Capacity of Wireless Networks with Multiple Types of Multicast Sessions

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ABSTRACT

A thorough understanding of capacity of wireless networks can help with effective design and efficient employment of wireless networks. Much effort has been spent on investigating capacity of multicast which is a popular communication model and generalization of unicast and broadcast. However, most previous works assume homogeneous traffic patterns, which is not meaningful for practical applications. This paper analyzes the capacity of wireless networks with multiple types of multicast sessions without the assumption of homogeneous traffic patterns. A new network model is proposed accommodating practical traffic patterns and the capacity is analyzed accordingly. A theoretical upper bound is derived, and a feasible transmission scheme with capacity lower bound is presented. Two bounds are asymptotically tight, that is, in the order of $\Theta(\frac{a^2 n_s}{\sum_{i=1}^{n_s} min\{\sqrt{k_i}\cdot R_i \cdot r, R_i^2\}} \cdot W)$, where a is the side length of the deployed region, r is the transmission range, n_s is the number of multicast sessions, and k_i and R_i are parameters of multicast session i. Furthermore, the variation of capacity towards different numbers of and distributions of destinations is illustrated.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication

Keywords

Capacity; multicast; heterogeneous traffic; scalability

1. INTRODUCTION

A thorough understanding of the capacity of wireless networks can help with effective design and efficient employment of wireless networks. Therefore, the investigation of the capacity of wireless networks is important and is also a challenging task.

Since the pioneering work [1] appeared, the capacity of wireless networks has been widely studied. These works assume various kinds of network models for different kinds of wireless networks in practice. For example, some of the works study static or mobile pure ad hoc networks [2, 3] while the others focus on hybrid networks [4]. From the view of traffic patterns, all the works can be classified into studies on unicast capacity [5], multicast capacity [2], broadcast capacity [6] and aggregation capacity [7, 8].

Multicast, which is the generalization of unicast and broadcast, has attracted more interests. The estimation of the achievable multicast capacity is required in many applications, such as the ones of sensor networks. Much more works focus on the multicast capacity recently [2][3][4][9][10][11]. Most of the previous studies on multicast capacity assume homogeneous traffic patterns, *i.e.*, the multicast sessions are identical in the number of destinations, and the destinations are randomly distributed in the whole region or within equal sized circles centered at each source node [9]. However, multicast sessions in real applications are often quite different in the number of destinations and the distribution of the destinations. The following are two examples.

Example 1. In a battlefield, military officers often send orders to their soldiers in defence areas via a wireless network. The numbers of the soldiers commended by officers are often various, which goes against the assumption that sessions are identical on the numbers of destinations. Meanwhile, the defence area guarded by each group of soldiers is often a sub-region of the whole battlefield, and the sizes of these sub-regions are usually various. Thus, the second assumption that destinations are randomly distributed in the whole region or equal sized circles is not meaningful.

Example 2. In wireless sensor networks, there may be non-identical multicast sessions at the same time. Some of the multicast sessions may be unicast or broadcast sessions rather than pure multicast sessions. In this case, the homogeneous assumption cannot be guaranteed and the previous results are not applicable.

A theoretical study on wireless networks with multiple types of multicast sessions can enhance the generality of the bounds on network capacity. As far as we know, none of the previous works totally looses the constraints on the identity of the sessions. These constraints can make the derived bounds on capacity more explicable and straightforward, however, it is at the cost of loosing a deeper understanding of the heterogeneity among sessions. This paper focuses on estimating the total traffic load brought by multicast sessions that are quite different from each other, and tries to derive the relationship between the capacity and the variety of sessions. Moreover, this paper aims to point out

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some open questions about network capacity that are worth considering.

In this paper, we study the multicast capacity of ad hoc wireless networks without the homogeneous traffic pattern assumption. There may exist multiple types of multicast sessions in a wireless network at the same time, and the sessions have different numbers of destinations and different distributions of destinations. The main contributions are as follows.

- We propose a new wireless network model to support the analysis of the capacity of wireless networks with multiple types of multicast sessions. In this model, the traffic pattern has two features: i) each multicast session in a wireless network can have any number of destinations without the constraint that the numbers must be identical, and ii) the destinations of each source node are uniformly distributed in a circle area centered at the source node with a radius of arbitrary length, rather than that the destinations are uniformly distributed in the whole region covered by the wireless network or within equal sized circles. Compared with the previous work [9], which still has some constraints for the identity of its multicast sessions for simplicity of analysis, our model is more applicable in real applications like the aforementioned two examples. Moreover, the model is a generalization of many previously proposed models.

- The capacity of wireless networks is analyzed towards the proposed model. A theoretical upper bound is derived which is $O(\frac{2a^2 \cdot n_s}{\sum_{i=1}^{n_s} min\{\sqrt{k_i} \cdot R_i \cdot r, {R_i}^2\}} \cdot W)$. In the proof, a new measurement metric, competitive intensity, of a wireless network is proposed. Competitive intensity denotes the number of node pairs or transmission pairs that cannot be scheduled simultaneously. Its bound can help with deriving the capacity bound of a wireless network. Competitive intensity is independent of network models, thus it can be used for the capacity analysis of any kind of wireless network. A feasible transmission scheme for the entire network is also determined with the lower bound of its capacity. The bound is $\Omega(\frac{2a^2 n_s}{3\sum_{i=1}^{n_s} min\{\sqrt{k_i} \cdot R_i \cdot r, R_i^2\}} \cdot W)$, which is asymptotically tight with the theoretical upper bound of the capacity of wireless networks.

- Some observations about the network model and the derived capacity bounds are discussed. First, the variation of the capacity for different numbers of destinations and the distribution area of destinations in multicast sessions is illustrated. Then, it is stated that the previous capacity bounds are the specialization of the capacity bounds derived in this paper when some constraints are added to our network model.

The rest of the paper is organized as follows. Section 2 reviews the related works. Section 3 introduces the network model and some basic definitions. The theoretical upper bound of the capacity of a wireless network is derived in Section 4. Section 5 presents a reachable lower bound on the capacity of a wireless network. In Section 6, some observations about the proposed capacity bounds and network model are discussed. Section 7 concludes the paper.

2. RELATED WORKS

There have been a lot of researches on capacity of wireless networks during the last decade. Gupta and Kumar first proved that the per-flow unicast capacity of random networks is in the order of $\Theta(\frac{W}{\sqrt{n \log n}})$ [1]. They also analyzed the capacity of wireless networks with arbitrary given traffic, that is, $\Theta(\frac{W}{\sqrt{n}})$. This result was then proved under a more general fading channel model in [12]. Later, Franceschetti proposed an optimal scheme to avoid the loss from the randomness using the percolation theory [5], which could achieve the capacity of $\Theta(\frac{W}{\sqrt{n}})$. Broadcast capacity was also studied in previous works. It is shown in [6] that the total broadcast capacity is only $\Theta(W)$. Grossglauser and Tse showed that the unicast capacity could be improved at the cost of larger delay in mobile wireless networks [13]. These works concentrate on either unicast or broadcast, which can be a special case of multicast.

