19	16	19	16	16	3	6

TABLE I. Appliance activation rates [% over 30000 users]

In the absence of any control, the appliances that are still active at time t are those turned on during the time interval $[t - D, t]$. Their number is then distributed as a Poisson random variable with parameter equal to the expected number of arrivals in $[t - D, t]$. Let $\text{Pois}(x)$ denote a Poisson random variable with parameter x, the instantaneous power consumption $P(t)$ at time t can then be calculated as

$$P(t) = \sum_{i=1}^{N(t)} X_i(t), \text{ where } N(t) \sim \text{Pois}\left(\int_{t-D}^{t} \lambda(\tau)\mathrm{d}\tau\right). \quad (1)$$

B. Activation Model under Probabilistic Control

We propose a control mechanism devised to modify the appliance activation process to a different non-homogeneous Poisson process with rate $\lambda_c(t)$, where $\lambda_c(t)$ is determined so that the corresponding power consumption $P_c(t)$ satisfies the probabilistic constraint imposed by the utility. Observe that $P_c(t)$ can be expressed through (1), simply replacing $\lambda(t)$ with $\lambda_c(t)$. We first describe our control, i.e., how the arrival rate will be modified from $\lambda(t)$ to $\lambda_c(t)$ and then we calculate how to set $\lambda_c(t)$.

In our framework the utility applies a time-varying activation control function $p(t)$ during a time interval $[T_{sc}, T_e] \supset [T_s, T_e]$. As we are going to discuss later, we consider $T_{sc} = T_s - D$. During this time interval, if the user turns on an appliance at time t, the appliance will actually start with probability $p(t)$, and with probability $1 - p(t)$ the decision about its activation will be postponed to time $t + T$. This simple algorithm can be implemented directly from the smart appliance, or from a device to be interposed between the plug and the appliance itself.

The scheme operation is depicted in Fig 1. The first time axis of the figure shows a sample of the point process ($\mathcal{P}$) of the time instants at which the user would like to turn on the appliance. The outcomes of the Bernoulli random variables drawn for every request determine two different point processes that are distinguished in the second time axis by two different marks, corresponding to the requests that are immediately accepted (point process $\mathcal{P}_a$ denoted by the checkmarks) and those that are deferred (point process $\mathcal{P}_d$ denoted by the crosses). The probability $p(t)$ is determined exogenously and independently from the given sample of the Poisson process $\mathcal{P}$, then both $\mathcal{P}_a$ and $\mathcal{P}_d$ are (non-homogeneous) Poisson

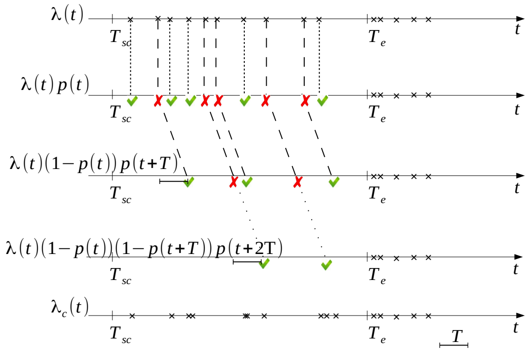

Fig. 1. Example of activation request point processes. The first time axis shows the time instants at which requests are first considered (and when they would be served without the control). The checkmarks in the second, third and fourth time axis respectively correspond to the requests that are immediately satisfied, deferred once and deferred twice. The last time axis shows the aggregate point process of the time instants at which the requests are satisfied.

processes respectively with rates $p(t)\lambda(t)$ and $(1-p(t))\lambda(t)$, and they are independent from each other [14, ch. 5]. The decision about the requests arrived in $\mathcal{P}_d$ is postponed by T time units as it is shown in the third time axis in Fig. 1. The shifted point process is still a Poisson one with rate $(1-p(t-T))\lambda(t-T)$ and independent from $\mathcal{P}_d$. At its turn, the shifted point process may be split in two point processes $\mathcal{P}_{d,a}$ and $\mathcal{P}_{d,d}$ respectively of the requests accepted at the second trial or further deferred. A similar reasoning leads to the conclusion that $\mathcal{P}_{d,a}$ and $\mathcal{P}_{d,d}$ are independent Poisson processes with rates respectively $p(t)(1-p(t-T))\lambda(t-T)$ and $(1-p(t))(1-p(t-T))\lambda(t-T)$ and they are also independent from $\mathcal{P}_a$.

