Robust Transmit Beamforming against Steering Vector Uncertainty in Cognitive Radio Networks

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***Abstract*—A robust transmit beamforming scheme is proposed for green cognitive radio networks in presence of incorrect steering vector estimations. The beamformer is designed using a stochastic optimization approach. The proposed beamforming scheme achieves three objectives: a) Probability for large-than-a-threshold for secondary user's received power in the steering direction is maximized; b) Probabilities for less-than-a-threshold interferences to the primary users in different locations are bounded; and c) Transmitted power along the directions of the moving secondary receiver is maintained as a constant. In addition, we present an algorithm to calculate the inverse Marcum Q function, and use it to solve the optimization problem for proposed beamforming scheme. Simulation results demonstrate that the proposed robust beamformer can achieve significant BER reductions for primary users while ensuring secondary user's BER is not sacrificed.**

I. Introduction

During the last few years, the demand for high speed broadband wireless data traffic has increased dramatically, leading to a large demand of radio frequency spectrum [1]. The radio spectrum is getting more and more crowded, and hence, there is an urgent need for techniques that can significantly improve the spectrum efficiency. Cognitive Radio (CR) [2], a promising paradigm in wireless communications that allows secondary (unlicensed) users (SUs) to share the spectrum with primary (licensed) users (PUs), has been demonstrated to provide highly efficient utilization of the scarce spectrum resource. There are two main approaches for accessing licensed spectrum in a CR: i) opportunistic access, also called overlay system, where SUs opportunistically transmit when the PUs are inactive, ii) concurrent access, also called underlay system, where SUs co-exist with the PUs as long as the interference to PUs are within an acceptable level. Previous works have shown that the access opportunities of SUs in the underlay system are much higher than that in an overlay system [3]. In this paper, we will focus on the underlay CR system.

Transmit beamforming is one of the popular ways of accessing spectrum in an underlay CR system. With transmit beamforming, the transmitters that are equipped with multiple antennas can direct the transmission towards the desired receiver while constraining interference in the directions of PUs by assigning proper weights to the antenna components. This efficient use of energy makes the transmit beamforming based underlay CR an enabling technique for green communications.

In our previous work [4], a transmit beamforming scheme was proposed for a mobile secondary user. Such beamforming technique maintains a uniform transmission power in the entire mobility range of the secondary receiver (SU-Rx) while limiting the interference to PU and other SUs. However, this scheme was based on an overly optimistic assumption – it required the perfect knowledge of steering vectors for primary and secondary receivers. In practical wireless environment, the estimates of steering vectors are error prone either due to incorrect direction estimates, or due to antenna-array imperfections. It has been shown in [5] and [6] that the effectiveness of a transmit beamformer can be destroyed even if a small mismatch arises in steering vector. In [7], a worst-case optimization method is proposed to tackle the steering vector uncertainty. In [8], a stochastic analysis approach is proposed to address the error in channel coefficients estimation. It has been shown in [8] that the stochastic approach is less conservative and provides more robustness than the worst-case optimization. However, these studies only consider stationary SUs. Such studies will not be complete unless we consider more general case where SU-Rx can either be stationary or mobile, and design a robust beamforming scheme while maintaining a uniform transmit power in the mobile range of SU-Rx. To the best of our knowledge, these issues have not been addressed yet.

The main contrubutions of this paper include:

- Based on the stochastic optimization method, we formulate and provide solutions to a transmit beamforming scheme that is robust against errors in steering vector estimations for the underlay CR system with mobile or stationary SU-Rx and multiple PUs in different locations.
- For mobile SU-Rx with known moving range of directions, we propose a scheme that maintains a constant power along the directions of SU's mobility. This ensures the same quality of communication for the moving SU-Rx in the entire range.
- Through computer simulations, we show the effectiveness of the proposed robust beamforming scheme. The simulation results demonstrate that the improved performance of PUs (compared to non-robust case) can be obtained without degrading the performance of SU.

The rest of the paper is organized as follows: Section II describes the system model and formulates the problem. Section III elaborates the proposed stochastic beamforming scheme that is robust against steering vector uncertainty. Section IV

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presents the simulation results. Finally, conclusions are made in Section V.

