Unlimited Cooperative Sensing with Energy Detection for Cognitive Radio

Sunghwan Bae and Hongseok Kim

Dept. of Electronic Engineering,
Sogang University, Seoul, Korea
Email: {usher731, hongseok}@sogang.ac.kr

***Abstract*—In this paper we investigate the fundamental performance limits of the cooperative sensing using energy detection by considering the unlimited number of sensing nodes. Although a lot of cognitive radio research so far proposed various uses of energy detection because of its simplicity, the performance limits of energy detection have not been well understood when a large number of sensing nodes exist. We show that when the sensing nodes see the i.i.d. channel conditions, then as the number of sensing nodes N goes to infinity, the OR rule of hard decision achieves zero probability of false alarm P_f for any given target probability of detection $\overline{P_d}$ irrespective of the non-zero received PU SNR γ. By contrast, when the AND rule of hard decision is used under the same condition, we show that P_f goes to 1 as N goes to infinity. Interestingly, however, there exists a lower bound of P_f.**

***Index Terms*—Cognitive radio (CR), in-band sensing, energy detection, hard decision, unlimited cooperative sensing.**

I. INTRODUCTION

To support 4G or beyond mobile networks, the 3rd Generation Partnership Project (3GPP) has implemented or is developing carrier aggregation, massive multiple-input multiple-output (MIMO), device-to-device (D2D), etc. [1]. However, these technologies still need more frequency spectrums, and the International Telecommunication Union (ITU) has estimated the additional spectrum demand of 1280 – 1720 MHz for mobile networks by 2020 [2]. Although spectrums are exclusive resources to licensed primary users (PU), cognitive radio (CR) technology allows unlicensed secondary users (SU) to access underutilized or unused spectrums of PU, provided that non-harmful interference to PU is guaranteed [3].

Moreover, the migration from analog to digital terrestrial television (DTT) transmission emphasizes the role of CR technology to utilize the unused TV spectrums, referred to as TV white space (TVWS) [4]. The regulations of using TVWS are ongoing in the regulatory bodies such as Federal Communications Commission (FCC) of the United State, Office of Communications (OFCOM) of United Kingdom, and Korea Communications Commission (KCC) of South Korea [5]. In addition, the standards developed by IEEE 802.22 and 802.11af recommend the use of TVWS opportunistically using CR technology [6]. Recently, a framework of LTE TDD system using CR was proposed to enhance the performance of both TV and TD-LTE system by exploiting the under-utilized spectrum [7]. An LTE-CR prototype based on LTE TDD proposed many potential application scenarios such as LTE with CR for backhaul, LTE small-cells with CR, and additional spectrum for capacity enhancement [8].

For the successful implementation of CR in wireless system, the fundamental task of SU is to detect the PU so that SU evacuate when PU appear. There are many sensing methods such as energy detection, cyclo-stationary feature detection, compressed sensing, matched filter detection, etc [9]–[12]. Among them, energy detection is most popular because of its low complexity of implementation that does not require a prior information about PU [13]. Even though the performance of energy detection is susceptible to the received PU SNR, the diversity gain of cooperative sensing can mitigate performance degradation [14]. In general, cooperative sensing techniques can be classified into two categories: *hard decision* where each cooperative sensing node feedbacks one-bit message about PU's presence, and *soft decision* where the actual values of sensing measurements from sensing nodes are combined to make a final decision about PU's presence.

In [15], given the high received PU SNR, when cooperative sensing with hard decision is applied, the system throughput improves in the number of cooperative sensing nodes. Cooperative sensing achieves better performance when nodes having higher received PU SNR participate in the cooperative sensing [16]. In [17], energy detection improves the performance of PU detection by using cooperative sensing to overcome shadowing or fading effects. As a new technique of cooperative sensing, a *softened hard decision* was proposed to achieve a good tradeoff between the performance and the complexity [18]. From a practical point of view, fair comparison between soft and hard decisions was provided under the common conditions such as the received PU SNR and the sensing time [19].

No matter what kind of cooperative sensing techniques are used, however, the performance bound of energy detection and the maximum achievable utilization of SU have not been well understood. Recent wireless systems such as hyper-dense deployment of heterogeneous and small cell networks (HetSNets) motivate many SU of CR to cooperate for successful PU detection [20]. For example, a number of small access points and/or mobile terminals can be a part of cooperative sensing nodes. Specifically, when a sufficiently large number of cooperative sensing nodes exist, it is of interest to see if it is possible to achieve 100% utilization for a wide range of the received PU SNR.

