Poster: The Construction of Reeb Graph and Its Applications in 3D Sensor Networks

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ABSTRACT

Existing algorithms for topology extraction focus on only one topology feature, either skeleton or segmentation, in 2D or 3D sensor networks, most of which requiring complete boundary information. As boundary information is not easily obtained, especially in sparse 3D sensor networks, and extracting these two features separately is very expensive, in this study, we propose to simultaneously extract the line-like skeleton of 2D/3D sensor networks and decompose the network into nice pieces, by constructing the Reeb graph.

The Reeb graph has been envisioned as a powerful tool for encoding the topology of an object, where the key is to select the right function f. Without using boundary information, we first construct a *cut* graph, and then regard the distance of a node to the nearest cut as the function f such that the corresponding Reeb graph is pose independent, based on which the skeleton extraction and network decomposition are simultaneously conducted. Some simulation results are presented to show the efficiency of the algorithm.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*wireless communication*

Keywords

3D Sensor Networks, Reeb Graph, Skeleton,Segmentation

1. INTRODUCTION AND MOTIVATIONS

Recently, there are growing interests on topology extraction in sensor networks. Among many proposals, line-like skeleton (also known as curve skeleton) extraction and network decomposition are two major topics, where the line-like skeleton of a 2D/3D object is defined as the collection of points whose geodesic shortest paths between their nearest boundary points can decompose the boundary into connected components [2]. Unfortunately, these studies mainly focus on one topological feature, either line-like skeleton or segmentation, of 2D or 3D sensor networks, and most of them require complete boundary information. In some scenarios, especially in sparse 3D sensor networks, boundary information is not easily obtained; most importantly, extracting these two features separately is rather costly. As such, in this paper, we conduct the first work on simultaneous extraction of line-like skeleton and network decomposition in 2D/3D sensor networks by constructing the Reeb graph, without reliance on location and boundary information.

Originally introduced in Morse theory, given a feature function $f : D \to R$ defined on the object D, the Reeb graph of D is obtained by continuously contracting to a single point each level set (or, contour). It has been envisioned as a powerful tool for surface segmentation and parametrization, topological simplification, etc., in computer graphics and computational geometry, since it has the capability to encode the topology of the object. Please see Fig. 1. The critical points, i.e., minima, saddles and maxima, of function f, correspond to the level sets where the topology changes. This can guide us to decompose the network into regularly shaped pieces. Besides, some studies in computer graphics have been done to compute the line-like skeleton by using Reeb graph. Note that the selection of the function f is of crucial importance to understand and represent the topology of the object. This is not straightforward as the Reeb graph is pose dependent (see Fig. 1(a)(b)), and the line-like skeleton (and segmentation result) computed based on different height functions can be very different (see Fig. 1(c)(d)).

2. RELATED WORK

There are many algorithms for topological feature extraction in sensor networks, and we only list a few of them in 3D cases. Zhou *et al.*[3] proposed to identify the bottlenecks and then decompose the network into pieces, and this algorithm works only when there exist bottlenecks. Jiang *et al.*[1] studied the segmentation of 2D/3D sensor networks based on some boundary nodes. Liu *et al.*[2] presented a unified framework for line-like skeleton extraction in 2D/3D sensor networks where boundary information is an input.

3. OUR APPROACH

We plan to construct a pose-independent Reeb graph without reliance on boundary and location information, and apply it for simultaneous line-like skeleton extraction and decomposition. As mentioned earlier, the feature function f for constructing the Reeb graph should be carefully selected.