Li studied the multicast capacity of ad hoc networks [2]. In this work, all the multicast sessions have identical number of destination nodes which are uniformly selected in the whole region. The achievable per-flow capacity given in this work is $\Theta(\frac{W}{\sqrt{kn \log n}})$, where k is the number of destination nodes. Shakkottai *et al.* studied the capacity of networks in which there are n^ϵ multicast sessions and each has $n^{1-\epsilon}$ destinations [3]. Their per-flow capacity is $\Theta(\frac{W}{\sqrt{n^\epsilon \log n}})$. Kozart studied the relationship between delay and capacity for multicast sessions [14]. Some other works focus on the multicast capacity of hybrid networks, investigating the effect of the number of base stations on capacity [4] and the traffic balancing between two transmission means [15]. Tang *et al.* focused on the capacity of hybrid networks with limited bandwidth between base stations [10]. All the above works assume that the sessions randomly choose the same number of destinations in the whole region. Our work does not enforce such an assumption.

Peng *et al.* studied the multicast capacity of networks with more general multicast sessions [9]. In their work, the source nodes pick their destinations according to a dispersion density function. These sessions share the same dispersion density function, and most of the sessions have the same numbers of destinations. The percolation theory is used in their work. The achieved capacity is $O(\frac{max\{1, a\delta_o\}W}{\sqrt{n_s}})$, where a is the side length of the whole square region, and δ_0 is the variance of the dispersion density function. Though the network model in [9] allows heterogeneous multicast sessions, one of its main improvements is for the case where the destinations are not uniformly picked from the whole region. The identities of the sessions are still required, which limits the commonality of the network model compared with ours.

Cooperative transmitting was introduced to improve multicast capacity in [11] and [16]. Rahul *et al.* developed a system [17] that could dynamically increase the capacity with user demand. These works take advantage of the extra coding technologies in increasing network capacity, while ours focuses on the network layer.

3. NETWORK MODEL

3.1 Networks

Assume n static wireless communication nodes, denoted as $M = \{m_1, m_2, \ldots, m_n\}$, are uniformly distributed in a region Ω. Ω is a square with side length $a = n^\alpha$, $\alpha \leq 1/2$.

These nodes form a random dense network, which means the node density increases as n grows. All the nodes transmit via the same channel, at a rate of no more than W $bits/s$. Two nodes u and v can transmit directly if $|u-v| \le r$, where $|u-v|$ is the Euclidean distance between u and v, and r is the transmission range of the sender. For simplicity, all the nodes have the same transmission range r. r is relatively large so that the whole network is connected with high probability, which means any pair of nodes can transmit either directly or in a multi-hop manner.

The protocol model for interference [1] is applied to define conflicts in a network: in each time slot, node i can successfully send a packet to node j if $|i-j| \le r$ and the Euclidean distance between any other concurrent transmitters and node j is bigger than $(1+\Delta)r$, where Δ is a positive constant number independent of nodes i and j, and it is set to $\Delta \ge 2$ [18] in this paper. Our model also works for $1 \le \Delta < 2$, which is proved in Section 6.3. Every transmission has a "guard zone" where no nodes can deliver packets simultaneously. This zone is a circle area around the destination with radius $(1+\Delta)r$. The guard zone of a multicast tree is the union of the guard zones of all the nodes in the tree. Two transmissions can occur concurrently if their senders are not within each other's guard zone. In a similar way, two multicast sessions can transmit simultaneously if their involved nodes fall outside of each other's guard zone.

Following are some other assumptions on a network: (1) the wireless channel is multiplexed by TDMA [18] and it is equally divided in a round robin manner when multiple transmissions conflict with each other, (2) the relay nodes only receive, store, and send packets in a multi-hop manner, and (3) the wireless channel is always in a good condition, *i.e.*, there is no packet loss if there is no conflict.

3.2 Traffic Pattern

Let ms_i denote the ith multicast session, $i = 1, 2, \ldots, n_s$, $n_s = \Theta(n^{\epsilon})$, and $1/2 < \epsilon \le 1$. The set of the source nodes is expressed by $S = \{s_1, s_2, \ldots, s_{n_s}\}$. For source node s_i, it randomly picks k_i destination nodes. The k_i destinations are all within a circle area with radius R_i centered at s_i. Each session ms_i independently and randomly determines its k_i and R_i. s_i transmits data to all its destinations via single or multi hops through a multicast tree T_i, which is a Steiner tree connecting the source node and all its destinations with the help of some intermediate nodes. A multicast session must follow the following restrictions: (1) k_i is less than the total number of nodes in its destinations' distributed area for ms_i, and (2) any edge in T_i is not greater than r.

3.3 Capacity

Let $V_s = \{\lambda_1, \lambda_2, \ldots, \lambda_{n_s}\}$ be an instance of the rate vector for all the multicast sessions. Here, λ_i is the transmission rate of source node s_i. Rate vector V_s is called feasible if there is a spatial and temporal scheme for scheduling the transmissions of these multicast sessions so that every source node s_i, $i = 1, 2, \ldots, n_s$, can transmit to all of its destination nodes at a rate of $v_i = \lambda_i$. It is assumed that the buffer at any intermediate node never overflows under this scheme, and thus no data is discarded. For a sufficiently long time T, all the destination nodes can separately receive $T \times \lambda_i$ bits from their corresponding source s_i.

λ_i is called the per-flow throughput capacity of both source node s_i and the multicast session ms_i. The aggregated multicast throughput capacity (capacity for short) of a feasible rate vector V_s is defined as $\Lambda_{n_s}(n) = \sum_{s_i \in S} \lambda_i$. The multicast capacity of random networks is defined as follows.

Definition 1. *(Capacity of Networks)[1]. The total multicast capacity of a network is in the order of $\Theta(f(n))$ bits/s if there is a constant $c > 0$ and $c < c' < +\infty$ such that*

$$\lim_{n\to\infty} Pr(\Lambda_{n_s}(n) = cf(n) \text{ is feasible}) = 1$$
$$\lim_{n\to\infty} Pr(\Lambda_{n_s}(n) = c'f(n) \text{ is feasible}) < 1$$

The definition of per-flow capacity of a network is similar.

4. UPPER BOUND FOR MULTICAST CAPACITY

In this section, part 1 analyzes the fundamental constraints that the proposed model must satisfy. Part 2 introduces the competitive intensity in wireless networks and estimates the intensity of a wireless network under the proposed network model. Part 3 derives the theoretical upper bound of the multicast capacity.

4.1 Constraints on Transmission Range and Destinations

Firstly, it is crucial to decide the minimum transmission range ensuring there is no isolated node in a network as n increases to infinity. All multicast sessions finish their transmissions via multicast trees under the new network model. However, when the transmission range r is relatively small, a node may fail to communicate with any other one and the multicast sessions choosing this node as a destination are infeasible. The minimum transmission range is also called the Critical Transmission Range (CTR). As proved in [1], when n nodes are uniformly distributed in a square region with side length a, the CTR is $\sqrt{\frac{\log n+\beta}{\pi n}}a$, where β is a constant. Thus, the actual transmission range must be bigger than CTR. Meanwhile, larger r leads to more conflicts [1], so the transmission range is:

$$\frac{r}{a} = \Theta\left(\sqrt{\frac{\log n}{\pi n}}\right) \tag{1}$$

Secondly, k_i and R_i of a multicast session are randomly chosen, and a simple constraint is that k_i must be less than the total number of nodes which fall inside of the circle area with radius R_i. The number of nodes in this circle area is denoted by C_{R_i}. C_{R_i} is independent of k_i because the nodes are uniformly distributed in the region. The following theorem helps quantify the constraint.