II. System Model and Problem Formulation

Consider an underlay CR network as shown in Fig. 1, where a SU pair co-exists with multiple primary receivers (PU-Rx) in the same frequency band. It is assumed that all PUs and secondary transmitter (SU-Tx) are stationary whereas SU-Rx can be stationary or mobile. In this model, SU-Tx obtains the direction estimates of PUs either by using sensing results, or by querying a geolocation database that provides direction information of all active users in the channel. The SU-Tx is assumed to be equipped with a smart channel selection mechanism where a shared channel that has enough separation between the directions of SU-Rx and PUs from SU-Tx is selected among all available channels. For the moving SU-Rx, the range of directions in which it is moving is assumed to be known at the SU-Tx. Moreover, all transmitters and receivers are assumed to have single omnidirectional antenna whereas the SU-Tx has N omnidirectional antennas. The antennas on SU-Tx are separated uniformly, forming a linear antenna array. Furthermore, it is reasonable to assume that SU-Tx can obtain the downlink channel state information (CSI) for stationary users precisely through either smart sensing or feedback mechanism. However, CSI for mobile SU-Rx changes continuously depending on its instantaneous position, and is difficult to estimate. Therefore, when SU-Rx is mobile the CSI is assumed unknown at SU-Tx.

As shown in Fig. 1, SU-Tx uses transmit beamforming and directs the signal towards SU-Rx's direction. Meanwhile, it must ensure that all PUs operating in the shared channel are protected from interference caused by its transmission. In case of stationary SU-Rx, it is desired that the width of the beam pattern be narrow in order to ensure a high SNR for a given transmit power of SU-Tx. However, in case of moving SU-Rx, the desired beam pattern should be wide enough to maintain the uniform transmitted signal strength along the entire range of SU-Rx's mobility region. This uniform signal power ensures the uniform quality of communication for the moving SU-Rx.

Define steering vector as a representation of the set of delays that a plane wave, originated from the antenna elements of SU-Tx, experiences in reaching a point in space. Also define direction of arrival, θ, as the angle made by the straight line connecting the receiver antenna and center of transmitter antenna array with the axis of the antenna array. For a given θ, steering vector, (α_θ), can be written as,

$$\boldsymbol{\alpha}(\theta) = [1, e^{-j\omega d \sin\theta/c}, \ldots, e^{-j(N-1)\omega d \sin\theta/c}] \tag{1}$$

where $\omega = 2\pi f_c$ is the angular frequency in radians/second, f_c is the carrier frequency in Hertz, d is the distance between adjacent transmitter antenna elements in meters, and c is the velocity of electromagnetic waves in meters/second.

Denote the multipath fading coefficients from SU-Tx to PU and stationary SU-Rx as $\boldsymbol{h_p}$ and $\boldsymbol{h_s}$, respectively. All channels are assumed to be Rayleigh fading channels, i.e., the channel coefficients in $\boldsymbol{h_s}$ and $\boldsymbol{h_p}$ are circularly symmetric complex

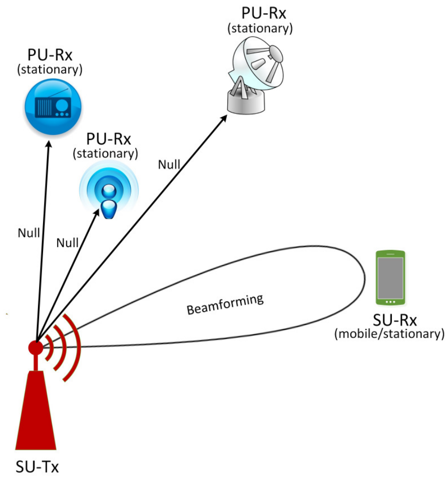

Fig. 1. System Model

Gaussian random variables. Let θ_s and θ_p denote the directions from SU-Tx to SU-Rx and PU respectively, and $\boldsymbol{w}$ denote the beamforming weight vector. If x denotes the message signal, then the signal received by SU-Rx, y_s, from SU-Tx, is given by,

$$y_s = \boldsymbol{H_s w} x + n_1 \tag{2}$$

where $\boldsymbol{H_s}$ is the element-wise multiplication of the channel coefficients vector ($\boldsymbol{h_s}$) and the steering vector for SU ($\boldsymbol{\alpha}(\theta_s)$) given by $\boldsymbol{H_s} = \text{diag}(\boldsymbol{h_s})\boldsymbol{\alpha}_s$, where $\boldsymbol{\alpha}_s = \boldsymbol{\alpha}(\theta_s)$ and diag($\boldsymbol{h_s}$) is a diagonal matrix of size $N \times N$ with elements of $\boldsymbol{h_s}$ as the diagonal elements. n_1 represents the additive white Gaussian noise with zero mean and variance σ_n^2.