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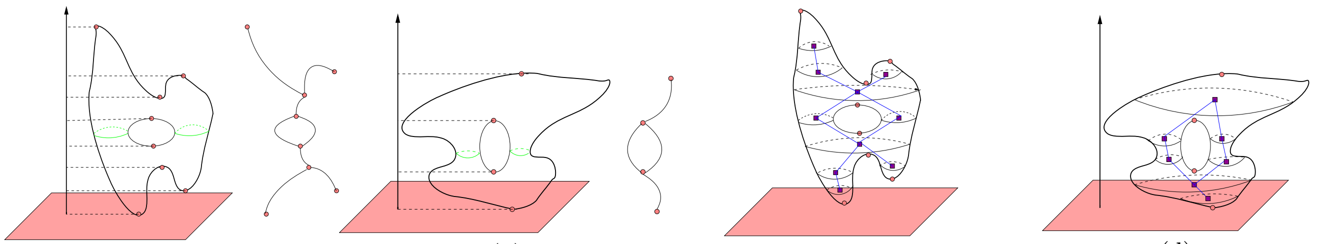
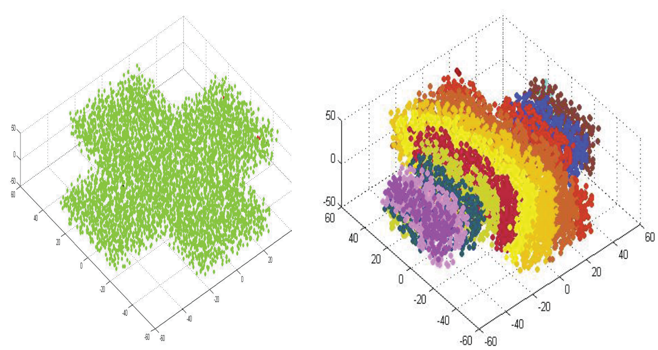


(a) (b) (c) (d)

Figure 1: Illustrative examples. Critical points are marked by solid circles. (a) A solid torus and its Reeb graph w.r.t the height function $f = z(x, y)$; (b) After rotation, the solid torus and its reeb graph w.r.t the function $f = y(x, z)$; (c) The line-like skeleton based on (a); (d) The line-like skeleton based on (b).



(a) (b)

Figure 2: (a) A Cross-shaped 3D network; (b) the cuts.

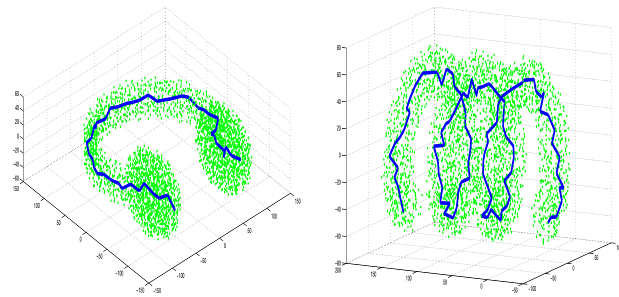
Note that for a sensor network with genus 0, each level set of its Reeb graph actually *cut* the network into two parts, which can be used to construct a feature function. Formally,

Definition 1. *For two nodes p and q in a network with genus zero, the cut between them is a connected component partitioning the network into two disjoint parts P, Q such that $p \in P, q \in Q$.*

Clearly, there are arbitrarily large number of cuts between two nodes. See Fig. 2. In order to guarantee that the extracted skeleton is medially placed, we propose to detect the minimum cut between nodes, which is defined below.

Definition 2. *A cut between p and q is the minimum cut if it has the smallest number of nodes.*

As such, we randomly select two nodes and find the minimal cut between them. Next, each node computes its distance to the cut. The distance function thus serves as a feature function f to construct the Reeb graph, where a level set represents a set of points having the same distance (in hops) to the cut. Similarly, for genus-n($>$0) networks, we find n minimum cuts, and the function f of a node is defined as its distance to the nearest minimum cut. With the constructed Reeb graph, we identify the line-like skeleton node within each level set. Clearly, the line-like skeleton node should locate medially in the level set, and the number of shortest paths passing it should be the largest among other nodes in the level set, which guide us for line-like skeleton node identification. Eventually, these skeleton nodes are orderly connected to derive the line-like skeleton.



(a) (b)

Figure 3: The line-like skeleton of the (a) headphone-shaped 3D network and (b) spiral-shaped 3D network.

4. RESULTS AND FUTURE WORK

We use the proposed method to extract the line-like skeleton of a headphone-shaped 3D network and spiral-shaped 3D network, respectively, as shown in Fig. 3. We can see that without boundary information, our algorithm can achieve a medially placed line-like skeleton, correctly capturing the topological features.

We currently studied the line-like skeleton extraction by constructing Reeb graph. Next we will propose to identify where the topology changes and design an algorithm to decompose the network into regular pieces, and conduct extensive simulations on 2D/3D sensor networks to validate the performance of the proposed algorithm.

5. ACKNOWLEDGEMENTS

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