In this regard we are motivated to extend the conventional

The full version of this paper is in [21].

cooperative sensing schemes using energy detection to the unlimited cooperative sensing, which have not been studied so far. We investigate the performance of unlimited cooperative sensing with energy detection, mainly by analyzing the probability of false alarm and the achievable utilization of SU. We prove that in the case of the OR rule of hard decision, given the probability of detection, the probability of false alarm of SU goes to zero irrespective of the non-zero received PU SNR as the number of sensing nodes goes to infinity. In the case of AND rule of hard decision, under the same condition, we also show that there exists a lower bound of probability of false alarm. However, as the number of sensing nodes goes to infinity, the probability of false alarm goes to 1.

II. SYSTEM MODEL

A. Cognitive Radio System in the Cellular Network

Fig. 1 illustrates a scenario of CR system where cellular networks spatially utilize TVWS. CR system requires SU to perform *out-of-band* or *in-band sensing* to detect PU. Out-of-band sensing implies that SU sense the spectrum bands that are not currently in use by SU. In-band sensing means that SU monitor the bands in use to check if PU newly appear. Hereafter, our unlimited cooperative sensing focuses on in-band sensing for TVWS in the cellular system because SU should not interfere PU's transmission and the periodic in-band sensing is mandatory. Of course, our work is applicable to out-of-band sensing, too.

In detecting PU signal, either base stations or mobile terminals can be cooperative sensing nodes. In case mobile terminals are cooperative sensing nodes, mobile terminals periodically send their sensing results to the fusion center or the base station that combines results and makes the final decision about the presence of PU. In case base stations are cooperative sensing nodes, base stations periodically report their sensing results to the central fusion center where the final decision is made. Although the wireless imperfect reporting channels have critical errors in the received local decision data, we assume that the proposed unlimited cooperative sensing scheme has error-free reporting channels for simplicity; the wired or reliable wireless backhaul network between base stations and a fusion center can be used. The final decision of PU existence will be discussed in detail in Section III.

B. Energy Detection and System Utilization

Let T be the total frame length and τ be the sensing time. We assume that τ is a continuous value. [1] Under the absence of PU (hypothesis $\mathcal{H}_0$), the received signal at the sensing node is given by

$$y(t) = n(t),$$

where $n(t)$ is the noise power at time t. Under the presence of PU (hypothesis $\mathcal{H}_1$), we have

$$y(t) = s(t) + n(t),$$

[1]Even though our model assumes continuous time, the unlimited cooperative sensing is still valid in the finite-sample region as long as the number of samples is large enough to model the test statistic as a Gaussian random variable.

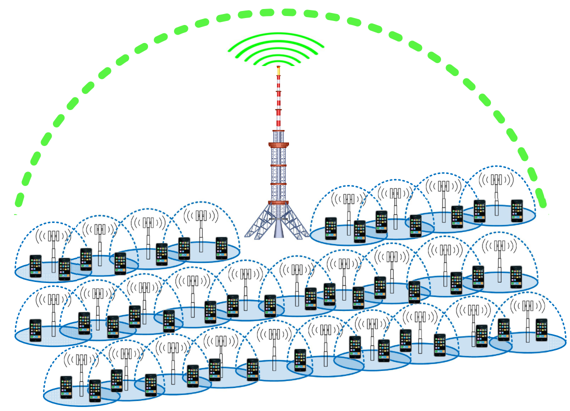

Fig. 1: A scenario for cognitive radio system with cooperative sensing in the cellular networks: one TV Tower with many base stations and mobile terminals.

where $s(t)$ is the received PU signal at time t. We assume that $n(t)$ is a white Gaussian noise with two-sided power spectral density N_0 *(AS I)*, and for the short time duration of in-band sensing, the channel can be assumed to be static *(AS II)*. According to [9], the test statistic using energy detection is given by

$$Y = \frac{1}{N_0}\int_0^\tau y(t)^2 dt.$$

Let γ be the received PU SNR given by $\gamma = \dfrac{P}{N_0 B}$, where P denotes the received power of PU at the sensing node, and B is the channel bandwidth. Under the assumptions of *(AS I)* and *(AS II)*, the received energy of PU signal is simply $P\tau$. When PU is absent, Y has the central chi-square distribution with the degrees of freedom equal to $2\tau B$. When PU is present, Y has the non-central chi-square distribution with $2\tau B$ degrees of freedom and non-centrality parameter $\lambda = \frac{P\tau}{N_0}$ equal to $\gamma\tau B$ [9]. Based on the central limit theorem (CLT), when $2\tau B$ is more than 250, Y under two conditions [9] can be approximated by Gaussian random variables,