A request can be deferred at most $K_{max} = \lceil (T_e - T_{sc})/T \rceil$ times. We can then build $\lceil (T_e - T_{sc})/T \rceil$ independent Poisson processes $\mathcal{P}_a, \mathcal{P}_{d,a}, \mathcal{P}_{d,d,a}, \ldots \mathcal{P}_{d,\ldots,d,a}$. Their superposition is still a Poisson process, whose points are the time instants at which the appliances become active. We denote by $\lambda_c(t)$ its rate, where:

$$\lambda_c(t) = p(t) \cdot \sum_{k=0}^{K_{max}} \lambda(t-kT) \prod_{i=1}^{k} (1 - p(t-iT)). \quad (2)$$

As we anticipated above, the effect of the probabilistic control is to transform the initial uncontrolled Poisson process $\mathcal{P}$ with rate $\lambda(t)$ into a Poisson process with rate $\lambda_c(t)$. All the requests arriving in the interval $[T_{sc}, T_e + T]$ are served in the same interval, then it follows that $\int_{T_{sc}}^{T_e+T} \lambda(\tau)\mathrm{d}\tau = \int_{T_{sc}}^{T_e+T} \lambda_c(\tau)\mathrm{d}\tau$. We can also define the point process $\mathcal{P}_{eq}$ as the sequence of time instants of all the requests, independently from them being accepted or deferred, whose rate is $\lambda_{eq}(t) = \sum_{k=0}^{K_{max}} \lambda(t-kT) \prod_{i=1}^{k} (1-p(t-iT))$. The rate of the controlled process can then be expressed simply as $\lambda_c(t) = p(t)\lambda_{eq}(t)$. We observe that $\mathcal{P}_{eq}$ is not in general a Poisson process (because for a point in t there is a point in $t+T$ with probability $1-p(t)$). Moreover a request can generate multiple points in $\mathcal{P}_{eq}$, then $\int_{T_{sc}}^{T_e+T} \lambda_{eq}(\tau)\mathrm{d}\tau \geq \int_{T_{sc}}^{T_e+T} \lambda(\tau)\mathrm{d}\tau$.

C. Tuning of the Activation Control Function

We now consider how $p(t)$ can be determined to guarantee that $\mathrm{Prob}\{P_c(t) > P_g\} \leq \epsilon$. The process $P_c(t)$ is completely characterized by the knowledge of the controlled arrival density $\lambda_c(t)$ and the power consumption density $f_X(.)$ for a single appliance. The degree of freedom which can be controlled by selecting $p(t)$ is the expected number $n_c(t)$ of appliances active at time t. It is possible to calculate $n_c(t)$ from $\mathrm{Prob}\{P_c(t) > P_g\} \leq \epsilon$ and then $\lambda_c(t)$ and $p(t)$ can be determined from $n_c(t) = \int_{t-D}^{t} \lambda_c(\tau)\mathrm{d}\tau$. For simplicity we develop the calculations for the case when $n_c(t)$, the expected number of appliances active at a given time instant, is large and the aggregated power consumption can be approximated by a normal distribution. This case is also probably the most relevant from a practical point of view, given that we are interested in controlling a large number of appliances. Being $P_c(t) = \sum_{i=1}^{N_c(t)} X_i$, with $N_c(t) \sim \mathrm{Pois}(n_c(t))$, it holds [15, ch.1]:

$$\mathrm{E}\left[P_c(t)\right] = n_c(t)\mathrm{E}\left[X\right], \text{ and } \mathrm{Var}\left(P_c(t)\right) = n_c(t)\mathrm{E}\left[X^2\right].$$

$P_c(t)$ can be approximated as a normal variable when $n_c(t)$ is large.[1] Under such approximation $\Pr\{P_c(t) > P_g\}$ is lower than ϵ if and only if $(P_g - n_c(t)\mathrm{E}\left[X\right])/\sqrt{n_c(t)\mathrm{E}\left[X^2\right]} \geq z_{1-\epsilon}$, where $z_{1-\epsilon}$ is the ϵ percentile of the standard normal distribution. The power consumption profile is then satisfied if:

$$n_c(t)\mathrm{E}\left[X\right] + z_{1-\epsilon}\sqrt{n_c(t)\mathrm{E}\left[X^2\right]} \leq P_g.$$