Theorem 1. *When n nodes are uniformly distributed in a square region with side length a, $Pr(C_{R_i} = \Theta(n \cdot \frac{R_i^2}{a^2})) \to 1$ when $n \to \infty$, where $Pr(T) \to 1$ means the occurrence probability of event T approaches to 1.*

Proof. Since nodes are uniformly distributed, for any node m_i,

$$Pr(m_i \text{ in circle area with } R_i) = \frac{\pi R_i^2}{a^2},$$

and $Pr(m_i$ *in circle area with* $R_i)$ is independent of all the nodes. It is obvious that $E(C_{R_i}) = \frac{\pi n R_i^2}{a^2}$, and according to the Chernoff's inequality [19], $Pr(|C_{R_i} - E(C_{R_i})| \ge$

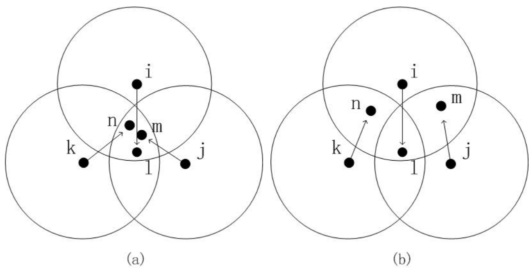

Figure 1: (a) Transmissions (i, l), (k, n), (j, m) conflict with each other. (b) Two transmissions pairs, (i, l), (k, n) and (i, l), (j, m), conflict with each other.

$\frac{E(C_{R_i})}{2}) \leq 2e^{-\frac{E(C_{R_i})}{12}}$. When $n \to \infty$, since $a^2 = n^{2\alpha} \leq n$, $2e^{-\frac{E(C_{R_i})}{12}} = 2e^{-\frac{\pi n {R_i}^2}{12a^2}} = 2e^{-\frac{\pi {R_i}^2 n^{1-2\alpha}}{12}} \to 0$. Thus, $Pr(\frac{E(C_{R_i})}{2} \leq C_{R_i} \leq \frac{3E(C_{R_i})}{2}) \to 1$ as $n \to \infty$. $\square$

From Theorem 1, we have $k_j = O(\frac{n{R_j}^2}{a^2})$ for every multicast session ms_j, $j = 1, 2, \ldots, n_s$.

4.2 Competitive Intensity

In a wireless network, all the source-destination pairs communicate via a common wireless channel. Because of the inherent destructive interference [1], two transmission pairs cannot transmit simultaneously if they are sufficiently close to each other. Then we say the two transmissions are competing for the channel with each other. Under the protocol model, a circle area of radius $(1+\Delta)r$ around a destination is called the guard zone. Any other transmitter inside this zone will cause a destructive interference when it delivers packets simultaneously, and the wireless channel has to be equally divided by all these transmitters. This competition, in fact, may be over-estimated, since some of the transmitters in this guard zone can be scheduled at the same time, as shown in Fig.1. In Fig.1(a), transmissions (i, l), (k, n), (j, m) conflict with each other, so node i can be scheduled once in three slots. But in Fig,1(b), transmissions (j, m) and (k, n) can be scheduled at the same time, so node i can actually be scheduled once in two slots. The competitive intensity is thus defined as the number of transmitters that a node or a transmission has to compete with one at a time.

Definition 2. ***Competitive Intensity*** *of node i, denoted as CI_i, is the size of the maximum set of nodes that cannot successfully transmit simultaneously with node i, and also any pair of nodes in the set fails to transmit at the same time.*

Multiple transmissions may choose same nodes as their transmitters, and they are obviously competitors for each other. Then the competitive intensity for a single transmission can be defined similarly.

The competitive intensity is a lower bound of the time intervals that a node or a transmission can be scheduled once. An upper bound of the transmission rate of a node or a transmission can be directly derived. Meanwhile, a multicast session is actually carried out by many pairwise transmissions, so the competitive intensity can help estimate the throughput of a session and then derive a bound on the multicast capacity. The competitive intensity of a single transmission under the proposed network model is estimated as follows.

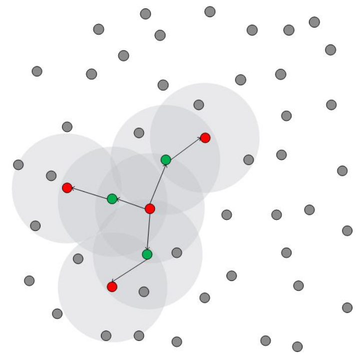

Figure 2: The union of the grey circles is the interference area. Green nodes stand for the intermediate nodes.

For an arbitrary session ms_i, source node s_i transmits to its destination nodes via a multicast tree T_i. All the nodes in the tree will receive a copy of the data originated by s_i. Meanwhile, all the nodes close to these nodes in the tree will overhear the data. They cannot participate in other sessions at the same time. Here, overhearing means a node is within the range $(1 + \Delta)r$ of any interior node in the tree of the current session. (Obviously, an overhearing at a node occurs when it is within the transmission range r of an interior node in the tree). Overhearing increases competitive intensity of these nodes, and leads to smaller transmission rate. In fact, the multicast tree forms an interference area $D(T_i)$, which is the union of the transmission areas of all the nodes in the tree. The nodes in this interference area overhear the data from the nodes in the tree. Fig.2 shows an example of the interference area, which is the union of the grey circles. Evaluating interference areas of all the sessions can help compute how many copies of data a single node will overhear, which is actually a lower bound of competitive intensity.

Consider an arbitrary multicast session ms_i. Its multicast tree T_i is composed of source node s_i, destinations $U = \{d_1, d_2, \ldots, d_{k_i}\}$ and some intermediate nodes. The whole tree falls in a circle region with radius R_i, centered at s_i. Define the total Euclidean length of a tree T's edges as $||T||$. $MST(s_i)$ is the minimum spanning tree connecting s_i and all its destinations. According to a series of theoretical studies [20], it is often believed that $||T_i|| \geq \beta_0 \cdot ||MST(s_i)||$, where β_0 is constant. The following lemma gives a lower bound of $||MST(s_i)||$.

Lemma 1. *For k_i randomly and uniformly chosen nodes in a circle region with radius R_i centered at node s_i,*

$$||MST(s_i)|| = \beta_1 \cdot c(2) \cdot \sqrt{k_i} \cdot R_i,$$

where β_1 and $c(2)$ are constant.

Proof. Based on the results in [21],

$$||MST(U)|| = c(2) \cdot \sqrt{k_i} \cdot \sqrt{\pi} R_i \tag{2}$$

$MST(U)$ is the minimum spanning tree connecting all the k_i destinations. When node s_i is added to $MST(U)$, it connects itself to some node in this tree, say d_u. The additional length is no more than R_i. So we have

$$||MST(s_i)|| \le ||MST(U)|| + R_i.$$

The new spanning tree may not be minimal, and the added node could possibly modify the original edges. But since the destinations are randomly and uniformly picked, when k_i is sufficiently large, the change only affects a small proportion of the tree. Thus,

$$||MST(s_i)|| \ge \beta_2||MST(U)||, 0 \le \beta_2 \le 1.$$

Combining the above two inequalities, we have

$$||MST(s_i)|| = \beta_1||MST(U)||.$$

β_1 is a constant and this can finish the proof when combined with Equation (2). □

For simplicity, let $\beta_3 = \beta_0 \cdot \beta_1 \cdot c(2)$, so $||T_i|| \ge \beta_3 \cdot \sqrt{k_i} \cdot R_i$. The bound of $||T_i||$ can help derive a lower bound on the size of the interference area $D(T_i)$. A similar problem has been considered in [2], which can be extended to the following lemma.