Similarly, the signal received by PU, y_p, from SU-Tx, is,

$$y_p = \boldsymbol{H_p w} x + n_2 \tag{3}$$

where $\boldsymbol{H_p}$ is the element-wise multiplication of the channel coefficients vector ($\boldsymbol{h_p}$) and the steering vector for PU ($\boldsymbol{\alpha}(\theta_p)$) given by $\boldsymbol{H_p} = \text{diag}(\boldsymbol{h_p})\boldsymbol{\alpha}_p$, where $\boldsymbol{\alpha}_p = \boldsymbol{\alpha}(\theta_p)$ and diag($\boldsymbol{h_p}$) is a diagonal matrix of size $N \times N$ with elements of $\boldsymbol{h_p}$ as the diagonal elements. n_2 represents the additive white Gaussian noise with zero mean and variance σ_n^2.

III. Robust Beamforming against Steering Vector Uncertainty

In an underlay CR system, the SU can utilize the PU's spectrum as long as the interference it generates at the PU-Rx remains below the interference threshold I_p, which is the maximum tolerable interference level at which the PU can still maintain reliable communication [9]. Considering this, the beamforming problem can be formulated as an optimization problem to calculate the optimum beamforming weight vector, $\boldsymbol{w}$, that maximizes the signal power along the direction of SU-Rx and constrains the interference in PU-Rx's direction. To make the notation simple, from now on, all the PU-Rxs will be denoted as PU.

In practical scenario, steering vector are error prone. The uncertainty of the steering vector can be modelled as a complex Gaussian noise [8]. The resulting steering vectors for SU-Rx and PU can be written as,

$$\begin{aligned} \boldsymbol{\alpha_s} &= \hat{\boldsymbol{\alpha_s}} + \Delta\boldsymbol{\alpha_s} \\ \boldsymbol{\alpha_p} &= \hat{\boldsymbol{\alpha_p}} + \Delta\boldsymbol{\alpha_p} \end{aligned} \tag{4}$$

where, $\hat{\boldsymbol{\alpha_s}}$ and $\hat{\boldsymbol{\alpha_p}}$ are the estimated steering vectors of SU-Rx and PU known at the SU-Tx with the corresponding estimation errors denoted by $\Delta\boldsymbol{\alpha_s}$ and $\Delta\boldsymbol{\alpha_p}$, respectively. Denote the variance of $\Delta\boldsymbol{\alpha_s}$ and $\Delta\boldsymbol{\alpha_p}$ as σ_s^2 and σ_p^2, the steering vector uncertainty can be represented by,

$$\begin{aligned} \Delta\boldsymbol{\alpha_s} &\sim \mathcal{CN}\left(0, \sigma_s^2 I\right) \\ \Delta\boldsymbol{\alpha_p} &\sim \mathcal{CN}\left(0, \sigma_p^2 I\right) \end{aligned} \tag{5}$$

In this study, SU-Rx can be either stationary or mobile. For mobile SU-Rx, the beamforming pattern should have a wide uniform beam directed towards SU-Rx so that the quality of communication of SU doesn't degrade when it moves within a given range of directions. To achieve such a uniform transmit power, we divide the range of mobility of SU-Rx, $(\theta_{s_1}, \theta_{s_2})$, into M uniformly spaced angles [4]. By randomly choosing one of these angles, an optimization problem is formulated such that the objective function maximizes the signal in that particular direction, and an additional constraint is added to ensure the same signal power in all other angles.

Based on above analysis, the beamforming weight vector, $\boldsymbol{w}$, can be obtained by solving the following optimization problem:

$$\begin{aligned} &\text{Maximize} \quad Pr\left(\left|\boldsymbol{H_s}^{(i)}\boldsymbol{w}\right|^2 \geq \gamma\right), \qquad i \in \{1, 2, \ldots, M\} \\ &\text{subject to} \quad Pr\left(\left|\boldsymbol{H_p}^{(j)}\boldsymbol{w}\right|^2 \leq I_p^{(j)}\right) \geq \epsilon_p^{(j)}, \quad j = 1, 2, \ldots, L \\ &\qquad \left|\boldsymbol{H_s}^{(k)}\boldsymbol{w}\right|^2 = \left|\boldsymbol{H_s}^{(k+1)}\boldsymbol{w}\right|^2, \qquad k = 1, 2, \ldots, M-1 \\ &\qquad ||\boldsymbol{w}||^2 \leq 1, \end{aligned} \tag{6}$$

where L is the total number of PUs operating in the shared channel. The second constraint ensures the same signal level in all M directions. The third constraint on the weights limits the total transmission power.

The objective of the optimization problem (6) is to maximize the service probability of the SU-Rx at a specified lower bound signal power γ. At the same time, the constraint is to control the interference level to the j-th PU below the threshold, $I_p^{(j)}$, within a probabilistic target $\epsilon_p^{(j)}$ for all PUs.