We can determine the maximum value n^* that satisfies this inequality, i.e. the solution of $n^*\mathrm{E}\left[X\right] + z_{1-\epsilon}\sqrt{n^*\mathrm{E}\left[X^2\right]} = P_g$ or equivalently of the quadratic equation

$$\left(n^*\mathrm{E}\left[X\right] - P_g\right)^2 = (1-\epsilon)^2 n^* \mathrm{E}\left[X^2\right].$$

The control $p_c(t)$ has to generate a controlled rate $\lambda_c(t)$ such that $n_c(t) \leq n^*$ for each $t \in [T_s, T_e]$. Note that in general we need to apply the control also before T_s, because we do not want to block an already active appliance. By applying the

[1]Consider $Y = \sum_{k=1}^{N} X_k$ where $N \sim \mathrm{Pois}(n)$, X_k are i.i.d. random variables with $\mathrm{E}\left[X_k\right] = \mu$ and $\mathrm{E}\left[X_k^2\right] = \nu^2$. Let $\varphi_X(t) := \mathrm{E}\left[\exp(iX_k t)\right]$ and $\varphi_Y(t) := \mathrm{E}\left[\exp(iY)\right]$ be respectively the characteristic functions of X_k and Y. It holds [15, ch.1] $\varphi_Y(t) = \exp\left(n \cdot (\varphi_X(t) - 1)\right)$. We want to prove that

$$Z_n = \frac{(\sum_{k=1}^{N} X_k) - n\mu}{\sqrt{n\nu^2}} \xrightarrow[d]{n\to\infty} \mathcal{N}(0,1),$$

where $\mathcal{N}(0,1)$ denotes the standard normal distribution. We will calculate the characteristic function of Z_n ($\varphi_{Z_n}(t)$) and show that, when n diverges, this function converges to that of $\mathcal{N}(0,1)$, i.e. to $\exp(-t^2/2)$.

$$\begin{aligned}
\varphi_{Z_n}(t) &= \mathrm{E}\left[\exp(iZ_n t)\right] = \mathrm{E}\left[\exp\left(it\frac{(\sum_{k=1}^{N} X_k) - n\mu}{\sqrt{n\nu^2}}\right)\right] \\
&= \exp\left(-it\frac{\mu\sqrt{n}}{\nu}\right)\mathrm{E}\left[\exp\left(it\frac{(\sum_{k=1}^{N} X_k)}{\sqrt{n\nu^2}}\right)\right] \\
&= \exp\left(-it\frac{\mu\sqrt{n}}{\nu}\right)\exp\left(n\left(\varphi_X\left(\frac{t}{\sqrt{n\nu^2}}\right) - 1\right)\right),
\end{aligned}$$

given that

$$\varphi_X\left(\frac{t}{\sqrt{n\nu^2}}\right) = 1 + i\frac{t}{\sqrt{n\nu^2}}\mu - \frac{t^2}{2n\nu^2}\nu^2 + o\left(\frac{t^2}{n\nu^2}\right)$$

then

$$\varphi_{Z_n}(t) = \exp\left(-\frac{t^2}{2}\right)\exp\left(n \cdot o\left(\frac{t^2}{n\nu^2}\right)\right) \xrightarrow[n\to\infty]{} \exp\left(-\frac{t^2}{2}\right)$$