LEMMA 2. *The size of interference area $D(T_i)$, denoted as $|D(T_i)|$, is at least $\beta_4 \cdot \sqrt{k_i} \cdot R_i \cdot r$ with high probability, when $k_i = O(\frac{R_i{}^2}{r^2})$, and $\beta_5 \cdot R_i{}^2$ when $k_i = \theta(\frac{R_i{}^2}{r^2})$. Here, β_4 and β_5 are constant, and r is the common transmission range.*

PROOF. This proof can be derived from Lemma 11 and Lemma 15 in [2]. We only need to change k to k_i and a to R_i.

Firstly, we build a new multicast tree T_i' based on T_i. Various methods [22, 23, 24] can be applied for tree construction. T_i' is a connected dominating set of the nodes in T_i, and it includes source node s_i. It has been proved by Lemma 11 in [2] that the maximum degree d of the nodes in T_i' is no more than 13 and $|D(T_i')| \ge \frac{|T_i'|\pi r^2}{6(d+1)}$, where $|T_i'|$ is the number of the nodes in T_i'.

Based on Lemma 1, $||T_i'|| + k_i \cdot r \ge \beta_0||MST(s_i)|| \ge \beta_3 \cdot \sqrt{k_i} \cdot R_i$, and $|T_i'| \ge \frac{||T_i'||}{r}$.

Since the set of the nodes in T_i' is a subset of the set of the nodes in T_i, we have

$$|D(T_i)| \ge |D(T_i')| \ge \frac{|T_i'|\pi r^2}{6(d+1)} \ge \frac{\frac{\beta_3 \cdot \sqrt{k_i} \cdot R_i - k_i \cdot r}{r}\pi r^2}{6(d+1)}.$$

When $k_i = O(\frac{R_i{}^2}{r^2})$,

$$|D(T_i)| \ge \frac{\sqrt{k_i} \cdot \pi r \cdot (\beta_3 R_i - O(R_i))}{6(d+1)} \approx \beta_4 \cdot \sqrt{k_i} \cdot R_i \cdot r,$$

where $\beta_4 = \frac{\pi \cdot \beta_3}{6(d+1)}$, and when $k_i = \theta(\frac{R_i{}^2}{r^2})$, $|D(T_i)| = \Theta(R_i^2)$. □

A limitation is that k_i must be less then C_{R_i}, thus the second assumption of k_i in Lemma 2 is feasible only when $C_{R_i} = \Omega(\frac{R_i{}^2}{r^2})$. $C_{R_i} = \Theta(n \cdot \frac{R_i{}^2}{a^2})$ according to Theorem 1, and since $\frac{r}{a} \ge \sqrt{\frac{\log n}{\pi n}}$, we have $C_{R_i} \ge \frac{R_i{}^2}{r^2}$.

It can be concluded that

$$|D(T_i)| = \begin{cases} \Omega(\sqrt{k_i} \cdot R_i \cdot r) & k_i = O(\frac{R_i{}^2}{r^2}) \\ \Theta(R_i{}^2) & k_i = \theta(\frac{R_i{}^2}{r^2}) \end{cases}$$

This bound can be used to estimate the probability that an arbitrary node m_j falls in $D(T_i)$:

$$Pr(m_j \text{ falls in } D(T_i)) = \frac{|D(T_i)|}{a^2} \quad (3)$$

Next lemma estimates the number of sessions whose interference area covers an arbitrary node m_j, denoted as I_j.

LEMMA 3. *For the wireless network and multicast sessions defined in Section 2,*

$$I_j \ge \frac{\sum_{i=1}^{n_s} min\{\sqrt{k_i} \cdot R_i \cdot r, R_i{}^2\}}{2a^2}, \text{ w.h.p..}$$

PROOF. Let $X_i = 1$ when ms_i covers m_j and 0 otherwise, $i = 1, 2, \ldots, n_s$. For a single multicast session ms_i, according to Equation (3),

$$Pr(X_i = 1) = \frac{|D(T_i)|}{a^2}.$$

Since the multicast sessions are generated independently, $I_j = \sum_{i=1}^{n_s} X_i$. So

$$E(I_j) = E(\textstyle\sum_{i=1}^{n_s} X_i) = \sum_{i=1}^{n_s} E(X_i) = \sum_{i=1}^{n_s} \frac{|D(T_i)|}{a^2}.$$

According to Lemma 2, $|D(T_i)| = min\{\beta_6\sqrt{k_i} \cdot R_i \cdot r, R_i{}^2\}$, where β_6 is a constant, so

$$E(I_j) = \frac{\sum_{i=1}^{n_s} min\{\sqrt{k_i} \cdot R_i \cdot r, R_i{}^2\}}{a^2}.$$

β_6 is omitted because it only changes $E(I_j)$ by constant times. Then

$$Pr(I_j \le \frac{E(I_j)}{2}) \le Pr(|I_j - E(I_j)| \ge \frac{E(I_j)}{2}) \le 2e^{-\frac{E(I_j)}{12}},$$

which is based on the Chernoff's inequality. Since $E(I_j) \to +\infty$ as the number of nodes goes to infinity, it is clear that $2e^{-\frac{E(I_j)}{12}} \to 0$. So $I_j \ge \frac{E(I_j)}{2}$ with high probability. □

Lemma 3 sets a lower bound on the number of sessions that compete with an arbitrary node m_j. But this may be an over-estimation of the competitive intensity, since some of these sessions can perhaps be scheduled together. Next lemma indicates that these sessions also conflict with each other.

LEMMA 4. *For two multicast sessions ms_i and ms_j, if $D(T_i)$ and $D(T_j)$ both cover a node m, then ms_i and ms_j cannot transmit simultaneously.*

PROOF. If $D(T_i)$ and $D(T_j)$ both cover a node m, obviously there must be nodes m_i and m_j in each session and their Euclidean distance to m is less than r. So the distance between m_i and m_j is less than $2r$. If they are both leaf nodes or non-leaf nodes, then the distance between one node and another's sender or receiver is always less than $3r$. m_i and m_j compete with each other since $\Delta \ge 2$. Otherwise, we assume m_i is a leaf node and m_j is an interior node, then m_j is in m_i's guard zone. ms_i and ms_j cannot transmit simultaneously either. □

Finally, a lower bound on the competitive intensity CI_j can be derived by combining Lemma 3 and Lemma 4:

$$CI_j \ge \frac{\sum_{i=1}^{n_s} min\{\sqrt{k_i} \cdot R_i \cdot r, R_i{}^2\}}{2a^2}$$

4.3 Upper Bound on Capacity

It is crucial to notice that the transmission rate of a multicast tree is no more than that of any node in the tree. Otherwise, the buffers of some nodes may overflow and packets will be dropped. Thus, when the lower bound of the scheduling time interval of a node in the tree is derived, an upper bound of the transmission rate of this multicast session is also obtained. An interval means the time between two assigned slots during which a node transmits for the same session.