$$Y \sim \begin{cases} \mathcal{N}\big(2\tau B, 4\tau B\big), & \text{under } \mathcal{H}_0, \\ \mathcal{N}\big(2\tau B + \gamma\tau B, 4\tau B + 4\gamma\tau B\big), & \text{under } \mathcal{H}_1. \end{cases} \quad (1)$$

where $\mathcal{N}\left(m, \sigma^2\right)$ represents the normal distribution with the mean m and the variance σ^2. From now on, we assume that τB is large enough so that the CLT condition is met, and Y follows the normal distribution. Let ϵ be the detection threshold, and $P_d^{(i)}$ denotes the probability of detection, i.e., $P_d^{(i)} = P\left(Y \geq \epsilon | \mathcal{H}_1\right)$ at the sensing node $i \in \{1, \cdots, N\}$, and given by

$$P_d^{(i)}(\epsilon, \tau, \gamma, B) = \mathcal{Q}\left(\frac{\epsilon - \big(2\tau B + \gamma\tau B\big)}{\sqrt{4\tau B + 4\gamma\tau B}}\right),$$

where $\mathcal{Q}(x) = \frac{1}{2\pi}\int_x^\infty \exp\left(\frac{-u^2}{2}\right) du$. In addition, $P_f^{(i)}$ denotes the probability of false alarm, i.e., $P_f^{(i)} = P\left(Y \geq \epsilon | \mathcal{H}_0\right)$ at the sensing node $i \in \{1, \cdots, N\}$, and given by

$$P_f^{(i)}(\epsilon, \tau, B) = \mathcal{Q}\left(\frac{\epsilon - 2\tau B}{\sqrt{4\tau B}}\right). \quad (2)$$

When the target probability of detection is set as $\overline{P_d}$, ϵ is accordingly given, and the probability of false alarm $P_f^{(i)}$ is related to $\overline{P_d}$ as follows [2]:

$$P_f^{(i)}(\tau,\gamma,B,\overline{P_d}) = \mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}(\overline{P_d}) + \frac{\gamma\sqrt{\tau B}}{2}\right). \quad (3)$$

Since B is fixed (e.g., 6 MHz) throughout this paper, hereafter we omit B in $P_f^{(i)}(\cdot)$ for simplicity. Then, the utilization of SU is given by

$$U\left(\tau, P_f^{(i)}(\tau,\gamma,\overline{P_d})\right) = \left(1-\frac{\tau}{T}\right)\left(1-P_f^{(i)}(\tau,\gamma,\overline{P_d})\right). \quad (4)$$

III. Unlimited Cooperative Sensing with Hard Decisions

In this section we present the achievable utilization of unlimited cooperative sensing using two types of hard decisions, mainly focused on the probability of false alarm. From (4), the unlimited cooperative sensing gives the following utilization of SU,

$$\lim_{N\to\infty} U\left(\tau, P_f(\tau,\gamma,\overline{P_d},N)\right) = \lim_{N\to\infty}\left(1-\frac{\tau}{T}\right)\left(1-P_f(\tau,\gamma,\overline{P_d},N)\right), \quad (5)$$

where $P_f(\tau,\gamma,\overline{P_d},N)$ refers to the final false alarm with N cooperative sensing nodes.

When we consider very short τ under CLT condition ($2\tau B > 250$), for example, where B = 6 MHz, $\tau > 20.8$ μsec, and T = 100 msec, the first term of the utilization $\left(1-\frac{\tau}{T}\right)$ is 0.9998 $\simeq$1. Hence, if we can show that $\lim_{N\to\infty} P_f = 0$, this implies that almost 100% utilization is achievable. Hereafter we mainly focus on P_f.

To better understand the improvement of utilization in the number of sensing nodes for various PU SNRs, we plot Fig. 2 as an example that motivates our work [15]. We notice that the utilization highly depends on the received PU SNR because the performance of the energy detection is sensitive to the received power in nature. As can be seen, the utilization slowly increases in the number of cooperative sensing nodes. However, we need to know if there exists the fundamental limitation of the energy detection or 100% utilization is achievable under the unlimited number of cooperative sensing nodes. To this end, we analyze the utilization with a sufficiently large number of cooperative sensing nodes for various received PU SNRs and for two different types of hard decisions: OR rule and AND rule.