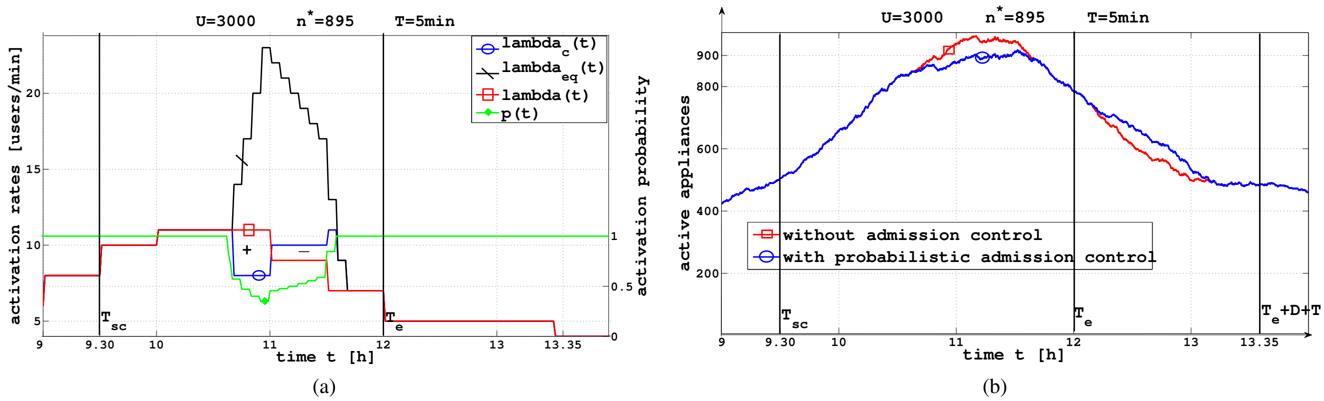

Fig. 2. Activation request rates and activation probability (a); instantaneous number of active appliances at a given time instant (b).

control starting from $T_{sc} = T_s - D$, we can be sure that for any expected number of active appliances at T_{sc}, it is possible to guarantee that $n_c(T_s) \leq n^*$. While respecting this constraint, we would like $n_c(t) = \int_{t-D}^{t} p(\tau)\lambda_{eq}(\tau)\mathrm{d}\tau$ to be as close as possible to n^*, in order to admit as many requests as possible and avoid useless delays. Then at time t the control should i) admit new activation requests if $n_c(t) < n^*$, ii) block them if $n_c(t) > n^*$, iii) try to admit new requests at the same rate at which the appliances are terminating their activation period if $n_c(t) = n^*$. This is implemented by the following control probability:

$$p(t) = \begin{cases} 0 & \text{if} \quad n_c(t) > n^* \\ \min(1, \frac{p(t-D)\lambda_{eq}(t-D)}{\lambda_{eq}(t)}) & \text{if} \quad n_c(t) = n^* \\ 1 & \text{if} \quad n_c(t) < n^* \end{cases} \tag{3}$$

for $t \in [T_{sc}, T_e]$. About the second case, we observe that $p(t-D)\lambda_{eq}(t-D) = \lambda_c(t-D)$ is the rate at which new appliances are activated at $t-D$ and then also the rate at which appliances terminate D time units later. We would like $p(t)\lambda_{eq}(t) = p(t-D)\lambda_{eq}(t-D)$ in order to maintain the expected number of active appliances equal to n^*, but this is not possible if $\lambda_{eq}(t) < p(t-D)\lambda_{eq}(t-D)$.

Practically speaking, the utility is not going to transmit a continuous time function to each appliance (or actuator device), but rather a sequence of probability values. Let us then consider for simplicity a discrete time version of the control so that $p(k)$ is the control probability applied during the time interval $[k, k+1)$. Moreover, let us assume that the activation time D corresponds exactly to d time steps, and T to exactly s time steps, and let $\Lambda(k) \triangleq \int_k^{k+1} \lambda(\tau)\mathrm{d}\tau$. From $\lambda_{eq}(t) = \sum_{k=0}^{K_{max}} \lambda(t-kT) \prod_{i=1}^{k}(1-p(t-iT))$, it follows that $\Lambda_{eq}(k) \triangleq \int_k^{k+1} \lambda_{eq}(\tau)\mathrm{d}\tau = \sum_{h=0}^{K_{max}} \Lambda(k-h\cdot s) \prod_{i=1}^{h}(1-p(k-i\cdot s))$. It holds:

$$n_c(k+1) = n_c(k) - p(k-d)\Lambda_{eq}(k-d) + p(k+1)\Lambda_{eq}(k+1). \tag{4}$$

As above the control probability sequence $p(k+1)$ will try to get $n_c(k+1) = n^*$ and can be obtained from the following compact equation that corresponds to Eq. (3) and can be easily calculated as $p(k+1) =$

$$\max\left\{\min\left\{\frac{n^* - n_c(k) + p(k-d)\Lambda_{eq}(k-d)}{\Lambda_{eq}(k+1)}, 1\right\}, 0\right\}. \tag{5}$$

Fig. 2 shows an example of the interplay of the different arrival rates and the activation control probability. We considered $U = 3000$ residential users whose laundry machines are controlled. We assumed the activation time of a laundry machine to be $D = 90$ minutes and its instantaneous power consumption to be constant[2] $X = 1.5$ kW. The utility would like to enforce a power consumption $P_g = 1.4$ MW between $T_s = 11$am and $T_e = 12$am that can be exceeded at most with probability $\epsilon = 0.1$. The power cap P_g corresponds to a maximum expected number of appliances active at the same time $n^* = 895$. The control starts at time $T_{sc} = T_s - D = 9.30$am.