A node may be an intermediate node of multiple multicast trees and be assigned multiple slots. The interval of a node is no less than its competitive intensity in a round robin manner. Then the transmission rate is bounded by the reciprocal of its competitive intensity for a single transmission. So the upper bound of the per-flow throughput capacity of a multicast session is given by the following theorem.

THEOREM 2. *For any multicast session ms_i, its per-flow throughput capacity v_i is less than $\frac{2a^2}{\sum_{i=1}^{n_s} min\{\sqrt{k_i}\cdot R_i\cdot r, {R_i}^2\}}\cdot W$ bits/second with high probability.*

PROOF. For multicast tree T_i, take the source node s_i into consideration. Its competitive intensity is at least

$$\frac{\sum_{i=1}^{n_s} min\{\sqrt{k_i}\cdot R_i\cdot r, {R_i}^2\}}{2a^2}$$

with high probability according to Lemma 3 and Lemma 4. Since the channel is equally divided for these sessions, the transmission rate of s_i is no more than $\frac{2a^2}{\sum_{i=1}^{n_s} min\{\sqrt{k_i}\cdot R_i\cdot r, {R_i}^2\}}\cdot W$. This is also an upper bound for session ms_i, then

$$v_i = O(\frac{2a^2}{\sum_{i=1}^{n_s} min\{\sqrt{k_i}\cdot R_i\cdot r, {R_i}^2\}}\cdot W)$$

□

Then the multicast capacity of a random network is bounded by

$$\Lambda_{n_s}(n) = O(\frac{2a^2\cdot n_s}{\sum_{i=1}^{n_s} min\{\sqrt{k_i}\cdot R_i\cdot r, {R_i}^2\}}\cdot W) \qquad (4)$$

5. AN ACHIEVABLE LOWER BOUND

This section provides a feasible transmission scheme for all the multicast sessions, and derives the lower bound of the multicast capacity achieved by this scheme. The basic idea of the scheme has also been used by lots of previous studies on network capacity [1][2][9].

5.1 A Scheme for Multicast

The proposed scheme is based on the idea of a backbone graph [25]. It partitions region Ω into equal-sized square grids $G = \{g_{1,1}, g_{1,2}, \ldots, g_{1,l}, g_{2,1}, \ldots, g_{l,l}\}$, each with side length $\frac{r}{\sqrt{5}}$, $l = \lceil\frac{a}{r/\sqrt{5}}\rceil$. When two grids share a common side, any two nodes in these two grids can communicate directly since their distance is always no bigger than r. In each grid, one node is randomly picked as the leading node. All the leading nodes form a connected graph. In each grid, other nodes can communicate with their leading node in a single hop. The leading nodes actually form a backbone graph for the network. All the sessions route on top of this backbone graph. A transmitter first forwards its packet to its leading node in the grid, then to the receiver's grid via the backbone graph, and finally to its receiver in the last step. The following two principles are proved before we introduce the comprehensive scheme:

1) No grid is empty when n tends to be infinite.

2) Each leading node can be scheduled in constant time intervals by a round robin manner.

LEMMA 5. *For any grid $g_{i,j}$, there is at least one node in the grid with high probability as n approaches to infinity.*

PROOF. For an arbitrary node m_i, $Pr(m_i$ in grid $g_{i,j}) = \frac{r^2}{5a^2}$, so

$$Pr(g_{i,j} \text{ is empty}) = (1-\frac{r^2}{5a^2})^n \le e^{-\frac{nr^2}{5a^2}}.$$

Since $r/a \ge \sqrt{\frac{\log n}{n}}$, when $n\to\infty$,

$$Pr(g_{i,j} \text{ is empty}) \le e^{-\frac{nr^2}{5a^2}} \le e^{-\log n} \to 0.$$

□

Lemma 5 ensures there is almost no empty grid, and the following lemma can support the second principle.

LEMMA 6. *On the backbone graph, every node can transmit once in every P time slots, $P \le 5\pi(3+\Delta)^2$.*

PROOF. If a transmitter u interferes with another node v, the Euclidean distance between u and v is less than $(1+\Delta)r + r$. It means the distance between one receiver and another source could be less than $(1+\Delta)r$. Thus the grid that u falls in is the circle of radius $(1+\Delta)r+r$ centered at v. Then, it is obvious that

$$P \le \frac{\pi((1+\Delta)r+2r)^2}{r^2/5} = 5\pi(3+\Delta)^2.$$

These P leading nodes can be scheduled by a round robin method, and each transmits once in every P time slots. □

The total number of nodes in each of these P grids is in the order of $\Theta(n\cdot\frac{r^2}{a^2}) = \Theta(\log n)$, and the proof is similar to Theorem 1.

The proposed scheme carries out the multicast sessions via multicast trees. The construction of the multicast tree for an arbitrary session ms_i includes three steps:

(1) Repeat steps (2) and (3) for $k = 2, 3, \ldots, k_i$.

(2) In the kth step, partition the circle region into no more than $k_i + 1 - k$ cells. Firstly, the partition is done by $\frac{R_i}{\sqrt{k_i+1-k}} + 1$ vertical lines. The first and last lines tangent to the circle, and others are between them with identical distance $\frac{R_i}{\sqrt{k_i+1-k}}$. Then the process is performed again with a group of horizontal lines.

(3) According to the pigeonhole principle, there is at least a cell containing two or more nodes. Two of the nodes in the same cell are connected by Manhattan Routing and will be considered as an integrated node in the next round. Thus, the number of nodes remained for connection decreases by one at the end of each round until all nodes are connected.

In step (3), when node u expects to transmit to node v, u first horizontally transmits its packet to the leading node of grid $g_{u,v}$. The leading node then vertically forwards the packet to v. Grid $g_{u,v}$ is a grid in the same row with u and same column with v. The routing strategy is called Manhattan Routing, as shown in Fig.3 where the red nodes are the transmitter and receiver, and the green nodes are the leading nodes. A multicast session involves three phases:

Phase 1 The source node in the multicast tree forwards its packets to the leading node of its own grid.

Phase 2 Packets are sent to the grids that contain their receivers via the backbone path of leading nodes. The Manhattan Routing is used in this phase.

Phase 3 Leading nodes deliver packets to their corresponding receivers, and the transmission is finished.

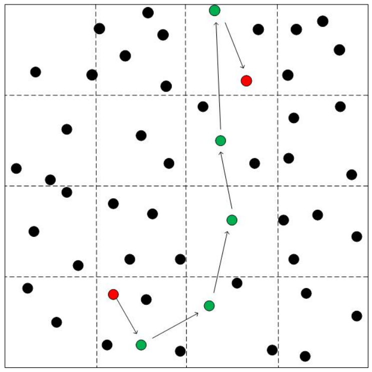

Figure 3: Red nodes are source-destination pair, green nodes are leading nodes, and the side length of a grid is $\frac{r}{\sqrt{5}}$.

It is important to notice that all the destinations connected by a backbone path are actually leaf nodes of the multicast tree, and they do not act as transmitters. Only the source node delivers packets in the first phase.