Mathematically manipulating the objective term in (6) gives

$$|\boldsymbol{H_s}\boldsymbol{w}|^2 = |\boldsymbol{h_s}\boldsymbol{\alpha_s}\boldsymbol{w}|^2 = |\boldsymbol{h_s}\left(\hat{\boldsymbol{\alpha}}_{\boldsymbol{s}}\boldsymbol{w} + \boldsymbol{\Delta\alpha_s}\boldsymbol{w}\right)|^2 \tag{7}$$

Since channel coefficient $\boldsymbol{h_s}$ is assumed to be perfectly known for stationary SU-Rx or a vector of value 1 for mobile SU-Rx without loss of generality, (7) is characterised as a non-cental Chi-square random variable with degree of freedom $n = 2$ and variance $\sigma^2 = \frac{||\boldsymbol{w}||^2\sigma_s^2}{2}$ with noncentrality parameter $s^2 = |\boldsymbol{h_s}\hat{\boldsymbol{\alpha_s}}\boldsymbol{w}|^2 = \left|\hat{\boldsymbol{H}}_{\boldsymbol{s}}\boldsymbol{w}\right|^2$, where $\hat{\boldsymbol{H}}_{\boldsymbol{s}}$ is the elementwise multiplication of channel coefficient vector, $\boldsymbol{h_s}$, and the known steering vector, $\hat{\boldsymbol{\alpha}}_{\boldsymbol{s}}$, given by $\hat{\boldsymbol{H}}_{\boldsymbol{s}} = \text{diag}(\boldsymbol{h_s})\hat{\boldsymbol{\alpha}}_{\boldsymbol{s}}$ [10]. Therefore, the objective function of (6) can be expressed as,

$$Pr\left(\left|\hat{\boldsymbol{H}}_{\boldsymbol{s}}\boldsymbol{w}\right|^2 \geq \gamma\right) = Q\left(\frac{s}{\sigma}, \frac{\sqrt{\gamma}}{\sigma}\right) \tag{8}$$

where $Q\left(.,.\right)$ represents the generalized Marcum's Q-function. Using similar approach, we can convert the probabilistic term from the constraints of (6) to the Marcum's Q-function. Hence, the first two lines of the optimization problem in (6) can be re-written as

$$\begin{aligned} &\text{Maximize} \ Q\left(\frac{\left|\hat{\boldsymbol{H}}_{\boldsymbol{s}}^{(i)}\boldsymbol{w}\right|}{\sqrt{\frac{||\boldsymbol{w}||^2\sigma_s^2}{2}}}, \frac{\sqrt{\gamma}}{\sqrt{\frac{||\boldsymbol{w}||^2\sigma_s^2}{2}}}\right), \quad i \in \{1, 2, \ldots, M\} \\ &\text{subject to} \ Q\left(\frac{\left|\hat{\boldsymbol{H}}_{\boldsymbol{p}}^{(j)}\boldsymbol{w}\right|}{\sqrt{\frac{||\boldsymbol{w}||^2\sigma_p^{2(j)}}{2}}}, \frac{\sqrt{I_p^{(j)}}}{\sqrt{\frac{||\boldsymbol{w}||^2\sigma_p^{2(j)}}{2}}}\right) \leq 1 - \epsilon_p^{(j)}, \end{aligned} \tag{9}$$

where $j = 1, 2, \ldots, L$. Optimization problem (9) is complicated to solve because both the objective and the constraint contain Marcum's Q function. It can be simplified as following by introducing the inverse of Marcum's Q function $\left(Q^{-1}\right)$ [8]

$$\begin{aligned} &\text{Maximize} \ Q\left(\frac{\left|\hat{\boldsymbol{H}}_{\boldsymbol{s}}^{(i)}\boldsymbol{w}\right|}{\sqrt{\frac{||\boldsymbol{w}||^2\sigma_s^2}{2}}}, \frac{\sqrt{\gamma}}{\sqrt{\frac{||\boldsymbol{w}||^2\sigma_s^2}{2}}}\right), \quad i \in \{1, 2, \ldots, M\} \\ &\text{subject to} \\ &\left|\hat{\boldsymbol{H}}_{\boldsymbol{p}}^{(j)}\boldsymbol{w}\right| \leq Q^{-1}\left(\frac{\sqrt{I_p^{(j)}}}{\sqrt{\frac{||\boldsymbol{w}||^2\sigma_p^{2(j)}}{2}}}, 1 - \epsilon_p^{(j)}\right)\sqrt{\frac{||\boldsymbol{w}||^2\sigma_p^{2(j)}}{2}}, \end{aligned} \tag{10}$$

where $j = 1, 2, \ldots, L$.
A simple algorithm that will be elaborated in subsection IIIC is proposed to find $\left(Q^{-1}\right)$.