In doing this, we mainly fix the target probability of detection $\overline{P_d}$ to protect PU and examine how the received PU SNR affects the probability of false alarm in the unlimited number of cooperative sensing nodes.

A. Hard decision - OR Rule

In OR rule, if at least one sensing node declares the presence of PU, then the final decision confirms that PU exist. The

[2]With a slight abuse of notation, $P_f^{(i)}$ in (3) has different variables from (2).

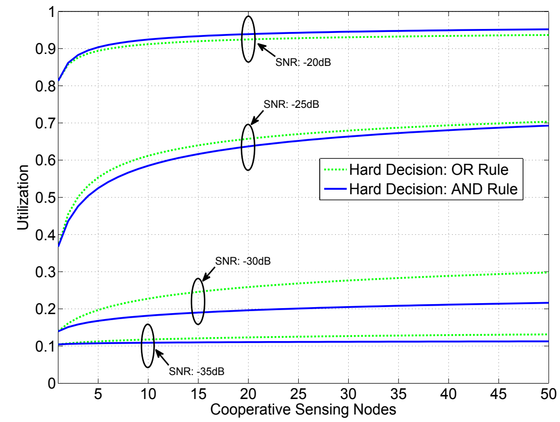

Fig. 2: Utilization of cooperative sensing with hard decisions: AND and OR rules.

probability of detection and the probability of false alarm are given by

$$P_d = 1-\prod_{i=1}^{N}\left(1-P_d^{(i)}\right), \quad (6)$$

$$P_f = 1-\prod_{i=1}^{N}\left(1-P_f^{(i)}\right). \quad (7)$$

Proposition 1: Suppose that all SU experience i.i.d. sensing channel condition, i.e., the same PU SNR γ, and the sensing variable Y follows the normal distribution as in (1). Then, in OR rule, for any $\overline{P_d}$, as N goes to infinity, P_f goes to zero irrespective of $\gamma > 0$.

Before providing the proof, we emphasize that the result of unlimited cooperative sensing using energy detection (OR rule) is not straightforward because the utilization with the very low γ (e.g., -35 dB or -30 dB) seems to saturate as can be seen Fig. 2. Moreover, when we consider relatively higher γ (e.g., -25 dB or -20 dB), there is no evidence that P_f goes to zero as N goes to infinity.

Proof: Since we assume that all sensing nodes see i.i.d. channel, let $P_f^{(i)}(\tau,\gamma,\overline{P_d}) \triangleq P_f^{(1)}$, $i \in \{1,\cdots,N\}$. From (3), (6) and (7), we obtain the probability of false alarm $\tilde{P_f}$ for the unlimited cooperative sensing using OR rule as follows:

$$\begin{aligned}
&\left(1-\tilde{P_f}\right)\\
&= \lim_{N\to\infty}\left(1-P_f(\tau,\gamma,\overline{P_d},N)\right)\\
&= \lim_{N\to\infty}\left(1-P_f^{(1)}(\tau,\gamma,\overline{P_d})\right)^N\\
&= \lim_{N\to\infty}\left\{1-\mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}\left(1-\left(1-\overline{P_d}\right)^{\frac{1}{N}}\right)+\frac{\gamma\sqrt{\tau B}}{2}\right)\right\}^N.
\end{aligned}$$

For analytical simplicity, we introduce $z \triangleq \dfrac{1}{N}$, and take logarithm of $\left(1-\tilde{P_f}\right)$.

Then, $\log\left(1-\tilde{P_f}\right) =$

$$\lim_{z\to 0}\frac{1}{z}\log\left\{1-\mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}\left(1-\left(1-\overline{P_d}\right)^z\right)+\frac{\gamma\sqrt{\tau B}}{2}\right)\right\}.$$

From the L'Hospital's rule, taking the derivative with respect to z gives us $\log\left(1-\tilde{P}_f\right)$

$$= \lim_{z\to 0} \frac{\mathcal{Q}'\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}\left(1-\left(1-\overline{P_d}\right)^z\right)+\frac{\gamma\sqrt{\tau B}}{2}\right)}{1-\mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}\left(1-\left(1-\overline{P_d}\right)^z\right)+\frac{\gamma\sqrt{\tau B}}{2}\right)} \times\sqrt{1+\gamma}\left(\mathcal{Q}^{-1}\right)'\left(1-\left(1-\overline{P_d}\right)^z\right)\left(1-\overline{P_d}\right)^z\log(1-\overline{P_d}).$$