5.2 Achievable Capacity of the Scheme

We build a feasible multicast tree T_i for a multicast session. This subsection estimates the transmission rate of T_i. For phase 1 and phase 3, it is easy to prove that each node can transmit or receive at the rate of $\Omega(\frac{1}{\log n} \cdot W)$ *bits/second*, since there are $\Theta(\log n)$ nodes in each grid, and a grid can be scheduled once every constant number of time slots. The transmission rate of each multicast session in phase 2 is analyzed below. The proof mainly focuses on estimating the number of sessions that a leading node has to serve. The transmission rate of the leading node is equally partitioned by these sessions.

LEMMA 7. *Given a grid g, the probability that a multicast session ms_i will use g is no more than $\beta_7\frac{\sqrt{k_i}\cdot R_i\cdot r}{a^2}$, where β_7 is a constant, saying ms_i uses g when the leading node of g is an interior node of T_i.*

PROOF. First of all, the leading node must fall in the distributed region of ms_i, which happens with probability $\frac{\pi R_i^2}{a^2}$.

According to the proposed scheme, in the kth step, $k = 1, 2, \cdots, k_i$, the probability that grid g is used by Manhattan Routing can be calculated as follow:

$$Pr(X_k = 1) = \frac{1}{k_i+1-k} \cdot p_s(\frac{R_i/\sqrt{k_i+1-k}}{r/\sqrt{5}})$$

Here, $X_k = 1$ means grid g is used in the kth step, and $\frac{1}{k_i+1-k}$ is the probability that the cell in which grid g falls contains more than two destinations. $p_s(\frac{R_i/\sqrt{k_i+1-k}}{r/\sqrt{5}})$ is the probability that grid g is on the transmitting path of the two destinations. L_i stands for the number of columns (rows) of grids in a cell in the ith round. Define $L_i = \frac{R_i/\sqrt{k_i+1-k}}{r/\sqrt{5}}$, then $p_s(L_i) = \frac{m-1}{L_i^2} \cdot \frac{L_i-m+1}{L_i} + \frac{n-1}{L_i^2} \cdot \frac{L_i-n+1}{L_i}$. Let u and v denote the two nodes to be connected in the ith round. The first part of the right side means that u is in the same row with grid g, and node u and v are on different sides of the column that grid g falls in. The second part means v is in the same column with grid g, node u and v are on different sides of grid g's row. Obviously,

$$Ps(L_i) \leq \frac{2}{L_i}$$

p is the probability that grid g is used by the multicast session ms_i when g is inside the distributed region of ms_i. Then for all the k_i steps,

$p \leq \sum_{k=1}^{k_i} Pr(X_k = 1) = \sum_{k=1}^{k_i} \frac{1}{k_i+1-k} \cdot p_s(\frac{R_i/\sqrt{k_i+1-k}}{r/\sqrt{5}}) \leq \frac{4\sqrt{10}}{5} \cdot \sqrt{k_i} \cdot \frac{r}{R_i}$.

Then

$$Pr(ms_i\ use\ g) \leq \frac{\pi R_i^2}{a^2} \cdot \frac{4\sqrt{10}}{5} \cdot \sqrt{k_i} \cdot \frac{r}{R_i} = \beta_7 \frac{\sqrt{k_i} \cdot R_i \cdot r}{a^2} \quad (5)$$

□

Since there are in total n_s independent multicast sessions, Lemma 8 can derive the upper bound of the number of sessions that use a grid g.

LEMMA 8. *For any grid g, the number of multicast sessions using g is less than $\frac{3\sum_{i=1}^{n_s} min\{\sqrt{k_i}\cdot R_i\cdot r, R_i^2\}}{2a^2}$ with high probability.*

PROOF. Let $Y_i = 1$ when ms_i uses g, and $Y_i = 0$ otherwise, $i = 1, 2, \ldots, n_s$. According to Lemma 7, $Pr(Y_i = 1) \leq \beta_7 \frac{\sqrt{k_i}\cdot R_i\cdot r}{a^2}$. Then

$$E(Y) = E(\textstyle\sum_{i=1}^{n_s} Y_i) = \sum_{i=1}^{n_s} E(Y_i),$$

also because $Pr(ms_i\ uses\ g) \leq \frac{\pi R_i^2}{a^2} \cdot 1$,

$$E(Y) \leq \frac{\sum_{i=1}^{n_s} min\{\sqrt{k_i}\cdot R_i\cdot r, R_i^2\}}{a^2}.$$

According to the Chernoff's inequality,

$$Pr(Y \geq \frac{3E(Y)}{2}) \leq Pr(|Y - E(Y)| \geq \frac{E(Y)}{2}) \leq 2e^{-E(Y)/12}$$

and then the lemma is proved since $E(Y)$ goes to infinity as n increases. □

Lemma 8 gives an upper bound of the number of sessions that a leading node must serve. According to the previous subsection, a leading node can transmit at a constant rate, and its rate is equally divided by these sessions. Then the per-flow throughput of these multicast sessions can be bounded.

THEOREM 3. *For any multicast session ms_i, its per-flow throughput capacity is more than*

$$\frac{2a^2}{3\sum_{i=1}^{n_s} min\{\sqrt{k_i}\cdot R_i\cdot r, R_i^2\}} \cdot W\ bits/second\ w.h.p..$$

PROOF. In the proposed scheme, phase 2 is carried out via the backbone path composed of leading nodes. According to Lemma 8, the transmission rate of phase 2 is bigger than $\frac{2a^2}{3\sum_{i=1}^{n_s} min\{\sqrt{k_i}\cdot R_i\cdot r, R_i^2\}} \cdot W$. This is because the wireless channel is equally divided for the sessions. Denote the average value of $\{\sqrt{k_1}R_1, \sqrt{k_2}R_2, \cdots, \sqrt{k_{n_s}}R_{n_s}\}$ as $\overline{\sqrt{k_i}R_i}$.

When $\overline{\sqrt{k_i}R_i} = \Omega(a)$, since $n_s = \Theta(n^{\epsilon}), 1/2 < \epsilon \leq 1$, the following inequality is true when n goes to infinity:

$$\frac{2a^2}{3\sum_{i=1}^{n_s} min\{\sqrt{k_i \cdot R_i \cdot r}, R_i^2\}} \cdot W \leq \frac{2a^2}{3 \cdot n_s \cdot a \cdot r} \cdot W \leq \frac{W}{\log n}.$$

$\Theta(\frac{W}{\log n})$ is an achievable transmission rate of phase 1 and phase 3.

The proof is a little complex when $\overline{\sqrt{k_i}R_i} = O(a)$, where $\overline{\sqrt{k_i}R_i} = \frac{\sum_{i=1}^{n_s}\sqrt{k_i}R_i}{n_s}$. Here is an intuitive description. The detailed proof is omitted due to space limitation. In phase 1 and phase 3, the leading node only serves the sessions whose sources or destinations fall inside its grid, while in phase 2, the leading node also acts as a relay node for other sessions besides receiving the packets towards its own grid. Then the number of packets a leading node sends or receives in phase 2 is larger than that of phase 1 and phase 3.

So the lower bound of the transmission rate for phase 2 is also a lower bound of the per-flow throughput capacity:

$$v_i = \Omega(\frac{2a^2}{3\sum_{i=1}^{n_s} min\{\sqrt{k_i} \cdot R_i \cdot r, R_i^2\}} \cdot W) \quad \square$$

Then a lower bound of multicast capacity is the sum of the achievable throughputs of all the sessions:

$$\Lambda_{n_s}(n) = \Omega(\frac{2a^2 n_s}{3\sum_{i=1}^{n_s} min\{\sqrt{k_i} \cdot R_i \cdot r, R_i^2\}} \cdot W)$$

6. DISCUSSION

In this section, some discussions of the network model and the derived bounds are presented.