Fig. 2(a) shows the activation rates $\lambda(t)$, $\lambda_c(t)$ and $\lambda_{eq}(t)$, as well as the control probability $p(t)$, while the Fig. 2(b) shows the instantaneous number of appliances active for one simulation of the request arrival process. At the beginning of the control period the expected number of active appliances is smaller than n^* so that there is no need to postpone activation times ($p(t) = 1$) and the controlled activation rate is equal to the uncontrolled one and to the equivalent activation request rate because no request is deferred ($\lambda(t) = \lambda_c(t) = \lambda_{eq}$), as it is shown in Fig. 2a. The activation rate $\lambda(t)$ increases to 11 arrivals per minutes at 10am. Slightly after 10.30am, the uncontrolled process would exceed the threshold n^*, so that the probabilistic control becomes effective and reduces $\lambda_c(t)$ to 8 arrivals per minutes, i.e. to the rate at which appliances are terminating, that is equal to the rate at which appliances were admitted $D = 90$ minutes earlier (as indicated in the second case of Eq. (3)). There are $3(= 11 - 8)$ requests every minute that are not satisfied and are postponed to $T = 5$ minutes later, increasing then $\lambda_{eq}(t)$ to 14 request per minute. Until 11am, $\lambda(t)$ and $\lambda_c(t)$ do not change, so the backlog of unsatisfied requests keeps increasing, as it is shown by $\lambda_{eq}(t)$ which increases of 3 requests per minute every 5 minutes. Due to the increase of $\lambda_{eq}(t)$, $p(t)$ has to decrease accordingly in order to keep $\lambda_c(t)$ constant. Starting from 11am $\lambda_c(t)$ becomes equal to 10 arrivals per minute, the same value it had 90 minutes earlier. At the same time $\lambda(t)$ decreases to 9 arrivals per minutes. The system is then able to reduce some of the backlog accumulated and indeed we see that $\lambda_{eq}(t)$ decreases. Similarly to what happened before, $p(t)$ now increases to maintain a constant $\lambda_c(t)$ with a decreasing $\lambda_{eq}(t)$. At 11.30am there is

[2]Note that in this case $P_c(t)$ is proportional to a Poisson random variable with parameter $n_c(t)$ and the normal approximation we consider is the usual normal approximation for a Poisson random variable.

a further increase of $\lambda_c(t)$ and a further decrease of $\lambda(t)$. A few minutes later the backlog is depleted and then $p(t) = 1$ and $\lambda(t) = \lambda_c(t) = \lambda_{eq}(t)$.

In the Fig. 2(b) we see how during the whole period when $p(t) < 1$ the instantaneous number of active appliances is smaller than it would have been without any control. In particular it never exceeds 933 that would generate an instantaneous power demand above $P_g = 1.4$ MW (remember that this can happen but with a probability smaller than 10%). Fig. 2(b) shows also how the control can have some effects after T_e: the deferred departure of some appliances during the time interval when $p(t) < 1$, i.e. roughly between 10.40am and 11.35am, leads to their deferred end, so that 90minutes later the number of active appliances is larger in the presence of the control.

D. Delay Analysis

We calculate now the average delay of the starting time for an appliance that would like to start in $[T_{sc}, T_e]$. Let $N(t)$ be the number of activation requests arrived for the first time by time t and $N_c(t)$ be the number of appliances started by time t. Fig. 3 shows a possible evolution of $N(t)$ and $N_c(t)$. Clearly $N(t) = N_c(t)$ for $t \in [0, T_{sc}]$ and for $t > T_e + T$, because no control is applied before T_{sc} and after T_e and all the requests deferred are finally accepted by $T_e + T$. The total delay experienced by all the appliances is

$$\int_{T_{sc}}^{T_e+T} (N(\tau) - N_c(\tau))\, \mathrm{d}\tau,$$

with expected value $\int_{T_{sc}}^{T_e+T} \int_{T_{sc}}^{\tau} (\lambda(x) - \lambda_c(x))\, \mathrm{d}x \mathrm{d}\tau$. The appliances arrived during the interval $[T_{sc}, T_e]$ are $N(T_e) - N(T_{sc})$ with expected value $\int_{T_{sc}}^{T_e} \lambda(x)\mathrm{d}x$.