For fixed power $||\boldsymbol{w}||^2 = p$, $0 < p \leq 1$, the terms related to Q and Q^{-1} can be simplified. Also taking into account that $Q\left(a, b\right)$ is non-decreasing with respect to a and non-increasing with respect to b [8], the optimization problem (6) can be reformulated as,

$$\begin{aligned} &\text{Maximize} \qquad \left|\hat{\boldsymbol{H}}_{\boldsymbol{s}}^{(i)}\boldsymbol{w}\right|, \quad i \in \{1, 2, \ldots, M\} \\ &\text{subject to} \quad \left|\hat{\boldsymbol{H}}_{\boldsymbol{p}}^{(j)}\boldsymbol{w}\right|^2 \leq I_p^{\prime(j)}, \quad j = 1, 2, \ldots, L \\ &\qquad \left|\hat{\boldsymbol{H}}_{\boldsymbol{s}}^{(k)}\boldsymbol{w}\right|^2 = \left|\hat{\boldsymbol{H}}_{\boldsymbol{s}}^{(k+1)}\boldsymbol{w}\right|^2, \quad k = 1, 2, \ldots, M-1 \\ &\qquad ||\boldsymbol{w}||^2 = p, \end{aligned} \tag{11}$$

where, $I_p^{\prime(j)} = Q^{-1}\left(\frac{\sqrt{I_p^{(j)}}}{\sqrt{\frac{p\sigma_p^{2(j)}}{2}}}, 1 - \epsilon_p^{(j)}\right)\sqrt{\frac{p\sigma_p^{2(j)}}{2}}$

In (11), the equality constraint of power makes the optimization problem difficult to converge. It can be solved by simply converting this equality constraint to inequality. However, as feasible solution to the optimization problem (11), some values of p which satisfies $0 < p \leq 1$ and $||\boldsymbol{w}||^2 = p$ must be found. Moreover, splitting the objective function into the equivalent real and imaginary parts as in [8], the final optimization problem becomes,

$$\begin{aligned}
\text{Maximize} \quad & \Re\left\{\hat{\boldsymbol{H}}_{\boldsymbol{s}}{}^{(i)}\boldsymbol{w}\right\}, \quad i \in \{1,2,\ldots,M\} \\
\text{subject to} \quad & \Im\left\{\hat{\boldsymbol{H}}_{\boldsymbol{s}}{}^{(i)}\boldsymbol{w}\right\} = 0, \\
& \left|\hat{\boldsymbol{H}}_{\boldsymbol{p}}{}^{(j)}\boldsymbol{w}\right|^2 \leq I_p^{'(j)}, \quad j = 1,2,\ldots L \\
& \left|\hat{\boldsymbol{H}}_{\boldsymbol{s}}{}^{(k)}\boldsymbol{w}\right|^2 = \left|\hat{\boldsymbol{H}}_{\boldsymbol{s}}{}^{(k+1)}\boldsymbol{w}\right|^2, \quad k = 1,2,\ldots,M-1 \\
& ||\boldsymbol{w}||^2 \leq p,
\end{aligned} \tag{12}$$

A. Optimization for Mobile SU-Rx

For mobile SU-Rx, actual channel coefficient, $\boldsymbol{h_s}$, is unknown. Without loss of generality, $\boldsymbol{h_s}$ is assumed to be a vector of values 1. Therefore, $\hat{\boldsymbol{H}}_{\boldsymbol{s}}$ is equal to $\hat{\boldsymbol{\alpha}}_{\boldsymbol{s}}$ and the optimization problem is,

$$\begin{aligned}
\text{Maximize} \quad & \Re\left\{\hat{\boldsymbol{\alpha}}_{\boldsymbol{s}}{}^{(i)}\boldsymbol{w}\right\}, \quad i \in \{1,2,\ldots,M\} \\
\text{subject to} \quad & \Im\left\{\hat{\boldsymbol{\alpha}}_{\boldsymbol{s}}{}^{(i)}\boldsymbol{w}\right\} = 0, \\
& \left|\hat{\boldsymbol{H}}_{\boldsymbol{p}}{}^{(j)}\boldsymbol{w}\right|^2 \leq I_p^{'(j)}, \quad j = 1,2,\ldots L \\
& \left|\hat{\boldsymbol{\alpha}}_{\boldsymbol{s}}{}^{(k)}\boldsymbol{w}\right|^2 = \left|\hat{\boldsymbol{\alpha}}_{\boldsymbol{s}}{}^{(k+1)}\boldsymbol{w}\right|^2, \quad k = 1,2,\ldots,M-1 \\
& ||\boldsymbol{w}||^2 \leq p,
\end{aligned} \tag{13}$$