We again change the variable, $y \triangleq \left(1-\overline{P_d}\right)^z$, and we have $\log\left(1-\tilde{P}_f\right)$

$$= \lim_{y\to 1} \mathcal{Q}'\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}(1-y)+\frac{\gamma\sqrt{\tau B}}{2}\right) \times\sqrt{1+\gamma}y\log(1-\overline{P_d})\left(\mathcal{Q}^{-1}\right)'(1-y).$$

To tackle the difficulty of dealing with the derivative of the inverse of $\mathcal{Q}$, we use the following derivative formula, which holds for any continuous and reversible analytic function f,

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}. \tag{8}$$

Then, we have $\log\left(1-\tilde{P}_f\right)$

$$= \lim_{y\to 1} \frac{\mathcal{Q}'\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}(1-y)+\frac{\gamma\sqrt{\tau B}}{2}\right)}{\mathcal{Q}'\left(\mathcal{Q}^{-1}(1-y)\right)}\sqrt{1+\gamma}y\log(1-\overline{P_d}).$$

Using

$$\mathcal{Q}'(x) = -\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right), \tag{9}$$

we have (10)[3].

Since $\lim_{y\to 1}\mathcal{Q}^{-1}(1-y)=\infty$, as y goes to 1, the exponent of the exponential function goes to negative infinity, and $\log(1-P_f)$ goes to 0, i.e., P_f goes to 0. ■

Remark 1: This result is perhaps surprising because of its independence of the received PU SNR γ as long as γ is positive.

Example 1: We assume that the channel bandwidth B of TVWS is 6 MHz. When the frame length is 100 msec, under CLT condition ($2\tau B > 250$), we fix the system sensing time $\tau = 1$ msec for in-band sensing, where $2\tau B = 12000 \gg 250$, so CLT condition is well established. Fig. 3 shows P_f of OR rule when $\overline{P_d}$ is 0.9. We notice that decreasing tendency of P_f significantly depends on both the received PU SNR γ and the number of cooperative sensing nodes N. When γ ranges from -15 dB to -10 dB, which is feasible for the general CR systems, P_f rapidly goes to zero as N increases. When γ is very low, e.g., -30 dB, P_f is initially $\overline{P_d}$. However, P_f *slowly* decreases as N grows.[4] In Fig. 4, we take a close look at P_f for the feasible γ (e.g., -15 dB $\sim$ -13 dB). As N increases, P_f quickly decreases. For example, when γ is -13 dB, P_f is already less than 0.005 when N is 100.

[3]Since (10) is a long equation, it is presented at the top of the next page.

[4]Even though P_f is proven to go to zero as N goes to infinity, its convergence speed substantially depends on γ, and thus in Fig. 3, we see that P_f is still large with $N = 10^{10}$ at $\gamma = -30$ dB.

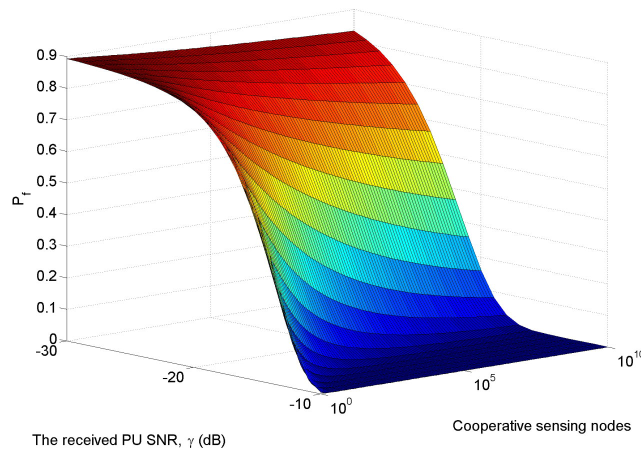

Fig. 3: Hard decision - OR rule: P_f with $\overline{P_d} = 0.9$.