6.1 Capacity Variation While R_i And k_i Change

R_i and k_i can decide the size of the interference area of multicast session ms_i based on Lemma 2. A bigger size causes larger traffic load due to the following reasons. More extra nodes serve as the relay nodes and more nodes incur collisions because of falling inside the interference area. Larger traffic load leads to smaller capacity since the channel resource for each session decreases. We are curious about how different R_is and k_is affect the total traffic load of a wireless network in our model, especially the groups of R_is and k_is that make the load sufficiently large or small. The result is shown by the relationship between the capacity and different R_is and k_is.

It is known that unicast and broadcast are two tails of multicast: unicast is the case where each session only contains single or few destinations, and broadcast is the case where the number of destinations of each session is in the same order of the total number of nodes in the network. The unicast capacity of a network is in the order $\Theta(W * \frac{a}{r})$, and the broadcast capacity is in the order $\Theta(W)$. We say that the traffic load of a wireless network is "heavy" when its capacity is in the same order of a broadcast network, and "light" for the unicast-like traffic load.

When R_is and k_is are both non-identical, define the average value of $\{\sqrt{k_1}R_1, \sqrt{k_2}R_2, \ldots, \sqrt{k_{n_s}}R_{n_s}\}$ as $\overline{\sqrt{k_i}R_i}$. Then i) If $\overline{\sqrt{k_i}R_i} = \Omega(\frac{a^2}{r})$, the capacity $\Lambda_{n_s}(n) = \Theta(W)$, which is derived from $\Lambda_{n_s}(n) = \Theta(\frac{a^2 n_s}{\sum_{i=1}^{n_s} min\{\sqrt{k_i} \cdot R_i \cdot r, R_i^2\}} \cdot W)$. The traffic load is heavy in the whole network, because most sessions expect to transmit either to many destinations (*i.e.*, large k_is) or to some destinations quite far away (*i.e.*, large R_is). In this case, the traffic load is like that of a broadcast network. ii) If $\overline{\sqrt{k_i}R_i} = O(a)$, the capacity $\Lambda_{n_s}(n) = \Theta(\frac{a}{r} \cdot W)$. The traffic load is light in the wireless network.

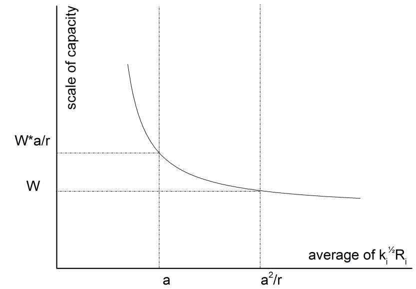

Figure 4: Relationship between $\overline{\sqrt{k_i}R_i}$ and multicast capacity.

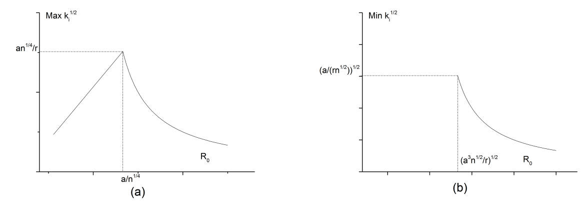

Figure 5: (a) $Max\ \overline{\sqrt{k_i}}$ keeps traffic load light. (b) $Min\ \overline{\sqrt{k_i}}$ keeps traffic load heavy.

The interference area of each session is only a small region. There are not many overlaps between these sessions' interference areas even when there are quite many multicast sessions (*i.e.*, $n_s = \Theta(n^\epsilon)$). In this case, the traffic load is like that of an unicast network.

Define the variance of $\{\sqrt{k_1}R_1, \sqrt{k_2}R_2, \ldots, \sqrt{k_{n_s}}R_{n_s}\}$ as $\delta(\sqrt{k} \cdot R)$. The changing of $\delta(\sqrt{k} \cdot R)$ will not affect the capacity, because the multicast capacity is bounded by the total traffic load, and the traffic load is determined by $\overline{\sqrt{k_i}R_i}$ and n_s according to our proofs. The relationship between $\overline{\sqrt{k_i}R_i}$ and $\Lambda_{n_s}(n)$ is shown in Fig.4.

Next, we discuss R_is and k_is separately. The primary question is how k_is (R_is) will affect multicast capacity when R_is (k_is) are set to be identical at some levels.

When R_is are set to be identical, the capacity $\Lambda_{n_s}(n) = \Theta(\frac{a^2 n_s}{\sum_{i=1}^{n_s} R_i \cdot min\{\sqrt{k_i} \cdot r, R_i\}} \cdot W)$, which is a function of $\overline{\sqrt{k_i}}$.

When $R_i = O(\frac{a}{n^{\frac{1}{4}}})$, since $k_i = O(n \cdot \frac{R_i^2}{a^2})$, $\overline{\sqrt{k_i}R_i} = O(\sqrt{n} \cdot \frac{R_i}{a} \cdot R_i) = O(\sqrt{n} \cdot \frac{1}{a} \cdot \frac{a^2}{\sqrt{n}}) = O(a)$. Then the traffic load in the network is always like that of an unicast network. The largest $\overline{\sqrt{k_i}}$ that can ensure that the traffic load is light equals $\frac{a}{R_i}$ if R_i keeps growing. The relationship between R_i and $\overline{\sqrt{k_i}}$ is shown in Fig.5(a).

On the other hand, with identical R_is, when $R_i = O(\sqrt{\frac{a^3}{r \cdot \sqrt{n}}})$, $\overline{\sqrt{k_i}R_i} = O(\sqrt{n} \cdot \frac{R_i}{a} \cdot R_i) = O(\sqrt{n} \cdot \frac{1}{a} \cdot a^2 \cdot \frac{a}{r \cdot \sqrt{n}}) = O(\frac{a^2}{r})$. Then no matter how k_i changes, the total traffic load is never as heavy as that of a broadcast network. The number of overlaps between sessions are relatively small due to the small distribution region of each session. When R_i keeps

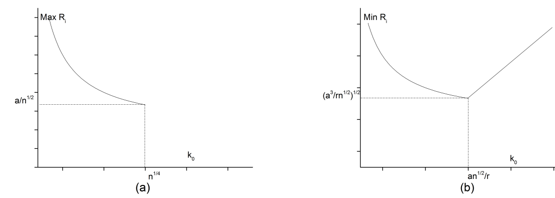

Figure 6: (a) The $max\ \overline{R_i}$ that keeps the traffic load light. (b) The $min\ \overline{R_i}$ that keeps the traffic load heavy.

growing, the smallest $\overline{\sqrt{k_i}}$ that makes the traffic load heavy in the network equals $\frac{a^2}{R_i \cdot r}$, as shown in Fig.5(b).

When $\sqrt{k_i}$s are set to be identical, the analysis is similar. So the result is directly shown here.

1) When $\sqrt{k_i} = O(n^{\frac{1}{4}})$, the biggest $\overline{R_i}$ that can keep the network having light traffic load equals $\frac{a}{\sqrt{k_i}}$. With larger $\sqrt{k_i}$, $\overline{R_i}$ must be large enough to contain these nodes, thus $\overline{\sqrt{k_i}R_i} = \Omega(a)$. The total traffic load is always not in the light tail. The result is shown in Fig.6(a).