B. Optimzation for Stationary SU-Rx

For stationary SU-Rx, M equals to 1 and the third constraint in (12) is not necessary. Therefore, the optimization problem can be rewritten as,

$$\begin{aligned}
\text{Maximize} \quad & \Re\left\{\hat{\boldsymbol{H}}_{\boldsymbol{s}}\boldsymbol{w}\right\} \\
\text{subject to} \quad & \Im\left\{\hat{\boldsymbol{H}}_{\boldsymbol{s}}\boldsymbol{w}\right\} = 0 \\
& \left|\hat{\boldsymbol{H}}_{\boldsymbol{p}}{}^{(j)}\boldsymbol{w}\right|^2 \leq I_p^{'(j)}, \quad j = 1,2,\ldots L \\
& ||\boldsymbol{w}||^2 \leq p,
\end{aligned} \tag{14}$$

C. Algorithm for Finding Inverse of Marcum's Q Function

Inverse of Marcum's Q function plays an important role in this work. A simple algorithm is proposed to find Q^{-1} numerically. Since Q^{-1} must follow the relation $Q\left(Q^{-1}(b,c), b\right) = c$ where $Q(a,b) = c$ therefore, $Q^{-1}(b,c) = a$ [8]. In this algorithm we guess a value $x = Q^{-1}(b,c)$ and check whether it is appropriate or not then we increase the x value with a very small Δx and repeat searching. Algorithm 1 shows the pseudocodes for finding the inverse of Marcum's Q function.

Algorithm 1 Finding Inverse of Marcum's Q function

1: ***Input*** $b, c, \Delta x$
2: Initialize $x = 0$
3: Calculate $t = marcumq(x, b)$
4: **if** $t = c$ **then**
5:     $Qinverse = x$
6:     Go to line 20
7: **else**
8:     **if** t >c **then**
9:         $Qinverse$=No Solution
10:     **else**
11:         initialize $d = 1$;
12:         **while** $(d \geq 0)$ **do**
13:             $x = x + \Delta x$
14:             $Qinverse = x$
15:             $t = marcumq(x, b)$
16:             $d = t - c$
17:         **end while**
18:     **end if**
19: **end if**
20: ***Output :*** $Qinverse$

Performance of Algorithm 1: We evaluate Algorithm 1 by comparing the estimated Q^{-1} with the actual Q^{-1}. We chosed three arbitrary values of b as 3, 6 and 9. The value of a is chosen as a constant increment from 0 to 10. Then, Matlab's inbuilt function, marcumq(a, b), is used for calculating $Q(a, b)$. Algorithm 1 finds the estimated value of a based on the corresponding b and c, i.e.; $a_{est} = Q^{-1}(b, c)$. Estimation error is calculated as $Estimation\ Error = a - a_{est}$. Fig. 2 shows the plot of Estimation Error Vs. Actual value of a. For $a \geq 5$, the estimation error is constant – we simply plot for a values upto 6. From the plot, we observe that the error magnitude is as small as in the range of 10^{-5} which justifies the acceptance of the Algorithm 1.

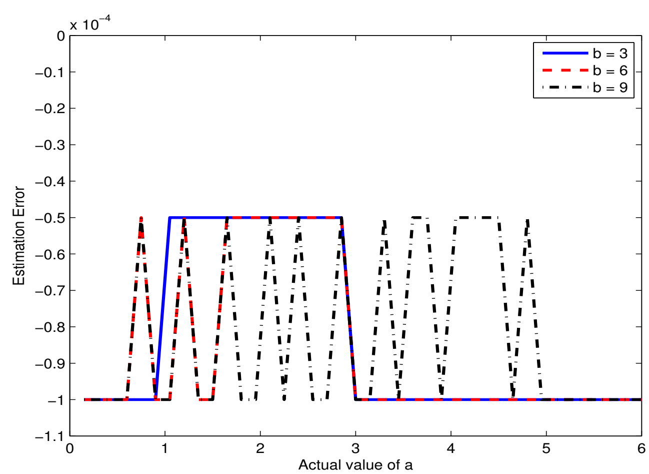

Fig. 2. Performance of Algorithm 1

IV. Simulation results

In this section, the simulation results of the proposed robust beamforming scheme are presented. Consider a CR network with a single SU pair and 3 PUs, i.e $L = 3$. The SU-Tx is equipped with N=9 omnidirectional antennas equally placed at half wavelength of the operating frequency at 3.5 GHz. Probabilistic targets of interference level, $I_p = -10$ dB, for all the PUs are $\epsilon_p = 90\%$. The target power threshold for the SU-Rx is γ=0 dB. For bit error rate (BER) calculations, binary phase shift keying (BPSK) modulation is used to transmit one million randomly generated bits. The BER performances for PUs and SU-Rx are compared for robust and non-robust cases when steering vector estimates are imperfect.