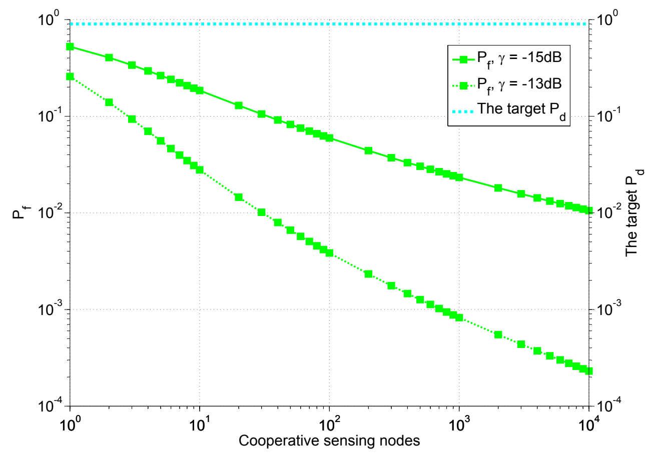

Fig. 4: Hard decision - OR rule: P_f with $\overline{P_d} = 0.9$.

B. Hard decision - AND Rule

In AND rule, if all sensing nodes declare the presence of PU, then the final decision confirms that PU exist. Thus, the probability of detection and the probability of false alarm are given by

$$P_d = \prod_{i=1}^{N} P_d^{(i)}, \tag{11}$$

$$P_f = \prod_{i=1}^{N} P_f^{(i)}. \tag{12}$$

The main difference of AND rule from OR rule is that P_f does not go to zero as N goes to infinity. Instead, there exists a lower bound of P_f.

Proposition 2: Suppose that all SU experience i.i.d. sensing channel condition, i.e., the same PU SNR, and the sensing variable Y follows the normal distribution as in (1). Then, in AND rule, for any $\overline{P_d}$, a lower bound of P_f exists such as ${P_f}^* = \left(\overline{P_d}\right)^{\sqrt{1+\gamma}\exp\left(\frac{1}{8}\gamma\tau B\right)}$. However, as N goes to infinity, P_f goes to 1.

Before providing the proof, we emphasize that our result is general in the sense that it is applicable not only to energy detection but also to any detection scheme that has the normal distributions for $\mathcal{H}_1$ and $\mathcal{H}_0$.

Proof: Since we assume that all sensing nodes see i.i.d.

$$\lim_{y\to 1}\log\left(1-P_f\right)=\lim_{y\to 1}\exp\left(-\frac{\gamma}{2}\left(\left(\mathcal{Q}^{-1}\left(1-y\right)\right)^2+\mathcal{Q}^{-1}\left(1-y\right)\sqrt{\left(1+\gamma\right)\tau B}+\frac{\gamma\tau B}{4}\right)\right)\sqrt{1+\gamma}y\log(1-\overline{P_d}). \quad (10)$$

channel, let $P_f^{(1)}(\tau,\gamma,\overline{P_d}) = P_f^{(i)}$, $i\in\{1,\cdots,N\}$ of (3). Then, from (11), (12) we obtain the probability of false alarm for cooperative sensing with N nodes using AND rule as follows.

$$\begin{aligned} P_f(\tau,\gamma,\overline{P_d},N) &= \left(P_f^{(1)}(\tau,\gamma,\overline{P_d})\right)^N \quad (13)\\ &= \mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}(\overline{P_d}^{\frac{1}{N}})+\frac{\gamma\sqrt{\tau B}}{2}\right)^N. \end{aligned}$$

Taking log of P_f gives us

$$\begin{aligned} &\log\left(P_f(\tau,\gamma,\overline{P_d},N)\right)\\ &= N\log\mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}(\overline{P_d}^{\frac{1}{N}})+\frac{\gamma\sqrt{\tau B}}{2}\right). \quad (14)\end{aligned}$$

For analytical purpose, we introduce $z\triangleq\dfrac{1}{N}$ and define

$$\begin{aligned} h(z) &\triangleq z\log P_f\left(\tau,\gamma,\overline{P_d},\frac{1}{z}\right)\\ &= \log\mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}(\overline{P_d}^{z})+\frac{\gamma\sqrt{\tau B}}{2}\right). \quad (15)\end{aligned}$$

Then, computing the value of $\log\left(P_f(\tau,\gamma,\overline{P_d},N)\right)$ as N goes to infinity is equal to the value of $\frac{h(z)}{z}$ as z goes to zero. So, we need to evaluate $\lim_{z\to 0}\frac{h(z)}{z}$. For analytic tractability we assume that z is continuous. Then, since $h(z)$ is a continuous and differentiable function, and $\lim_{z\to 0}h(z)=0$ from (15), we have $\lim_{z\to 0}\frac{h(z)}{z}=h'(z)|_{z=0}$, where $h'(z)$ is given by

$$\begin{aligned} h'(z) = &\frac{\mathcal{Q}'\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}\left(\overline{P_d}^{z}\right)+\frac{\gamma\sqrt{\tau B}}{2}\right)}{\mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}\left(\overline{P_d}^{z}\right)+\frac{\gamma\sqrt{\tau B}}{2}\right)}\\ &\times\sqrt{1+\gamma}\left(\mathcal{Q}^{-1}\right)'\left(\overline{P_d}^{z}\right)\overline{P_d}^{z}\log\overline{P_d}. \end{aligned}$$