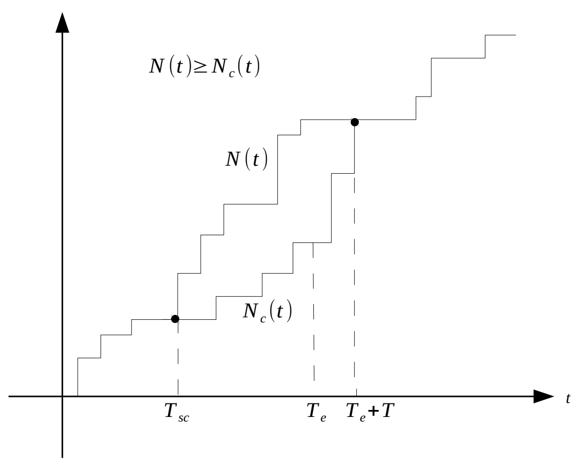

Fig. 3. Total number of activation requests satisfied by time t without control ($N(t)$) and with the probabilistic control ($N_c(t)$)

Let us now imagine S samples of the request arrival process $N^{(i)}(t)$ and the corresponding $N_c^{(i)}(t)$. The average delay can be defined as

$$\frac{\sum_{i=1}^{S} \int_{T_{sc}}^{T_e+T} \left(N^{(i)}(\tau) - N_c^{(i)}(\tau)\right) \mathrm{d}\tau}{\sum_{i=1}^{S} \left(N^{(i)}(T_e) - N^{(i)}(T_{sc})\right)}.$$

The expected average delay is then

$$\begin{aligned} \mathrm{E}[W] &= \lim_{S\to\infty} \frac{\sum_{i=1}^{S} \int_{T_{sc}}^{T_e+t} \left(N^{(i)}(\tau) - N_c^{(i)}(\tau)\right) \mathrm{d}\tau}{\sum_{i=1}^{S} \left(N^{(i)}(T_e) - N^{(i)}(T_{sc})\right)} \\ &= \frac{\int_{T_{sc}}^{T_e+T} \int_{T_{sc}}^{\tau} (\lambda(x) - \lambda_c(x))\, \mathrm{d}x\mathrm{d}\tau}{\int_{T_{sc}}^{T_e} \lambda(x)\mathrm{d}x}, \end{aligned}$$

because of the renewal theorem. Then the average delay experienced by a user who would like to start its appliance in $[T_{sc}, T_e]$ can be obtained dividing the expected total delay by the expected number of requests in the interval $[T_{sc}, T_e]$.

E. Complexity

The envisioned control scheme has a very limited complexity both at the utility side (for evaluating the admission control function $p(t)$) and at the user side (for evaluating the admission of a novel activation request of the controlled appliance). As discussed in III-C, for practical reasons the function $p(t)$ has to be evaluated as a discrete-time function $p(k)$. Being M the number of discrete time intervals in $[T_{sc}, T_e]$, the utility controller has to evaluate M times: i) $n_c(k)$ by applying Eq. (4) which requires two multiplications, ii) $p(k)$ by applying Eq.(5) which requires two comparisons, one multiplication and one division; iii) $\Lambda_{eq}(k)$, that can be conveniently computed as $\Lambda(k) + \Lambda_{eq}(k-s)[1-p(k-s)]$ with one more multiplication. Without considering partial storing of the results (which can reduce the number of overall operations), it follows that the total number of operations is linear in M. With a discrete time interval of 1 minute and the usual control interval $[9.30\text{am}, 12\text{am}]$, such a complexity is of the order of hundreds of operations (thus resulting really negligible).

By assuming to send a quantized value of b bytes for each discrete-time probability $p(k)$, it is also necessary to send a broadcast message of $M \cdot b$ bytes to all the users enrolled in the control program. Finally, at the user side, when a new activation request is generated at the discrete time k, during the control interval it is only required to extract an average number of random numbers equal to $\sum_{i=0}^{K_{max}} (i+1)p(k+i \cdot s) \prod_{h=0}^{i-1}(1-p(k+h \cdot s))$.

IV. Numerical Results

In order to quantify the impact of the probabilistic control mechanism on the achievable load reduction and user discomfort, we applied our model for evaluating the average activation delay as a function of different tunable parameters of the scheme.