A. When SU-Rx is Stationary

Assume that the SU-Rx is stationary at 30^o and PUs are stationary at 70^o, 80^o and 90^o. Optimization problem (14) is solved to get the robust beamforming weights. Fig. 3 shows the resulting beamforming pattern. It is obvious that a large portion of the transmit power is steered towards the direction of SU-Rx while nulls are created towards the directions of PUs. Comparisons of BER performance between robust and non-robust beamforming schemes are shown in Fig. 4 and Fig. 5 for PU and SU-Rx respectively. In Fig. 4, a significant improvement in BER can be observed with robust beamformer. For a BER requirement of 0.03, an improvement (in terms of SNR gain) of approximately 7 dB is achieved for all the PUs. Fig. 5 reveals that, the BER performance for SU-Rx is not affected when the proposed robust beamforming scheme implemented. This is an important result: the improvement in BER performance for PU does not tradeoff the BER of SU-Rx.

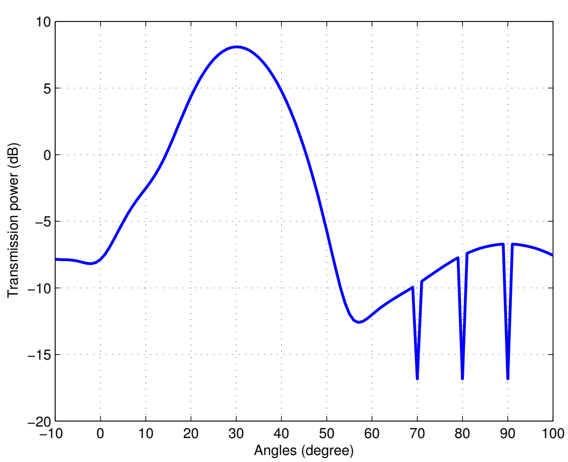

Fig. 3. Log plot of beam pattern for stationary SU at 30^o and PUs at 70^o, 80^o and 90^o

B. When SU-Rx is Mobile

In this case, the SU-Rx is mobile in the directions between 30^o and 40^o, and PUs are stationary at 70^o, 80^o and 90^o. The optimization problem (13) is solved to find the beamforming