Note that $h'(0)$ cannot be directly evaluated. However, since $h'(z)$ is continuous, we have $h'(0)=\lim_{z\to 0}h'(z)$. So from now on, we focus on $h'(z)$, and we will see that $\lim_{z\to 0}h'(z)=0$, which implies P_f goes to 1 as N goes to infinity. However, this result is neither interesting nor useful. Instead, we are more interested in either the minimum or lower bound of $\log\left(P_f(\tau,\gamma,\overline{P_d},N)\right)$ that is achieved for some sufficiently large N. To this end, we need to keep track of $\log\left(P_f(\tau,\gamma,\overline{P_d},N)\right)$ as N grows, or equivalently $\frac{h(z)}{z}$ as z approaches to zero.

However, $\frac{h(z)}{z}$ is yet tractable, so we need to detour again. Since $h(z)$ is a continuous and differentiable function, we can apply the Mean Value Theorem on the interval $[0,z]$. Then, we can always find a $\tilde{z}\in[0,z]$ such that $h'(\tilde{z})=\frac{h(z)}{z}$. Then, one can easily show that the minimum of $h'(\tilde{z})$ on $[0,1]$ servers as the *lower* bound of $\frac{h(z)}{z}$ on $[0,1]$.

To investigate a lower bound of $\frac{h(z)}{z}$, we compute the minimum value of $h'(\tilde{z})$ on $[0,1]$. To do this, we define $y\triangleq\overline{P_d}^{\tilde{z}}$, and let $q(y)\triangleq h'(\tilde{z})=\log\left(P_f(N)\right)$, where $q(y)$ is given by

$$\begin{aligned} q(y) = &\frac{\mathcal{Q}'\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}\left(y\right)+\frac{\gamma\sqrt{\tau B}}{2}\right)}{\mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}\left(y\right)+\frac{\gamma\sqrt{\tau B}}{2}\right)}\\ &\times\sqrt{1+\gamma}\left(\mathcal{Q}^{-1}\right)'(y)\,y\log\overline{P_d}. \end{aligned}$$

To tackle the difficulty of dealing with the derivative of the inverse of $\mathcal{Q}$ in the second term, we again use the formula of (8). Then, we have

$$q(y)=\frac{\mathcal{Q}'\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}\left(y\right)+\frac{\gamma\sqrt{\tau B}}{2}\right)\sqrt{1+\gamma}y\log\overline{P_d}}{\mathcal{Q}'\left(\mathcal{Q}^{-1}\left(y\right)\right)\mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}\left(y\right)+\frac{\gamma\sqrt{\tau B}}{2}\right)}. \quad (16)$$

Applying (9) into (16) and rearranging the equation gives us (17)[5]. Note that $\lim_{y\to 1}q(y)=0$ because $\lim_{y\to 1}\mathcal{Q}^{-1}\left(y\right)=-\infty$, and thus, the numerator composed of exponential function of the negative quadratic of $\mathcal{Q}^{-1}\left(y\right)$ goes to zero, and the denominator goes to 1 as y goes to 1. Hence, $\lim_{N\to\infty}\log P_f=0$ and P_f goes to 1.

However, we are more interested in the minimum of $q(y)$, i.e., a lower bound of $\log P_f$, which will be denoted by $\log {P_f}^*$. Obviously, the numerator is minimized at y^* such that

$$\mathcal{Q}^{-1}\left(y^*\right)=-\frac{1}{2}\sqrt{\left(1+\gamma\right)\tau B}. \quad (18)$$

Note that y^* is typically very close to 1. In addition, at this value of y^*, plugging (18) into the denominator of (17) gives us $\mathcal{Q}\left(-\frac{\sqrt{\tau B}}{2}\right)$, which can be considered as a constant (i.e., 1) around $y=1$ when τB satisfies the CLT condition. Thus, it is sufficient to consider only the numerator to find out $\log {P_f}^*$. Finally, ${P_f}^*$ at y^* is given by

$${P_f}^* = \left(\overline{P_d}\right)^{\sqrt{1+\gamma}\exp\left(\frac{1}{8}\gamma\tau B\right)}. \quad (19)$$

■

Remark 2: As can be seen from (19), ${P_f}^*$ goes to zero *exponentially of exponentially fast* as γ increases.