Fig. 4 shows the average activation delay for various desired load limits P_g for a population ranging from 2500 to 3500 users. All the other parameters are the same than in the previous numerical example: the duration of the activation interval D of each appliance is fixed to 90 min with a fixed required power equal to $X = 1.5$ kW, the control mechanism is applied from T_{sc} =9.30am to T_e =12am, the probability to violate the power bound is $\epsilon = 0.1$ and a refused request is postponed by T =5 minutes. Without any control, we found that the 0.9 percentile (P_a) of the aggregated power request during the peak hour is equal to about 1185 kW, 1428 kW, and 1650 kW respectively for $U = 2500$, $U = 3000$, and

$U = 3500$. From the figure, it is evident that the average activation delay is negligible (e.g. lower than 10 minutes) for $P_g > P_a - 200$ kW.

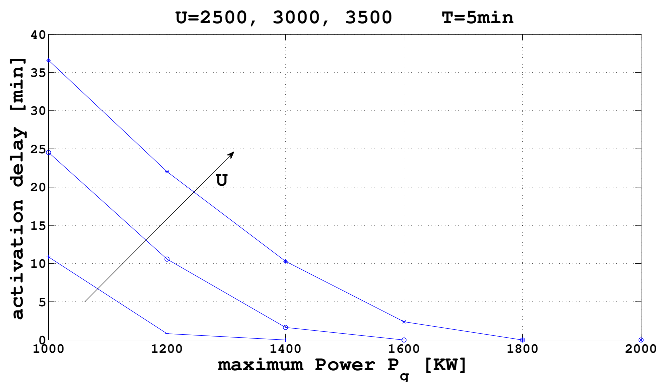

Fig. 4. Average activation delay as a function of the power limit P_g.

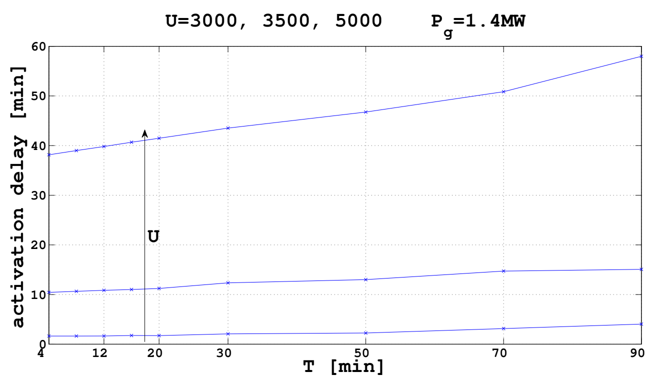

Fig. 5. Average activation delay as a function of the deferral interval T.

We also evaluated the impact of the deferral interval T. Fig. 5 plots the activation delay, obtained when P_g is fixed to 1.4 MW, as a function of T for $U = 3000$, $U = 3500$ and $U = 5000$. As expected, the delay is generally an increasing function of T. However, we also observe that tuning T is not very critical when the control mechanism is operating on a user population lower than 3500 users (i.e. when the power limitation is not too far from the power request). For example, for $U = 3500$, the activation delay is about the same for T in the range $[4, 20]$ min.

V. Conclusions and Future Works

The role of direct load control in modern power grids has been shown to be beneficial for several applications. However, in the case of small individual energy loads, these benefits can be appreciable only if a large number of users are involved in the control process. The main contribution of this paper is proposing a load control mechanism whose deployment requires minimal communication overhead in order to allow a prompt user penetration. The idea is to work on deferrable loads whose activation requests are admitted by a local energy controller on the basis of a probabilistic admission function. This function is periodically signaled by the energy utility according to the expected load demand and desired power limit. Although we evaluate the admission control function for a fixed power limit, we think it is possible to generalize the derivation for a time-varying power limit function $P_g(t)$ and we plan to work on it in the near future. An extension to the case of a non-deterministic activation time is probably much more cumbersome analytically, but we do not expect the results to be strongly dependent on the assumption of a deterministic activation time. This is also another axis that deserves further investigation.

In the current scheme, we assume that the expected load demand is simply characterized by collecting historical data, quantifying the appliance arrival rate in different intervals of the day, and assuming that these rates do not change day by day. An interesting model extension, that we are considering as a future work, is coupling the proposed control scheme with a mechanism for estimating the actual time-varying arrival rate of activation requests from the instantaneous aggregated load.

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