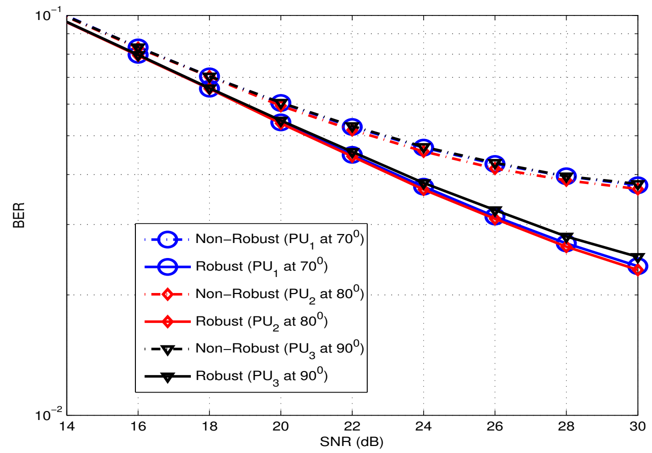

Fig. 4. BER of PUs with and without robust beamforming when SU is stationary

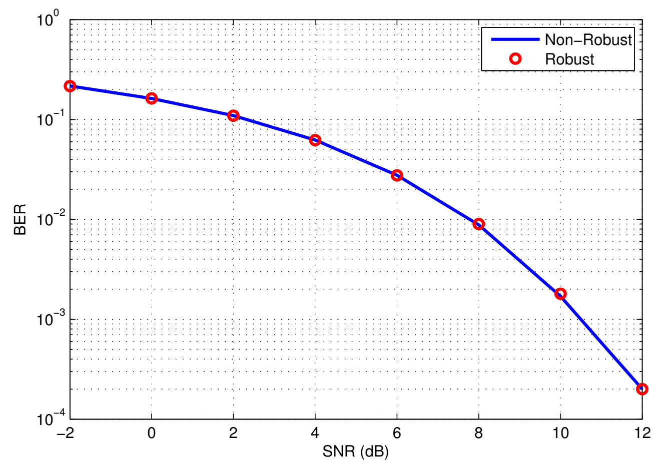

Fig. 5. BER of SU with and without robust beamforming when SU is stationary

weights with $M = 6$. Fig. 6 shows the beamforming patterns for mobile SU-Rx with different values of i in the objective function of the optimization problem (13). It is clear that the power transmission is constant within the range of SU-Rx's mobility, no matter which i is selected to solve (13). Fig. 7 and Fig. 8 show the BER performance of PU-Rx and SU-Rx. A noticeable improvement in BER (SNR gain of about 7 dB for a BER requirement of 0.025) is achieved using the robust beamformer for all PUs, while there is no degradation in BER performance for SU-Rx.

V. Conclusion

In this paper, a robust beamforming scheme for stationary as well as mobile SU-Rx is presented under both PUs' and SU's steering vector uncertainty. A stochastic robust approach is used to maximize the service probability of SU, maintain

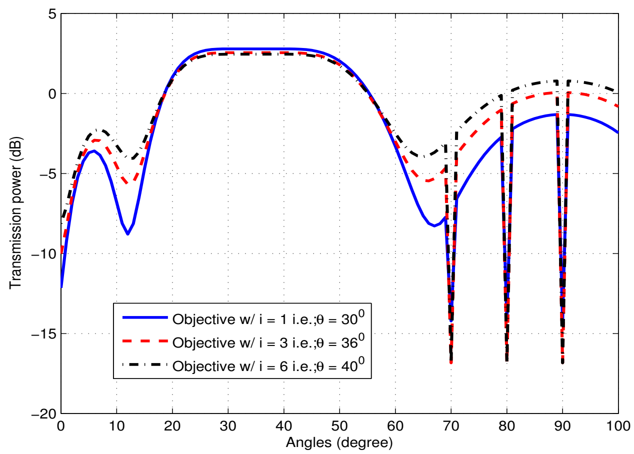

Fig. 6. Log plot of beam pattern when SU is moving within 30^o and 40^o, PUs are stationary at 70^o, 80^o and 90^o

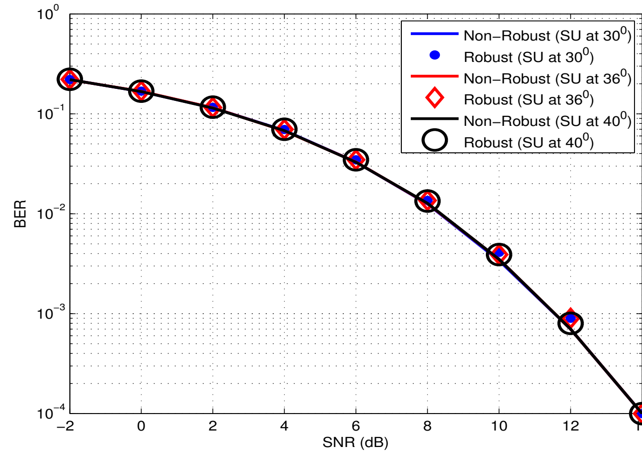

Fig. 8. BER of SU with and without robust beamforming when SU is moving

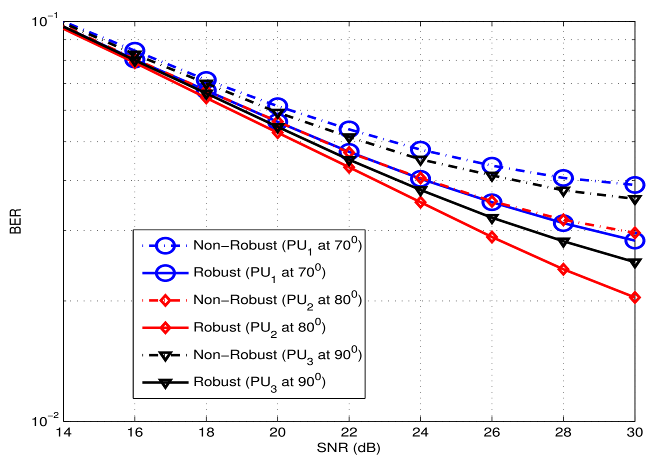

Fig. 7. BER of PUs with and without robust beamforming when SU is moving

the uniform transmission power in all moving directions of SU-Rx, and constrain the probabilities of interference level to PUs. Simulation results show that the proposed robust beamforming scheme significantly improves the reliability of PUs' communication (compared to the non-robust case) without any loss of SU's performance.

Acknowledgment

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