Example 2: Fig. 5 represents P_f of AND rule with $\overline{P_d}=0.9$. When γ is from -15 dB to -10 dB, P_f of AND rule quickly drops as N grows. However, when γ is very low, P_f is typically large because ${P_f}^*$ in (19) is not negligible. When AND rule is used, Fig. 6 represents ${P_f}^*$ in (19). Given $\overline{P_d}=0.9$ and $B=6$ MHz, ${P_f}^*$ depends on both the received PU SNR and the sensing time. When we select 1 msec as the sensing time, ${P_f}^*$ rapidly drops at the feasible γ above -25 dB, but ${P_f}^*$ is noticeable, e.g., almost $\overline{P_d}$ at the very low γ below -30 dB; unlikely with the case of OR rule, ${P_f}^*$ of AND rule exists. In addition, note that ${P_f}^*$ also depends on the sensing time and the channel bandwidth as can be seen in (19).

[5] Eq. (17) is on the next page.

$$q(y) = \frac{\exp\left(-\frac{\gamma}{2}\left(\left(\mathcal{Q}^{-1}(y)+\frac{1}{2}\sqrt{(1+\gamma)\tau B}\right)^2-\frac{1}{4}\tau B\right)\right)\sqrt{1+\gamma}y\log\left(\overline{P_d}\right)}{\mathcal{Q}\left(\sqrt{1+\gamma}\mathcal{Q}^{-1}(y)+\frac{\gamma\sqrt{\tau B}}{2}\right)}. \tag{17}$$

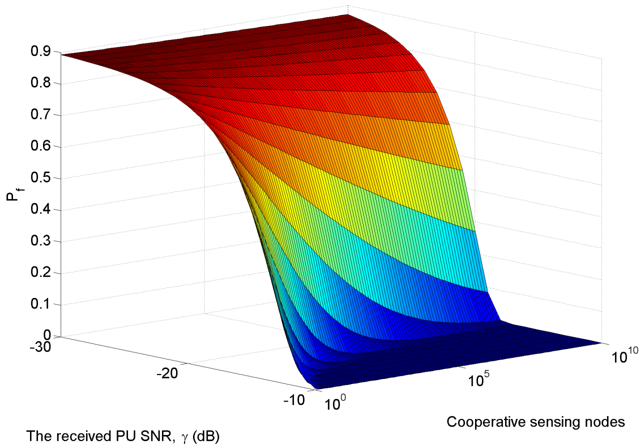

Fig. 5: Hard decision - AND rule: P_f with $\overline{P_d} = 0.9$.

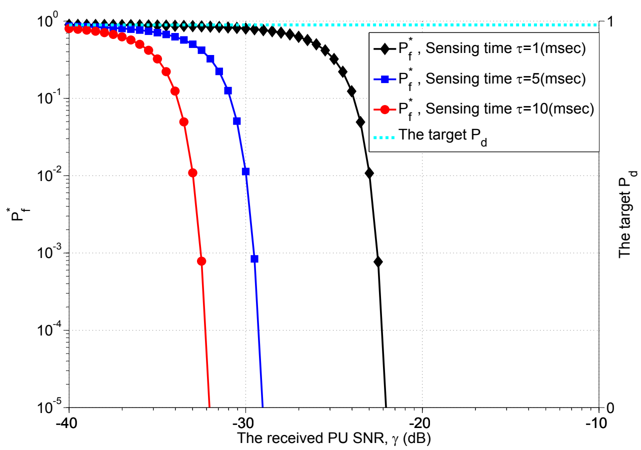

Fig. 6: Hard decision - AND rule: a lower bound of P_f.

IV. CONCLUSION

In this paper, we showed that when the OR rule of hard decision using the energy detection is considered, given the target $\overline{P_d}$, the probability of false alarm P_f of SU goes to zero irrespective of the received PU SNR γ as the number of sensing nodes N goes to infinity. In the case of the AND rule of hard decision, under the same condition, we showed that P_f does not go to zero as N grows, but there exists a lower bound of P_f. Moreover, interestingly, P_f finally goes to 1 as N goes to infinity. Our analyses were verified by numerical examples. Our future work will be to investigate the way of achieving 100% utilization in the case of soft decision and to analyze the impact of the fading channels on the unlimited cooperative sensing.

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