2) When $\sqrt{k_i} = O(\sqrt{\frac{a}{r} \cdot \sqrt{n}})$, the smallest R_i that keeps the network's traffic load heavy equals $\frac{a^2}{r \cdot \sqrt{k_i}}$. When $\sqrt{k_i} = \Omega(\sqrt{\frac{a}{r} \cdot \sqrt{n}})$, the traffic load is always like that of a broadcast network, as shown in Fig.6(b).

In the proposed network model, $a = n^{\alpha}, \alpha \leq 1/2$ and $\frac{a}{r} \leq \sqrt{\frac{n}{\log n}}$. Then the following theorem is given to show some bounds of the multicast capacity presented by n. Other results are similar, thus omitted.

Theorem 4. *For the proposed network model, when $R_i = O(n^{\frac{1}{4}})$, $\Lambda_{n_s}(n) = \Theta(\sqrt{\frac{n}{\log n}} \cdot W)$, or when $k_i = \Omega(\frac{n}{\sqrt{\log n}})$, $\Lambda_{n_s}(n) = \Theta(W)$.*

Finally, the multicast capacity of wireless networks under the proposed network model is concluded in Theorem 5.

Theorem 5. *For the proposed network model,*
1) When $R_i = \Theta(a)$ and $k_i = k_0, \forall i = 1, 2, \cdots, n_s$:

$$\Lambda_{n_s}(n) = \begin{cases} \Theta(\frac{a}{\sqrt{k_0} \cdot r} \cdot W) & when\ \sqrt{k_0} = O(\frac{a}{r}) \\ \Theta(W) & when\ \sqrt{k_0} = \omega(\frac{a}{r}) \end{cases}$$

2) When R_is and k_is are both independently chosen:

$$\Lambda_{n_s}(n) = \begin{cases} \Theta(W) & when\ \overline{\sqrt{k_i}R_i} = \Omega(\frac{a^2}{r}) \\ \Theta(\frac{a}{r} \cdot W) & when\ \overline{\sqrt{k_i}R_i} = O(a) \\ \Theta(\frac{a^2}{\overline{\sqrt{k_i}R_i \cdot r}} \cdot W) & others \end{cases}$$

3) When R_is are identical, and $R_i = R_0, \forall i = 1, 2, \cdots, n_s$: $\Lambda_{n_s}(n) =$

$$\begin{cases} \Theta(\frac{a}{r} \cdot W) & when\ R_0 = O(\frac{a}{n^{\frac{1}{4}}}) \\ \Theta(W) & when\ R_0 = \Omega(\sqrt{\frac{a^3}{r \cdot \sqrt{n}}}), \overline{\sqrt{k_i}} = \Omega(\frac{a^2}{R_0 \cdot r}) \\ \Theta(\frac{a^2}{\overline{\sqrt{k_i}} \cdot R_0 \cdot r} \cdot W) & others \end{cases}$$

4) When k_is are identical, and $k_i = k_0, \forall i = 1, 2, \cdots, n_s$: $\Lambda_{n_s}(n) =$

$$\begin{cases} \Theta(\frac{a}{r} \cdot W) & when\ k_0 = O(n^{\frac{1}{2}}), \overline{R_i} = O(\frac{a}{\sqrt{k_0}}) \\ \Theta(W) & when\ k_0 = \Omega(\frac{a}{r} \cdot \sqrt{n}) \\ \Theta(\frac{a^2}{\overline{R_i} \cdot \sqrt{k_0} \cdot r} \cdot W) & others \end{cases}$$

6.2 Generality of the Network Model

The multicast capacity of wireless networks under the proposed network model is in the order of $\Theta(\frac{a^2 n_s}{\sum_{i=1}^{n_s} min\{\sqrt{k_i} \cdot R_i \cdot r, R_i^2\}} \cdot W)$. Firstly, when setting $R_i = \frac{a}{\sqrt{2}}$ and $k_i = k_0$ for all ms_i, all sessions are identical on the number of destinations and randomly pick destinations from the whole region. This is the traffic pattern used by most previous works. The capacity can be changed to $\Lambda_{n_s}(n) = \Theta(\frac{a}{\sqrt{k_0} \cdot r} \cdot W)$ when $k_0 = O(\frac{a^2}{r^2})$ and $\Lambda_{n_s}(n) = \Theta(W)$ when $k_0 = \Omega(\frac{a^2}{r^2})$. It is the same with the result in [2], so our network model is a proper generalization of the previous ones both intuitively and theoretically.

6.3 Applicability of Competitive Intensity

When $1 \leq \Delta \leq 2$, the competitive intensity is still in the order of $\Omega(\frac{\sum_{i=1}^{n_s} min\{\sqrt{k_i} \cdot R_i \cdot r, {R_i}^2\}}{2a^2})$. Such a Δ is for some indoor applications. The sketch of the proof is given below.

Consider the interference area of a multicast tree T_i,

$$|D(T_i)| = \begin{cases} \Omega(\sqrt{k_i} \cdot R_i \cdot r) & k_i = O(\frac{{R_i}^2}{r^2}) \\ \Theta({R_i}^2) & k_i = \theta(\frac{{R_i}^2}{r^2}) \end{cases}$$

Let $D'(T_i)$ denote the interference area of T_i's non-leaf nodes. Then $|D'(T_i)| \geq |D(T_i)| - k_i * r^2$ since some destinations may also serve as relay nodes.

(1) When $k_i = O(\frac{{R_i}^2}{r^2})$, $|D'(T_i)| = \Omega(\sqrt{k_i} \cdot R_i \cdot r - k_i * r^2) = \Omega(\sqrt{k_i} \cdot r(R_i - O(\frac{R_i}{r}) * r)) = \Omega(\sqrt{k_i} \cdot R_i \cdot r)$.

(2) When $k_i = \Omega(\frac{{R_i}^2}{r^2})$, the union of the circle areas with radium r, centering at each destination, is in the order of $\Theta({R_i}^2)$. This is based on the assumption that the destinations are randomly picked. Then there must be a transmitter within or on the edge of each circle area that delivers packets to the destination, and a constant proportion of the circle area is covered by the transmitter. This property holds for all the destinations, thus $|D'(T_i)| = \eta \cdot {R_i}^2 = \Theta({R_i}^2)$, where η is a constant.

Lemma 3 is then proved by replacing $|D(T_i)|$ with $|D'(T_i)|$. All the non-leaf nodes are both transmitters and receivers, thus Lemma 4 holds with $1 \leq \Delta \leq 2$, and the proof is finished.

The proposed network model needs to be extended when the traffic load is heterogeneous around the network. But the competitive intensity is still workable, and the throughput of a single session is no more than the transmission rate of the interior node with the largest competitive intensity.

7. CONCLUSION

This paper studies the capacity of wireless networks with multiple types of multicast sessions. A new network model is proposed ignoring the assumption of traffic patterns being homogeneous. Based on the proposed network model, a theoretical upper bound and a reachable lower bound of the multicast capacity are derived, and two bounds are asymptotically tight. The proposed network model and derived capacity bounds generalize the previous network models and the capacity bounds. Meanwhile, they are more practical and applicable to practical applications.

Acknowledgements

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